

EE312 Macroeconomic Theory

Chapter 7

A Two-Period Model:

The Consumption-Savings Decision

Agenda

- **Two-period model set-up and optimality conditions**
- Implications for behavioral responses of consumption
- Credit market equilibrium

Two-Period Model

- The consumer makes **intertemporal choice** between current consumption and future consumption.
 - Savings (lending) and dissaving (borrowing).
 - **The real interest rate** is the relative price of future consumption in terms of current consumption.
- Decisions to be affected by changes in the real interest rate and in current and future incomes.

The Consumer

- Assume the consumer receives **exogenous income (endowment economy)**.
- The consumer's budget constraint:
 - c = current consumption goods;
 - s = current **savings**;
 - y = current real income;
 - t = current lump-sum taxes.
 - The current disposable income ($y - t$) equals current consumption plus savings ($c + s$).

The consumer's budget constraint

$$c + s = y - t$$

- Assuming bonds directly traded in the credit market.
- If $s > 0$, the consumer is **a lender**.
- If $s < 0$, the consumer is **a borrower**.

- Assume one single real interest rate (r) for borrowers and lenders.
- One unit of **current consumption** can be exchanged for $(1+r)$ units of **future consumption goods** in the credit market.
 - The relative price of future consumption in terms of current consumption is $1/(1+r)$.

Future budget constraint

- The consumer's future budget constraint:
 - c' = future consumption;
 - y' = future income;
 - t' = future taxes.
- If $s < 0$, the consumer pays the interest and principal on loan.

$$c' = y' - t' + (1 + r)s$$

Lifetime (consolidated) budget constraint

$$c' = y' - t' + (1+r)s$$

$$s = \frac{c' - y' + t'}{(1+r)}$$

but $c + s = y - t$

$$c + \frac{c' - y' + t'}{(1+r)} = y - t$$

$$c + \frac{c'}{(1+r)} = y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}$$

- The PV of **lifetime consumption** equals PV of **lifetime income** minus PV of **lifetime taxes**.
- The lifetime disposable income is the same as **lifetime wealth** (we)

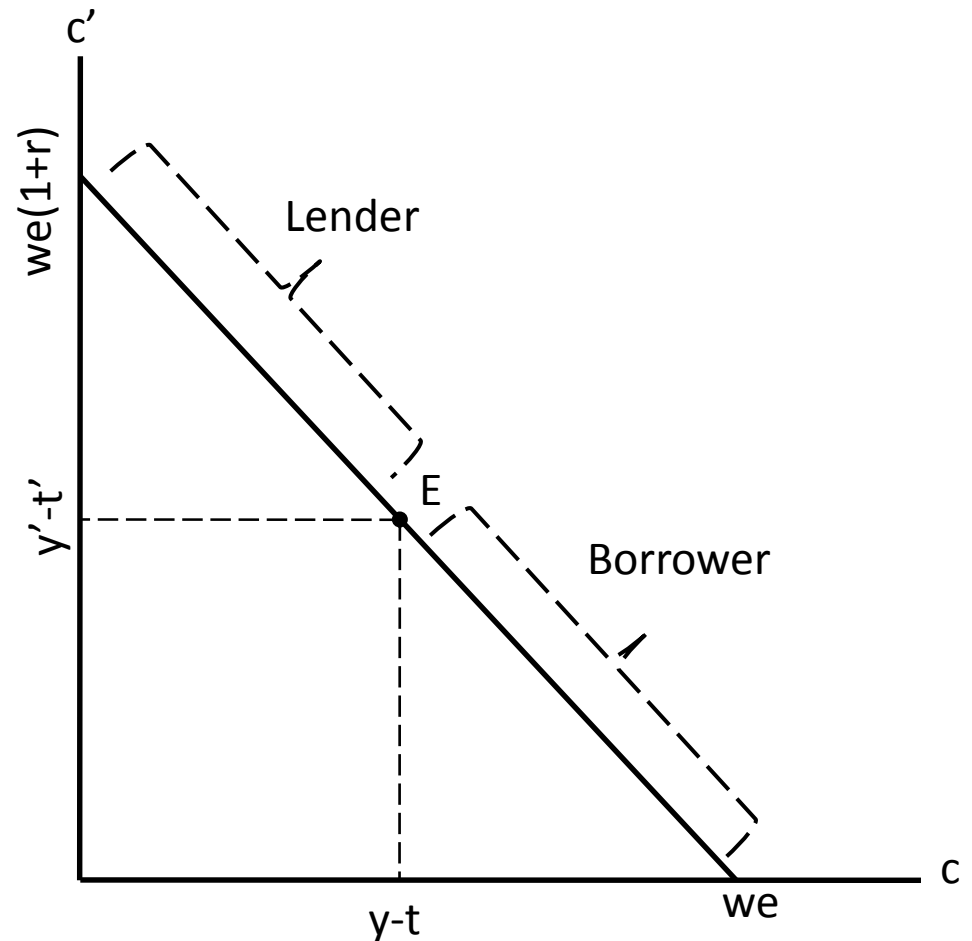
$$we = y + \frac{y'}{(1+r)} - t - \frac{t'}{(1+r)}$$

