B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2017

Practice Problem Set 2 (Topic 4) - Answers

Question 1 Basic operations on matrix

1.1

$$\text{Find } A^{-1}B \quad \text{when} \quad \mathrm{A} = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix} \ \mathrm{B} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Answer:

$$A^{-1} = -\frac{1}{32}\begin{bmatrix} -50 & 23 & 11 \\ 54 & -21 & -17 \\ 12 & -10 & -2 \end{bmatrix}$$

$$A^{-1}B = -\frac{1}{32}\begin{bmatrix} -50 & 22 & -4 \\ 54 & -34 & 24 \\ 24 & -4 & -8 \end{bmatrix}$$

1.2

$$\text{If A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} -2 & 3 \\ 4 & 0 \end{bmatrix}, \mathrm{C} = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} \text{ Find } (ABC)^T.$$

$$(ABC)^T = \begin{bmatrix} 6 & 12 \\ 3 & 18 \end{bmatrix}$$

Answer:

1.3

If A $= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ Find $A^{-1}B$.

Answer

$$A^{-1} = \begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -0.4 & 0.6 \\ 0.8 & -0.2 \end{bmatrix}$$

1.4

Let A $= \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ Find determinant of A

Answer : det(A) = -32

1.5

If A $= \begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 1 \end{bmatrix}$ B $= \begin{bmatrix} 2 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$ Find $[A^T B]^{-1}$.

Answer: $\left[[A^T B]^{-1} = \frac{1}{13} \begin{bmatrix} 0 & -13 \\ 1 & 12 \end{bmatrix} \right]$

Question 2

Consider a simple macroeconomic model.

$C = a + bY_d$; $0 < b < 1$
$I = I_a + iY$; $0 < i < 1$
$G = G_0$
$T = T_0 + tY$; $0 < t < 1$
$R = R_0$
$Y_d = Y - T + R$

where R is the government transfer and G is the government purchase. All the remainings are defined as usual.

a) Determine *all* the endogenous and exogenous variables in the model.
Y, C, I, T, Y_d
b) State the condition that characterizes the equilibrium of this model.
Y = C + I + G
c) Simplify the model into a 3-variable system of equations that only includes on Y, C and I.
Y = C + I + G_0
$C = a + b(Y - T + R) = a + b(Y - T_0 - tY + R_0) = a - bT_0 + bR_0 + (1-t)bY$
$I = I_a + iY$

d) Rewrite the system of equations in 2.3 in the form of matrix.

$$\begin{bmatrix} 1 & -1 & -1 \\ -(1-t)b & 1 & 0 \\ -i & 0 & 1 \end{bmatrix} \begin{bmatrix} Y \\ C \\ I \end{bmatrix} = \begin{bmatrix} G_0 \\ a - bT_0 + bR_0 \\ I_a \end{bmatrix}$$

$$A \qquad x = \qquad d$$

e) Solve for the solution of Y, C and I. Use the Cramer's rule method.

$\det(A) = 1 - i - (1 - t) * b$

$$\frac{\begin{vmatrix} G_0 & -1 & -1 \\ a - bT_0 + bR_0 & 1 & 0 \\ I_a & 0 & 1 \end{vmatrix}}{\det(A)}$$

Y =

$$Y = \frac{G_0 + I_a + (a - bT_0 + bR_0)}{det(A)}$$

$$\frac{\begin{vmatrix} 1 & G_0 & -1 \\ -(1-t)*b & a - bT_0 + bR_0 & 0 \\ -i & I_a & 1 \end{vmatrix}}{\det(A)}$$

C =

$$C = \frac{G_0 + I_a + (a - bT_0 + bR_0)}{det(A)}$$

$$\frac{\begin{vmatrix} 1 & -1 & G_0 \\ -(1-t)*b & 1 & a - bT_0 + bR_0 \\ -i & 0 & I_a \end{vmatrix}}{\det(A)}$$

I =

$$I = \frac{I_a (1 - b(1-t) + i*(a - bT_0 + bR_0) + G_o * i}{det(A)}$$

f) Compare the multipliers of G and R. Which one has a bigger impact? Why?

Multiplier of G is $\frac{1}{1-i-b(1-t)}$

Multiplier of R is $\frac{b}{1-i-b(1-t)}$

Multiplier of G > Multiplier of R. An increase in R doesn't cause an increase in output in the first round as the increase in G does. So, the effect of an increase in R is discounted by "b" time of the effect of an increase in G.

