CHAPTER 4

COST-VOLUME-PROFIT RELATIONSHIPS

AGENDA

1. Basics
2. CVP relationships
3. Contribution margin (CM) ratio.
4. Target profit analysis.
5. Break-even analysis.
6. Margin of safety.
7. Cost structure and operating leverage.
8. Multiproduct break-even analysis.
9. Major assumptions

BASICS

Cost-volume-profit (CVP) analysis is concerned with the effects on net operating income of:

- Selling prices.
- Sales volume.
- Unit variable costs.
- Total fixed costs.
- The mix of products sold.

The contribution approach

A contribution format income statement is very useful in CVP analysis because it highlights cost behavior.

EXAMPLE: Last month's contribution income statement for Nord Corporation, a manufacturer of exercise bicycles, follows:

	Total	*Per Unit*
Sales (*500 bikes*)	$250,000	$500
Variable expenses	150,000	300
Contribution margin.....	100,000	$200
Fixed expenses.....	80,000	
Net operating income.....	$ 20,000	

CONTRIBUTION MARGIN

- The amount that sales (net of variable expenses) contributes toward covering fixed expenses and then toward profits.
- The unit contribution margin remains constant so long as the selling price and the unit variable cost do not change.

Volume changes and net operating income

Contribution income statements are given on this and the following page for monthly sales of 1, 2, 400, and 401 bikes.

	Total	*Per Unit*
Sales (*1 bike*)	$ 500	$500
Variable expenses	300	300
Contribution margin	200	$200
Fixed expenses	80,000	
Net operating income (loss)	$(79,800)	

	Total	*Per Unit*
Sales (*2 bikes*)	$ 1,000	$500
Variable expenses	600	300
Contribution margin	400	$200
Fixed expenses	80,000	
Net operating income (loss)	$(79,600)	

	Total	*Per Unit*
Sales (*400 bikes*)	$200,000	$500
Variable expenses	120,000	300
Contribution margin	80,000	$200
Fixed expenses	80,000	
Net operating income (loss)	$ 0	

	Total	*Per Unit*
Sales (*401 bikes*)	$200,500	$500
Variable expenses	120,300	300
Contribution margin	80,200	$200
Fixed expenses	80,000	
Net operating income (loss)	$ 200	

Contribution margin

Each additional unit sold increases net operating income by the amount of the unit contribution margin.

CVP RELATIONSHIPS

Equation form

The contribution format income statement can be expressed in equation form as follows:

$$\text{Profit} = (\text{Sales} - \text{Variable expense}) - \text{Fixed expense}$$

When a company has a single product, we can further refine the equation as follows:

$$\text{Profit} = (P \times Q - V \times Q) - \text{Fixed expense}$$

EXAMPLE: This equation can be used to compute Nord Company's net operating income if it sells 401 bikes:

$$\begin{aligned}\text{Profit} &= (\$500 \times 401 - \$300 \times 401) - \$80{,}000 \\ &= (\$500 - \$300) \times 401 - \$80{,}000 \\ &= (\$200) \times 401 - \$80{,}000 \\ &= \$80{,}200 - \$80{,}000 = \$200\end{aligned}$$