$$c + \frac{c'}{(1+r)} = we$$

$$c' = -(1+r)c + we(1+r)$$

Consumer's lifetime budget constraint

- E = endowment point.
 - If $c = y - t$ and $c' = y' - t'$, then $s = 0$.
 - Above E, the consumer is a **lender**.
 - Below E, a **borrower**.

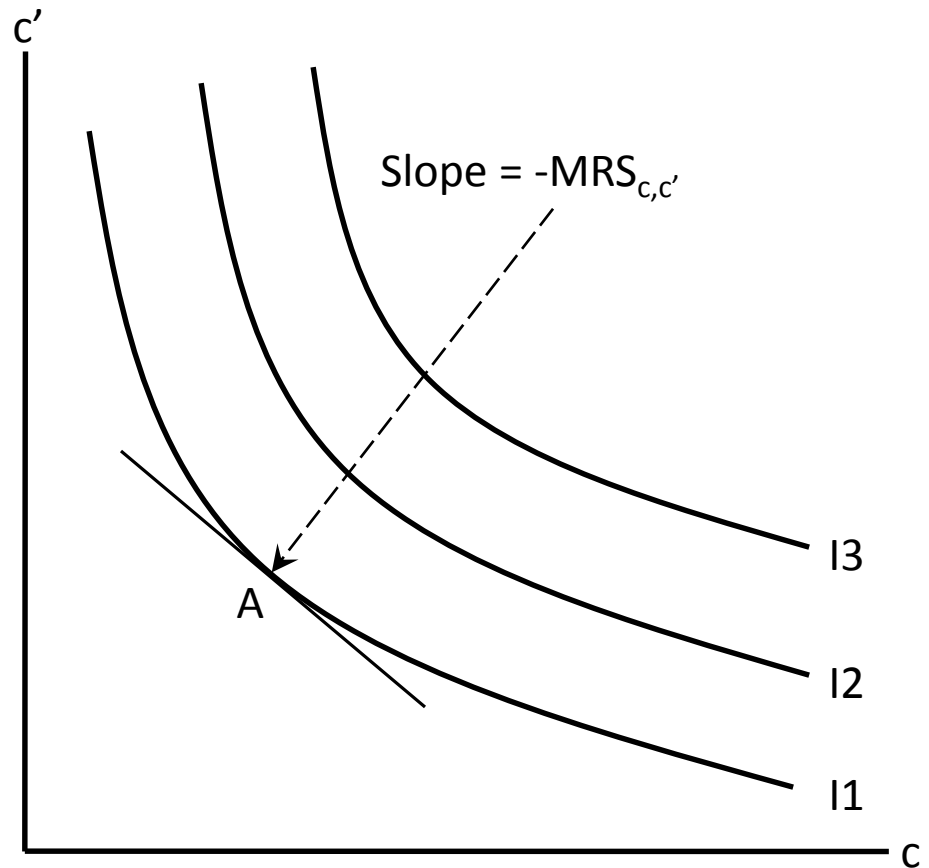


The Consumer's Preferences

- **A consumption bundle** is a combination of current and future consumption goods.
- Properties of consumer preferences:
 - More is preferred to less.
 - **Diversity** in the consumption bundle is preferred (*consumption smoothing*).
 - Current and future consumption goods are **normal goods**.

