



EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

Table 1

Student	Y_i	X_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$x_i y_i$
1	2.8	63	-14.625	-0.4125	6.0328125
2	3.4	72	-5.625	0.1875	-1.0546875
3	3.0	78	0.375	-0.2125	-0.0796875
4	3.5	81	3.375	0.2875	0.9703125
5	3.6	87	9.375	0.3875	3.6328125
6	3.0	75	-2.625	-0.2125	0.5578125
7	2.7	75	-2.625	-0.5125	1.3453125
8	3.7	90	12.375	0.4875	6.0378125
Σ	25.7	621			$\Sigma = 17.4375$

$$\bar{y} = 3.2125 \quad \bar{x} = 77.625 \quad \Sigma x_i^2 = 511.875$$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \quad \hat{\beta}_2 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = 3.2125 - 0.0340659(77.625) = \frac{17.4375}{511.875} = 0.0340659$$

$$= 0.568132$$

$$\hat{y}_i = 0.568132 + 0.0340659(x_i)$$

With the exam score increase by 1 unit, on average, GPA will increase by 0.0340659 units. With no score on exam, student will get GPA equal to 0.568132.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

$$\begin{array}{llllll} \hat{Y}_1 = 2.714284 & \hat{Y}_2 = 3.020677 & \hat{Y}_3 = 3.225452 & \hat{Y}_4 = 3.32765 & \hat{Y}_5 = 3.532045 & \hat{Y}_6 = 3.123255 \\ \hat{u}_1 = 0.085716 & \hat{u}_2 = 0.379123 & \hat{u}_3 = -0.225452 & \hat{u}_4 = 0.17235 & \hat{u}_5 = 0.067955 & \hat{u}_6 = -0.123255 \end{array}$$

$$\begin{array}{ll} \hat{Y}_7 = 3.123255 & \hat{Y}_8 = 3.634243 \\ \hat{u}_7 = -0.423255 & \hat{u}_8 = 0.065757 \end{array} \quad \Sigma \hat{u}_i = 0.001061 \approx 0$$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{0.4348931}{6} = 0.0724822 = 6^2$$

$$var(\hat{\beta}_1) = \frac{\Sigma x_i^2}{n \Sigma x_i^2} 6^2 = \frac{48717}{8(511.875)} (0.0724822) = 0.862299$$

$$var(\hat{\beta}_2) = \frac{6^2}{\Sigma x_i^2} = \frac{0.0724822}{511.875} = 0.0001416$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

2. Data is listed in the table

$$\text{mean: } 20 \quad 9.1$$

$$\Sigma = 200 \quad 91$$

$$\Sigma = 394$$

$$\Sigma x_i^2 = (100+64+36+16+4) \times 2 = 440$$

$$\Sigma x_i^2:$$

X_i	Y_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$x_i y_i$
10	0	-10	-9.1	91
12	2	-8	-7.1	56.8
14	5	-6	-4.1	24.6
16	6	-4	-3.1	12.4
18	7	-2	-2.1	4.2
22	10	2	0.9	1.8
24	10	4	0.9	3.6
26	15	6	5.9	35.4
28	16	8	6.9	55.2
30	20	10	10.9	109

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\Sigma \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y ?

