

Macroeconomics

Lecture 6

Asset Prices and Consumption

- To apply recursive structure for the construction of consumption and asset price theories.

Asset Prices and Consumption

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad 0 < \beta < 1, \quad (1)$$

$$s.t. \quad A_{t+1} = R_t (A_t + y_t - c_t), \quad A_0 \text{ given} \quad (2)$$

$\{R_t\}$ is a random process,

R_t becomes known to the agent only at the beginning of period $(t+1)$.

$\{y_t\}$ is also a random process.

The Euler equation is

$$U'(c_t) = E_t \beta R_t U'(c_{t+1}), \quad t = 0, 1, \dots \quad (3)$$

Asset Prices and Consumption

- Special cases of the Euler equation (3) provide the basis for several theories:
- 1) Hall's Random Walk Theory of Consumption;
- 2) The Random Walk Theory of Stock Prices;
- 3) Lucas's Model of Asset Prices.

Hall's Random Walk Theory of Consumption

Suppose that $R_t = R > 1$, then (3) implies that

$$E_t U'(c_{t+1}) = (\beta R)^{-1} U'(c_t), \quad (4)$$

The marginal utility of consumption follows a univariate first-order Markov process and that no other variables in the information set help to predict $U'(c_{t+1})$, once lagged $U'(c_t)$ has been included.

For example, in the case of constant relative risk aversion utility function $U(c_t) = c_t^\gamma / \gamma$, equation (4) becomes

$$E_t c_{t+1}^{\gamma-1} = (\beta R)^{-1} c_t^{\gamma-1}$$

The Random Walk Theory of Stock Prices

- To interpret the asset as a share of an enterprise that sells for price p_t , measured in consumption goods at period t per share during period t .
- This asset pays d_t , which is a nonnegative random dividends, to the owner of the share at the beginning of t .

The Random Walk Theory of Stock Prices

- Assume that d_t is governed by a Markov process, with time invariant transition density $f(d',d)$ where $\text{prob}(d_{t+1} \leq d' \mid d_t = d) = \int_0^{d'} f(s,d) ds$.
- s_t denote the number of share owned by the consumer at the beginning of t and let
- $(p_t + d_t)s_t = A_t$, then from eq.(2)

$$(p_{t+1} + d_{t+1})s_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} [(p_t + d_t)s_t + y_t - c_t] \quad (5)$$

where $\frac{p_{t+1} + d_{t+1}}{p_t} = R_t$

Then Euler equation (3) can be rewritten as

$$1 = \beta E_t \left(\frac{p_{t+1} + d_{t+1}}{p_t} \right) \frac{U'(c_{t+1})}{U'(c_t)} \quad (6)$$

$$\because E_t(xy) = E_t x E_t y + \text{cov}_t(x, y), \quad \text{cov}_t \equiv E_t(x - E_t x)(y - E_t y),$$

$\therefore$ Eq. (6) becomes

$$1 = \beta E_t \left(\frac{p_{t+1} + d_{t+1}}{p_t} \right) E_t \frac{U'(c_{t+1})}{U'(c_t)} + \beta \text{cov}_t \left[\frac{U'(c_{t+1})}{U'(c_t)}, \frac{p_{t+1} + d_{t+1}}{p_t} \right] \quad (7)$$

To obtain the random walk theory of stock prices, it is necessary

to assume, first, that $E_t \frac{U'(c_{t+1})}{U'(c_t)}$ is a constant and, second that

$$\text{cov}_t \left[\frac{U'(c_{t+1})}{U'(c_t)}, \frac{p_{t+1} + d_{t+1}}{p_t} \right] = 0.$$

For convenience, suppose that $E_t \frac{U'(c_{t+1})}{U'(c_t)} = 1$

and

$$\text{cov}_t \left[\frac{U'(c_{t+1})}{U'(c_t)}, \frac{p_{t+1} + d_{t+1}}{p_t} \right] = 0.$$

If $U(c_t)$ is linear in c_t , so that $U'(c_t)$ is independent of c_t .

Eq (7) implies that

$$E_t(p_{t+1} + d_{t+1}) = \beta^{-1} p_t \quad (8)$$

- Eq (8) states that, when the share price is adjusted for dividends and discounting, it follows a first-order univariate Markov process.

The Random Walk Theory of Stock Prices

- The stochastic difference equation (8) has the following solution

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t d_{t+j} + \gamma_t \left(\frac{1}{\beta} \right)^t, \quad (9)$$

where $\{\gamma_t\}$ is any random process that obeys

$$E_t \gamma_{t+1} = \gamma_t.$$

If $\gamma_t = 0$, eq.(9) becomes a formula linking share prices with expected future dividends.

From eq.(8),

$$p_t = \beta E_t \left[\left\{ \beta E_{t+1} (p_{t+2} + d_{t+2}) \right\} + d_{t+1} \right]$$

$$= \beta^2 E_t \cdot E_{t+1} (p_{t+2}) + \beta^2 E_t d_{t+2} + \beta E_t d_{t+1}$$

$$= \beta^2 E_t \cdot E_{t+1} (p_{t+2}) + \sum_{j=1}^2 \beta^j E_t d_{t+j}$$

$$= \beta^3 E_t \cdot E_{t+1} E_{t+2} (p_{t+3}) + \sum_{j=1}^3 \beta^j E_t d_{t+j}$$

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$$= \lim_{j \rightarrow \infty} \left\{ E_t \cdot E_{t+1} \dots E_{t+j-1} (p_{t+j} \cdot \beta^j) \right\} + \sum_{j=1}^{\infty} \beta^j E_t d_{t+j}$$

$$= \left[\left(\frac{1}{\beta} \right)^t \times \lim_{j \rightarrow \infty} \left\{ E_t \cdot E_{t+1} \dots E_{t+j-1} (p_{t+j} \cdot \beta^{t+j}) \right\} \right] + \sum_{j=1}^{\infty} \beta^j E_t d_{t+j} \dots \dots \dots (9)$$