The consumer's indifference curves

- The ICs are downward-sloped and **convex**.
- Slope = $-MRS_{c,c'}$ = **the marginal rate of substitution** of c for c' is falling.



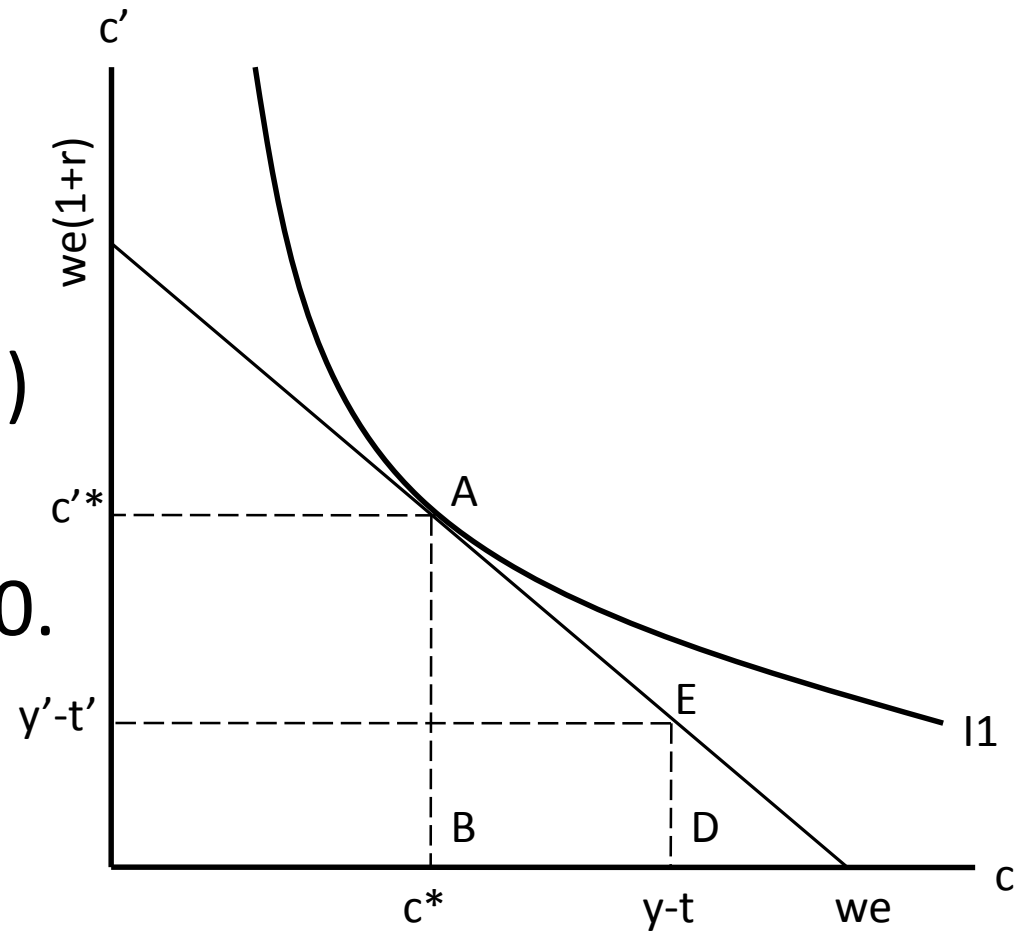
Consumer optimization

$$MRS_{c,c'} = 1 + r$$

- The consumer is optimizing where the marginal rate of substitution of c for c' equals the relative price of c in terms of c' .
 - The willingness to trade off c for c' equals the market rate of trading c for c' .
- The optimal consumption bundle is $(c, c') = (c^*, c'^*)$.