Question 3

Assume that we have three markets in the economy. Each market can be characterized by demand and supply equations as given below.

$$\text{Demand for goods A}: q_A^d = 3 - P_A + P_B \qquad \text{Supply for goods A}: q_A^s = P_A - 2$$
$$\text{Demand for goods B}: q_B^d = 3 - 2P_B + P_C \qquad \text{Supply for goods B}: q_B^s = P_B - 1$$
$$\text{Demand for goods C}: q_C^d = 6 + 2P_A - P_C \qquad \text{Supply for goods C}: q_C^s = 2P_C - 2$$

a. State the equilibrium conditions for this multi-market economy, and write the model in the form of matrix.

$q_A{}^d = q_A{}^S, \quad q_B{}^d = q_B{}^S, \quad q_c{}^d = q_c{}^S$

Ax = d

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ P_C \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -8 \end{bmatrix}$$

det(A) = -16

b. Solve for the equilibrium solution by using the *Cramer's rule*.

$P_A = \frac{-90}{-16} = 5$

$P_B = \frac{-90}{-16} = 5$

$P_C = \frac{-96}{-16} = 6$

$[q_A = 3, q_B = 4, q_C = 10 \text{and} P_A = 5, P_B = 5, P_C = 6]$

Question 4

Consider a simple global economy model. There are but two countries, namely A and B. Model equations for each country are given below.

$$\begin{aligned} \text{Country A} : C_A &= 150 + 0.4Y_A \\ M_A &= 0.6Y_A \\ I_A &= 250 \end{aligned}$$

$$\begin{aligned} \text{Country B} : C_B &= 100 + 0.5Y_B \\ M_B &= 0.4Y_B \\ I_B &= 200 \end{aligned}$$

where Y is national income, C is consumption, M is import, and I is investment.

4.1) Let "X" be a new variable representing export of a country. That is, when I write X_A, this is referred to the value of export of country A. Based on the information given above, can we find the export function of both country A and country B?

$$X_A = M_B = 0.4Y_B$$

$$X_B = M_A = 0.6Y_A$$

4.2) State all the endogenous and exogenous variables.

Y_A, Y_B, C_A, C_B, X_A, X_B, M_A, M_B

4.3) State the equilibrium condition<u>s</u> for the global economy model.

Equilibrium in country A: $Y_A = C_A + I_A + X_A - M_A$

Equilibrium in country B: $Y_B = C_B + I_B + X_B - M_B$

4.4) Simplify the model into a 2-variable system of equations where only Y_A *and* Y_B are included. Then rewrite the simplified model in the matrix form.

$$Y_A = C_A + I_A + X_A - M_A$$

$$Y_A = 150 + 0.4Y_A + 250 + 0.4Y_B - 0.6Y_A$$

$$1.2Y_A - 0.4Y_B = 400$$

$$Y_B = C_B + I_B + X_B - M_B$$

$$Y_B = 100 + 0.5Y_B + 200 + 0.6Y_A - 0.4Y_B$$

$$-0.6Y_A + 0.9Y_B = 300$$

$$\begin{bmatrix} 1.2 & -0.4 \\ -0.6 & 0.9 \end{bmatrix} \begin{bmatrix} Y_A \\ Y_B \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \end{bmatrix}$$

4.5) Solve for the equilibrium income using the *inverse matrix method*.

$$A = \begin{bmatrix} 1.2 & -0.4 \\ -0.6 & 0.9 \end{bmatrix} ==> A^{-1} = \begin{bmatrix} 1.07 & 0.47 \\ 0.71 & 1.42 \end{bmatrix}$$

$$Y_A^* = 571.43 \quad and \quad Y_B^* = 714.29$$

4.6) Under the equilibrium, how much is the *net export* in both countries?

$$X_A - M_A = 0.4Y_B - 0.6Y_A = 0.4 * 714.29 - 0.6 * 571.43 = 285.71 - 342.85 = -57.14$$

$$X_B - M_B = 0.6Y_A - 0.4Y_B = 0.6 * 571.43 - 0.4 * 714.29 = 57.14$$

Question 5

Given the following supply and demand functions:

$$Q_D = 100 - 3P$$

$$Q_S = 80 + 2P$$

a) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

Ans. Eq'm condition: $Q_D = Q_S$.

System of equations:

$$\begin{aligned} Q_D - Q_S &= 0 \\ Q_D \quad + 3P &= 100 \\ Q_S - 2P &= 80 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix}$$

b) Use <u>matrix inversion</u> to solve for the equilibrium quantity and equilibrium price.

Ans.

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \rightarrow \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

c) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Use Cramer's rule to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

Ans. (i) $P_d^* = P_s^* - 5 = \$7 - \$5 = \$2$

(ii) $P_s^* = \$7$

(iii) $Q^* = 94$; $S^* = 94x\$5 = \$470$

Question 6

Examine for what values of the constants *a* and *b* the system of equations

$ax + y = 3$
$x + z = 2$
$y + az + bu = 6$
$y + u = 1$

has a unique solution in the unknown *x, y, z,* and *u*. Find the unique solution (expressed in terms of *a* and *b*).