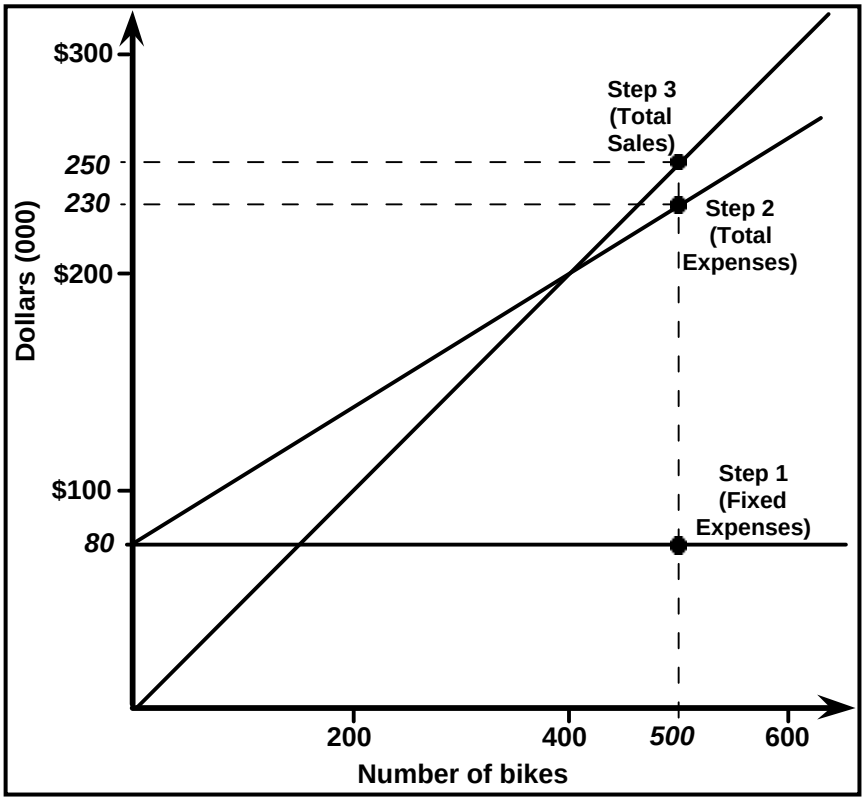
It is often useful to express the simple profit equation in terms of the unit contribution margin as follows:

$$\text{Profit} = \text{Unit CM} \times Q - \text{Fixed expense}$$

This equation can also be used to compute Nord Company's net operating income if it sells 401 bikes:

$$\begin{aligned}\text{Profit} &= (\$200 \times 401) - \$80{,}000 \\ \text{Profit} &= \$80{,}200 - \$80{,}000 = \$200\end{aligned}$$

Graphic form



Contribution margin ratio

The contribution margin (CM) ratio is the ratio of contribution margin to total sales:

$$\text{CM ratio} = \frac{\text{Contribution margin}}{\text{Total sales}}$$

If the company has only one product, the CM ratio can also be computed using per unit data:

$$\text{CM ratio} = \frac{\text{Unit contribution margin}}{\text{Unit selling price}}$$

EXAMPLE: For Nord Corporation, the CM ratio is 40%, computed as follows:

The relation between profit and the CM ratio can also be expressed using the following equation:

$$\text{Profit} = \text{CM ratio} \times \text{Sales} - \text{Fixed expense}$$

EXAMPLE: Nord Company's profit if it sells 401 bikes can be computed as follows:

Changes in variable costs, fixed costs and sales volume

The CM ratio shows how the contribution margin will be affected by a given change in total sales.

Assume that Nord Corporation's sales increase by $150,000 next month. What will be the effect on (1) the contribution margin and (2) net operating income?

(1) Effect on contribution margin:

Increase in CM = Increase in sales * CM ratio

(2) Effect on net operating income:

If fixed expenses do not change, the net operating income for the month will also increase by $60,000.

	Present	*Expected*	*Change*
Sales (in units).....	500	800	300
Sales (in dollars).....	$250,000	$400,000	$150,000
Variable expenses	150,000	240,000	90,000
Contribution margin	100,000	160,000	60,000
Fixed expenses	80,000	80,000	0
Net operating income	$ 20,000	$ 80,000	$ 60,000

What is the profit impact if the Nord Corporation can increase unit sales from 500 to 540 by increasing the monthly advertising budget by $10,000?

What is the profit impact if the Nord corporation can use higher quality raw materials, thus increasing variable costs per unit by $10, to generate an increase in unit sales from 500 to 580?

What is the profit impact if the Nord Corporation: (1) cuts its selling price by $20 per unit, (2) increases its advertising budget by $15,000 per month, and (3) increases sales from 500 to 650 units per month?