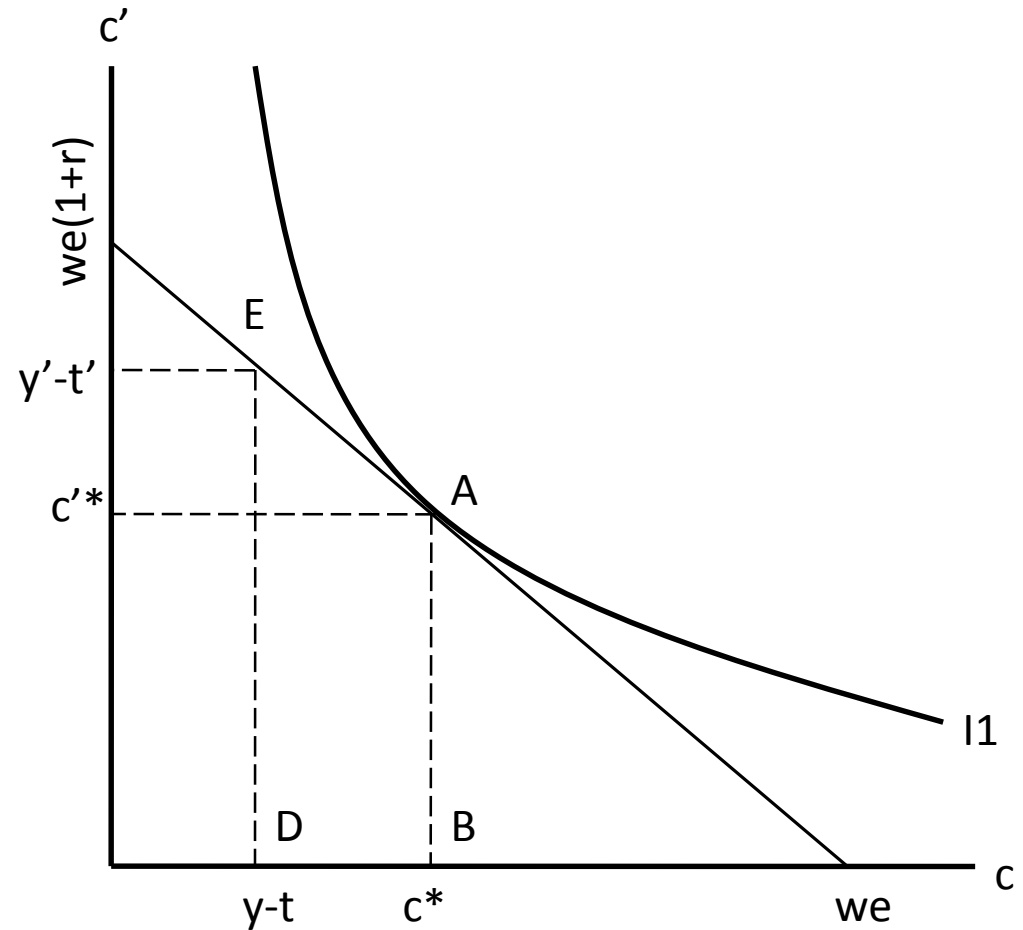
The consumer is a lender.

- At A, $(c, c') = (c^*, c'^*)$
and $c^* < (y - t)$.
- $s = y - t - c^* = BD > 0$.



The consumer is a borrower.

- At A, $c^* > (y - t)$.
- $-s = c^* - y + t =$
 $BD < 0$.



Agenda

- ~~Two period model set-up and optimality condition~~

- **Implications for behavioral responses of consumption**

- Credit market equilibrium

An increase in current income

- An increase in **current income** results in an increase in **lifetime wealth**.
 - A pure positive income effect.
 - The budget line shifts *horizontally* to the right.

$$we_1 = y_1 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$we_2 = y_2 + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$\Delta we = we_2 - we_1 = y_2 - y_1$$

- Both current and future consumptions increase (**normal goods**).
- The increase in c is **smaller** than the increase in y .
- savings increase; hence, c' increases.
- The consumer prefers **diversity** in the consumption bundle --- **consumption smoothing**.