2.5 Find $\text{var}(\hat{u}_i), \text{var}(\hat{\beta}_1), \text{var}(\hat{\beta}_2)$

$$2.1) \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$

$$= 9.1 - (17.909091)$$

$$= -8.809091.$$

$$= \frac{394}{440} = 0.895455$$

$$\hat{Y}_i = -8.8091 + 0.8955(X_i)$$

$$2.2) \hat{Y}_1 = 0.1459$$

$$\hat{u}_1 = -0.1459$$

$$\hat{Y}_2 = 1.9369$$

$$\hat{u}_2 = 0.0631$$

$$\hat{Y}_3 = 3.7279$$

$$\hat{u}_3 = 1.2721$$

$$\hat{Y}_4 = 5.5189$$

$$\hat{u}_4 = 0.4811$$

$$\hat{Y}_5 = 7.3099$$

$$\hat{u}_5 = -0.3099$$

$$\hat{Y}_6 = 9.0919$$

$$\hat{u}_6 = -0.8919$$

$$\hat{Y}_7 = 10.8829$$

$$\hat{u}_7 = -2.6829$$

$$\hat{Y}_8 = 12.6739$$

$$\hat{u}_8 = 0.5261$$

$$\hat{Y}_9 = 14.4649$$

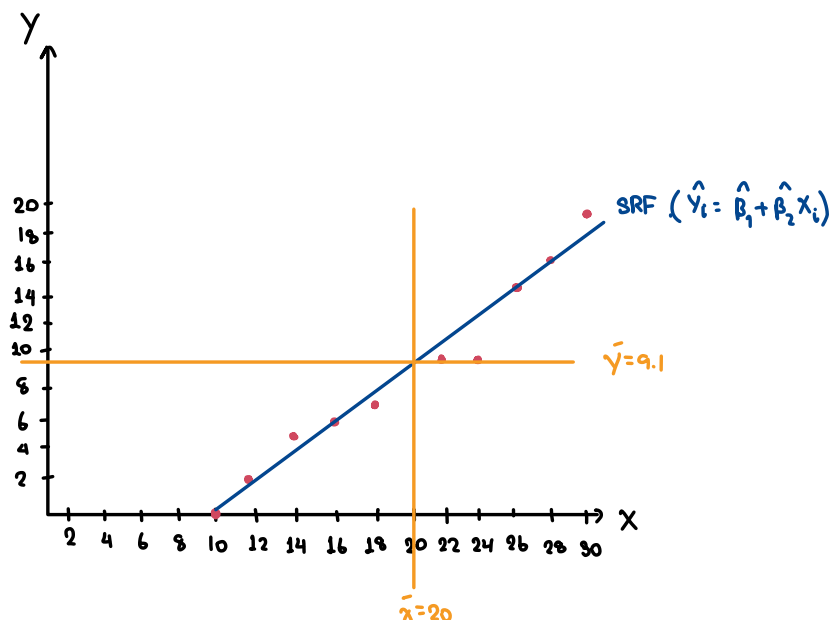
$$\hat{u}_9 = -0.2649$$

$$\hat{Y}_{10} = 16.2559$$

$$\hat{u}_{10} = 1.9441$$

$$\Sigma \hat{u}_i = -0.009 \approx 0$$

2.3)



$$\hat{Y}_i = -8.8091 + 0.8955(20) = 9.1009 \approx 9.1 = \bar{Y}$$

2.4) $X_i = 18, \hat{Y}_i = 7.3099$ from 2.2

$$2.5) \text{Var}(\hat{u}_i) = \sigma^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{14.09092}{8} = 1.761365$$

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{1.761365}{440} = 0.004$$

$$\text{Var}(\hat{\beta}_1) = \frac{\Sigma x_i^2}{n} \cdot \frac{\sigma^2}{\Sigma x_i^2} = \frac{440}{10} (0.004) = 1.776$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2) \rightarrow \text{assumption 1, linear regression model.}$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

As $\hat{\beta}_1$ is an estimator of β_1 , $E(\hat{\beta}_1)$ should be equal to β_1

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}, \quad \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \quad \text{let } k = \frac{x_i}{\sum x_i^2}$$

$$\hat{\beta}_1 = \bar{Y} - \sum k y_i \bar{X} \quad \text{and } y_i = \beta_1 + \beta_2 x_i + u_i$$

$$\hat{\beta}_1 = \frac{\sum y_i}{n} - \sum k y_i \bar{X} = \sum \left(\frac{1}{n} - k \bar{X} \right) y_i$$

$$= \sum \left(\frac{1}{n} - k \bar{X} \right) (\beta_1 + \beta_2 x_i + u_i)$$

$$= \sum \left(\frac{\beta_1}{n} + \frac{\beta_2 x_i}{n} + \frac{u_i}{n} - k \bar{X} \beta_1 - k \bar{X} \beta_2 x_i - k \bar{X} u_i \right)$$

$$= \frac{\sum \beta_1}{n} + \beta_2 \frac{\sum x_i}{n} + \frac{\sum u_i}{n} - \bar{X} \beta_1 \sum k_i - \beta_2 \bar{X} \sum k_i x_i - \bar{X} \sum k_i u_i$$

$$\hat{\beta}_1 = \beta_1 + \beta_2 \bar{X} + \frac{\sum u_i}{n} - \beta_2 \bar{X} - \bar{X} \sum k_i u_i$$

take $E(\cdot) \rightarrow E(\hat{\beta}_1) = E(\beta_1) + \bar{X} E(\sum k_i u_i) \rightarrow \text{Assumption 3: Zero mean value of disturbance } u_i$
treat x as given $E(u_i | x_i)$

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{X} \sum k_i E(u_i) \quad \text{as } k \text{ is } f(x), k \text{ would be constant.}$$

$\sum k_i = \text{constant}$

$E(\hat{\beta}_1) = \beta_1$

$\therefore \hat{\beta}_1$ is an unbiased estimator of actual β_1