What is the profit impact if Nord: (1) pays a $15 sales commission per bike sold instead of paying salespersons flat salaries that currently total $6,000 per month, and (2) increases unit sales from 500 to 575 bikes?

If Nord has an opportunity to sell 150 bikes to a wholesaler without disturbing sales to other customers or fixed expenses, what price would it quote to the wholesaler if it wants to increase monthly profits by $3,000?

TARGET PROFIT ANALYSIS

Summary of Nord Corporation Data:

	Per Bike	*Percent*	*Per Month*
Selling price	$500	100%	
Variable expenses	300	60%	
Contribution margin	$200	40%	
Fixed expenses			$80,000

EXAMPLE: Assume that Nord Corporation's target profit is $70,000 per month. How many exercise bikes must it sell to reach this goal?

Equation method (Unit sales)

Q = Number of bikes to attain the target profit

Profit = Unit CM × Q – Fixed Expenses

Equation method (Sales dollars)

What if Nord wanted to know how much sales revenue needed to be generated to achieve a target profit of $70,000? We can compute the answer two ways. First, we can multiply the answer from above by the selling price per bike:

Formula method (Unit sales)

Unit sales to attain a target profit = (Target profit + Fixed expense)/Unit CM

Formula method (Dollar sales)

Dollar sales to breakeven = (Target profit + Fixed expense)/CM ratio

BREAK-EVEN ANALYSIS

Equation method (Unit sales)

Profit = Unit CM × Q – Fixed expense

Equation method (Dollar sales)

Profit = CM ratio × Sales – Fixed Expenses

Formula method (Units sales)

Unit sales to breakeven = Fixed expense/Unit CM

Formula method (Dollar sales)

Dollar sales to breakeven = Fixed expense/CM ratio

MARGIN OF SAFETY

The margin of safety is the excess of budgeted (or actual) sales over the break-even sales. The margin of safety can be expressed either in dollar or percentage form. The formulas are:

Margin of safety in dollars = Total sales – Breakeven sales

Margin of safety percentage = Margin of safety in dollars/Total sales

Margin of safety percentage = 1 – Breakeven percentage

	Company X		*Company Y*	
Sales	$500,000	100%	$500,000	100%
Variable expenses	350,000	70%	100,000	20%
Contribution margin	150,000	30%	400,000	80%
Fixed expenses	90,000		340,000	
Net operating income	$ 60,000		$ 60,000	
Break-even point:				
$90,000 ÷ 0.30	$300,000			
$340,000 ÷ 0.80			$425,000	
Margin of safety in dollars:				
$500,000 – $300,000	$200,000			
$500,000 – $425,000			$75,000	
Margin of safety percentage:				
$200,000 ÷ $500,000	40%			
$75,000 ÷ $500,000			15%	

COST STRUCTURE AND OPERATING LEVERAGE

Operating leverage measures how a given percentage change in sales affects net operating income.

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Net operating income}}$$

	Company X		*Company Y*	
Sales	$500,000	100%	$500,000	100%
Variable expenses	350,000	70%	100,000	20%
Contribution margin	150,000	30%	400,000	80%
Fixed expenses	90,000		340,000	
Net operating income	$ 60,000		$ 60,000	
Degree of operating leverage	**2.5**		**6.7**	

If the degree of operating leverage is 2.5, then a 10% increase in sales should result in a 25% (= 2.5 × 10%) increase in net operating income.

EXAMPLE: Assume that both company X and company Y experience a 10% increase in sales:

	Company X		*Company Y*	
Sales	$550,000	100%	$550,000	100%
Variable expenses	385,000	70%	110,000	20%
Contribution margin	165,000	30%	440,000	80%
Fixed expenses	90,000		340,000	
Net operating income	$ 75,000		$100,000	
Increase in net operating income	25%		67%	

The degree of operating leverage is not constant—it changes with the level of sales.