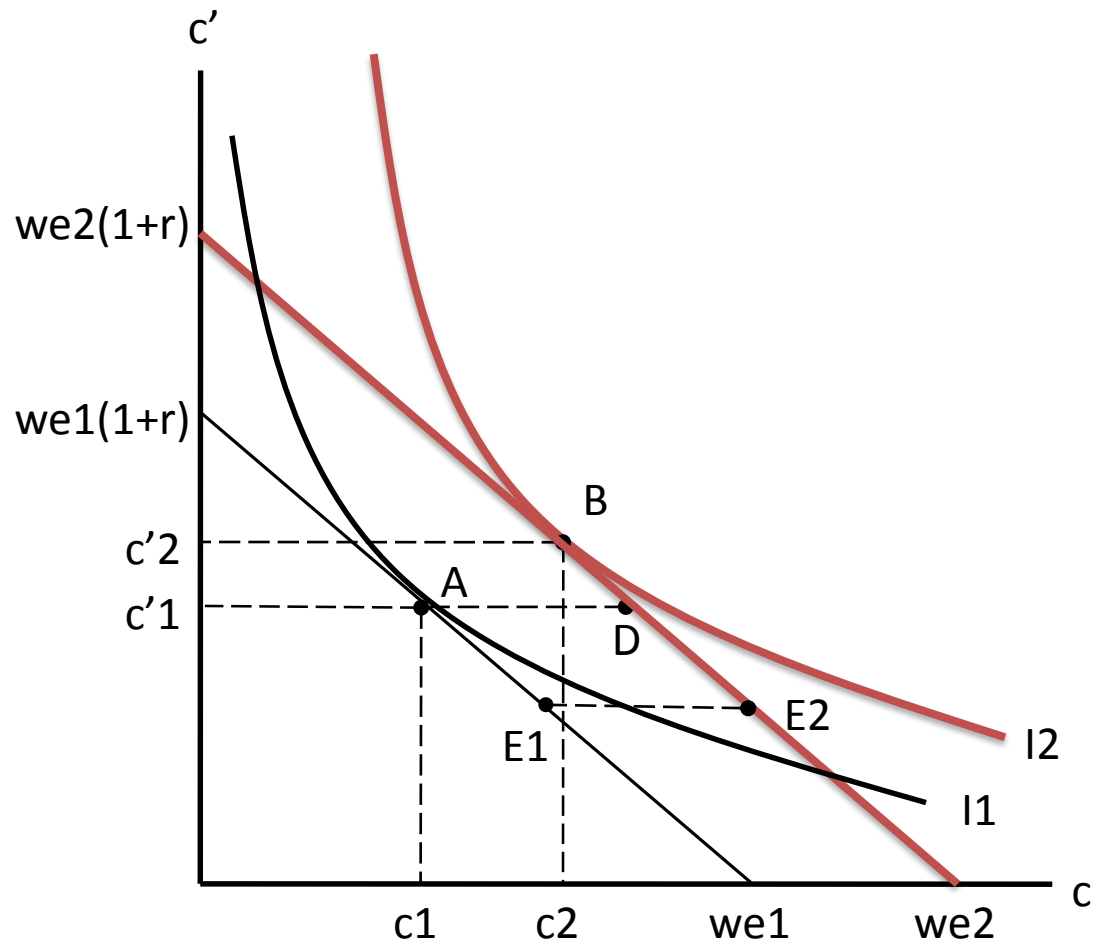
$$\Delta s = \Delta y - \Delta t - \Delta c$$

$$\Delta t = 0; \Delta y > \Delta c$$

$$\Delta s > 0$$

Increase in current y for a lender.

- Both c and c' increase (A to B).
- $\Delta c = c_2 - c_1 < \Delta Y = AD$; $\Delta s > 0$.
- So c' increases = $c'_1 c'_2$, given r and t .



An increase in future income

- An increase in **future income** results in an increase in **lifetime wealth**.
 - The budget line shifts *vertically* to the top.

$$we_1 = y + \frac{y_1'}{1+r} - t - \frac{t'}{1+r}$$

$$we_2 = y + \frac{y_2'}{1+r} - t - \frac{t'}{1+r}$$

$$\Delta we = we_2 - we_1 = \frac{y_2' - y_1'}{1+r}$$

- Future consumption increases, but by a lesser amount than the increase in future income.
 - **savings** decrease; **current consumption** increases.
 - The increase in future income is smoothed into increases in **both** future and current consumption.