Ans. There is a unique solution provided that $a(b-2) \neq 0$.

Question 7

Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.

Ans. Y = 800, r = 0.12.

Question 9: (*Timbergen 1952: "On the theory of economic policy"*)

This question guides you to understand an important foundation on the principle of economic policy laid down by Jan Timbegen. (Timbergen was the first Noble laureate in economics science. His main contribution is the advancement in developing methodological frameworks for the centrally-planned policy and government interventions analysis.) In his classical manuscript published in 1952, he argues that authority might have multiple economic objectives/targets, given the policy instrument on hands. (Target is generally defined a variable that authority ultimately would like to achieve. Instrument is defined as a variable that authority can control with its high precision, and has certain relationship with the targets.) He claims that *the number of targets should be equal to number of instruments; otherwise, all the targets cannot simultaneously achieved.*

Following the model described in class, consider the following macroeconomic model that takes into account a dynamic feature of

$$Y = C + I + G + X - M \qquad (a)$$

$$C = 0.75Y_d \quad \text{(b)}$$

$$Y_d = 0.75(Y - T) \quad \text{(c)}$$

$$T = G \quad \text{(d)}$$

$$G = G_c + G_I \quad \text{(e)}$$

$$I = 0.25(Y - Y_{-1}) + 0.1G_I \quad \text{(f)}$$

$$M = 0.2C + 0.08I + 0.06G_I + 0.03X \quad \text{(g)}$$

$$N = 0.6Y \quad \text{(h)}$$

$$B = P_x X - P_M M = 1.05X - 1.07M \quad \text{(j)}$$

where Y – GDP, Y_{-1} – GDP of the previous year, C – Private Consumption, I – Private Investment, M – Imports, X – Exports, Yd – Disposable Income, T – Taxes, G–Government Expenditure, G_c –Public Consumption Expenditures, G_I – Public Investment Expenditures, N – Employment, B – Current Account of the Balance of Payments. All variables, except N , are measured in $ billion. Employment (N) unit of measurement is 1,000 persons.

The first three equations are three standard equations used in Keynesian cross model. The fourth equation (d) suggests that government always use the balanced budget policy. Equation (e) explains about the composition of government spendings which can be divided into government consumption and government investment. Equation (f) and (g) represent the investment function and the import function. The assumed form of the investment function is a generalization of the commonly known accelerator-based investment behavior developed by Samuelson. Equation h links that level of employment to the level of real economic activity

(Y). Lastly, the equation (j) provides the definition for the balance of payment where economy has only opened up the international trade account.

Suppose the following data is given: $Y_{-1} = 200$ and X = 200. Consider the following questions.

i. Identify the endogenous and exogenous variables of this model.

Y; C; I; G; M; N and B are endogenous;

G_c, G_I, T *are exogenous.*

ii. Suppose that the government wants to target balance of payments and employment. Is the structure of the model suitable for this purpose? Why?

Yes, both are endogenous variables.

iii. Can government use private investment (I) as a policy instrument? Why?

No, because it is an endogenous variable. It can not directly be controlled by the policy maker.

iv. Can government use only public investment to reach its *balance of payments* and *employment* targets? Why?

No, Tinbergen's theorem asserts that the number of instruments should be equal to the number of targets. See (vi).

v. Following (iv), eliminate the irrelevant endogenous variables and express the remaining target variables in terms of the exogenous ones.

Through substitution one gets

$B = -0.375G_I - 0.268G_c + 317;$

$N = 6.789G_I + 5.829G_c - 1,874;$

vi. Suppose that the government aims at achieving $160 billion surplus in the balance of payments and an employment level of 1,100 by using public consumption and public investment. How much should government spend?

Notice that the equations given in (v) can be expressed as

$$y = b + Ax$$

where y is the vector of target variables, x is the vector of instruments, A is the coefficients matrix and b is the vector of intercept terms that are fixed. A unique solution to the above system can be obtained, if A^{-1} exists. The matrix A has an inverse if it is a square matrix, i.e. the number of rows (which is equal to the number of targets) should be equal to the number of columns (which is equal to number of instruments). ***This condition is Tinbergen's theorem.*** Secondly, A should have full rank, i.e. target variables, as well as instruments, should not be linearly dependent, i.e. they must be different. For the given values of target variables the solution of the above system is obtained by calculating

$$x = A^{-1}(y - b)$$

which gives $G_I = 325.64$ billion and $G_C = 131.02$ billion

i. Y; C; I; G; M; N and B are endogenous, G_C ; G_I , Y_{-1}; X are exogenous. ii. Yes, both are endogenous variables.

iii. No, because it is an endogenous variable. It can not be controlled by the policy maker.