EXAMPLE: At the higher level of sales, the degree of operating leverage for Company X decreases from 2.5 to 2.2 and for Company Y from 6.7 to 4.4.

	Company X (000s)		*Company Y (000s)*	
Sales	$500	$550	$500	$550
Variable expenses	350	385	100	110
Contribution margin	150	165	400	440
Fixed expenses	90	90	340	340
Net operating income	$ 60	$ 75	$ 60	$100
Degree of operating leverage	2.5	2.2	6.7	4.4

Ordinarily, the degree of operating leverage declines as sales increase.

Sales mix

When a company has multiple products, the overall contribution margin (CM) ratio is used in break-even analysis.

Overall CM ratio = Total contribution margin/Total sales dollars

Dollar sales to break even = Fixed expenses/CM ratio

	Product A		*Product B*		*Total*	
Sales	$100,000	100%	$300,000	100%	$400,000	100.0%
Variable expenses	70,000	70%	120,000	40%	190,000	47.5%
Contribution margin	$ 30,000	30%	$180,000	60%	210,000	52.5%
Fixed expenses					141,750	
Net operating income					$ 68,250	

The relative proportions in which the products are sold is called the sales mix. If the sales mix changes, the overall contribution margin ratio will change.

Example: Assume that total sales remain unchanged at $400,000. However, the sales mix shifts so that more of Product A is sold than of Product B.

	Product A		*Product B*		*Total*	
Sales	$300,000	100%	$100,000	100%	$400,000	100.0%
Variable expenses	210,000	70%	40,000	40%	250,000	62.5%
Contribution margin	$ 90,000	30%	$ 60,000	60%	150,000	37.5%
Fixed expenses					141,750	
Net operating income					$ 8,250	

MAJOR ASSUMPTIONS OF CVP ANALYSIS

1. Selling price is constant. The price does not change as volume changes.
2. Costs are linear and can be accurately split into fixed and variable elements. The total fixed cost is constant and the variable cost per unit is constant.
3. The sales mix is constant in multi-product companies.
4. In manufacturing companies, inventories do not change. The number of units produced equals the number of units sold.

Voltar Company manufactures and sells a specialized cordless telephone for high electromagnetic radiation environments. The company's contribution format income statement for the most recent year is given below:

	Total	Per Unit	Percent of Sales
Sales (20,000 units)	$1,200,000	$60	100%
Variable expenses	900,000	45	? %
Contribution margin	300,000	$15	? %
Fixed expenses	240,000		
Net operating income	$ 60,000		

Management is anxious to increase the company's profit and has asked for an analysis of a number of items.

Required:

1. Compute the company's CM ratio and variable expense ratio.
2. Compute the company's break-even point in both units and sales dollars. Use the equation method.
3. Assume that sales increase by $400,000 next year. If cost behavior patterns remain unchanged, by how much will the company's net operating income increase? Use the CM ratio to compute your answer.
4. Refer to the original data. Assume that next year management wants the company to earn a profit of at least $90,000. How many units will have to be sold to meet this target profit?
5. Refer to the original data. Compute the company's margin of safety in both dollar and percentage form.
6. *a.* Compute the company's degree of operating leverage at the present level of sales.
   b. Assume that through a more intense effort by the sales staff, the company's sales increase by 8% next year. By what percentage would you expect net operating income to increase? Use the degree of operating leverage to obtain your answer.
   c. Verify your answer to (*b*) by preparing a new contribution format income statement showing an 8% increase in sales.
7. In an effort to increase sales and profits, management is considering the use of a higher-quality speaker. The higher-quality speaker would increase variable costs by $3 per unit, but management could eliminate one quality inspector who is paid a salary of $30,000 per year. The sales manager estimates that the higher-quality speaker would increase annual sales by at least 20%.
   a. Assuming that changes are made as described above, prepare a projected contribution format income statement for next year. Show data on a total, per unit, and percentage basis.
   b. Compute the company's new break-even point in both units and dollars of sales. Use the formula method.
   c. Would you recommend that the changes be made?