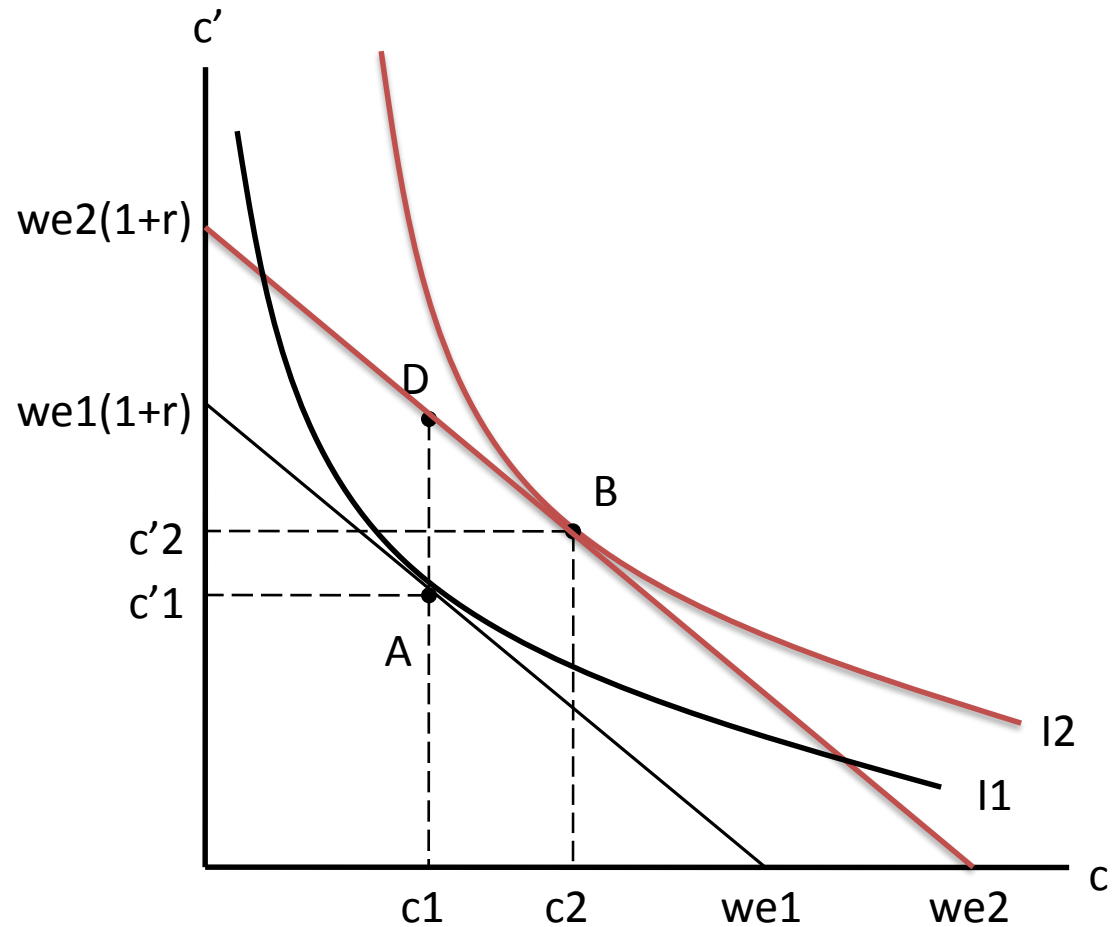
$$\Delta s = \Delta y - \Delta t - \Delta c$$

$$\Delta t = \Delta y = 0;$$

$$\text{So } \Delta s < 0; \Delta c > 0$$

Increase in future income

- Both c and c' increase (A to B).
- $\Delta c' = c'_2 - c'_1 < (\Delta y' = AD)$;
- $\Delta s < 0$.



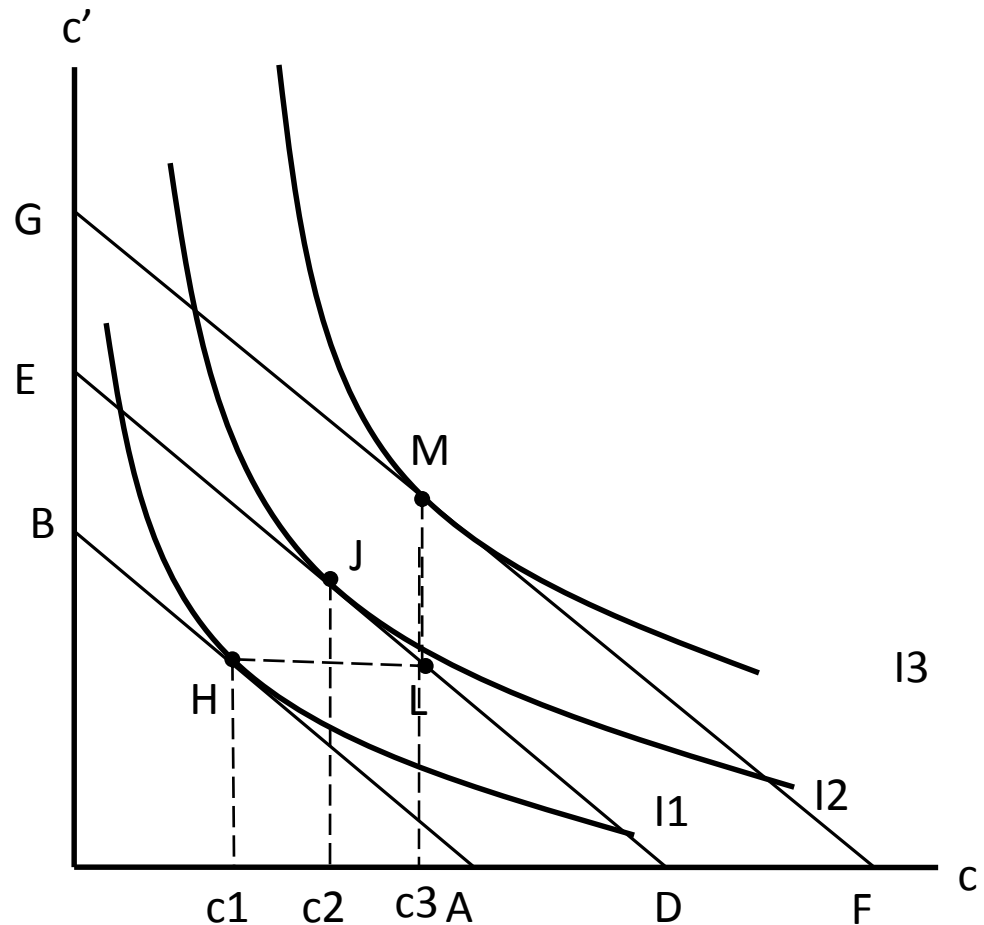
Temporary and permanent increases in income

- **A permanent increase** in income has a larger positive effect on lifetime wealth and current income than **a temporary increase**.
 - **Milton Friedman's 'permanent income hypothesis'**: the level of current consumption depends on the level of permanent income'.
- **The consumer will tend to save most of the temporary increase in income.**

- **A temporary increase** in $y = HL$: the budget line shifts from AB to ED.
 - The consumption bundle rises from H to J.
 - Current consumption rises less than current income; savings increase --- **consumption smoothing.**
- **A permanent increase** in $y = y_2 - y_1 = y'_2 - y'_1$: the budget line shifts from AB to GF.
 - $y_2 - y_1 = HL = y'_2 - y'_1 = LM$.
 - The consumption bundle rises from H to K.
 - Larger effect on current consumption: $c_1 c_3$

Bigger response under permanent Δy than temporary

- HJ = effect of **temporary (one-period) rise** in y .
- HM = effect of **permanent rise** in y .
 - So, saving does not change!
- **Effect on saving in then ambiguous**



Effect of a tax cut?

- The effect of the government's tax cut on consumption depends on whether the cut is temporary or permanent.
 - **If temporary**, the increase in consumption will be small; *most of the increased income is saved.*
 - **If permanent**, the increase in consumption will be large.

An increase in the real interest rate

- $1/(1+r)$ is the relative price of future consumption in terms of current consumption.
 - The slope of the budget line is $-(1+r)$.
 - A change in the real interest rate causes the budget line to rotate around the endowment point.
 - Changes in **intertemporal decision** between current and future consumption goods.
 - Analysis of **the substitution effect** and **income effect**.

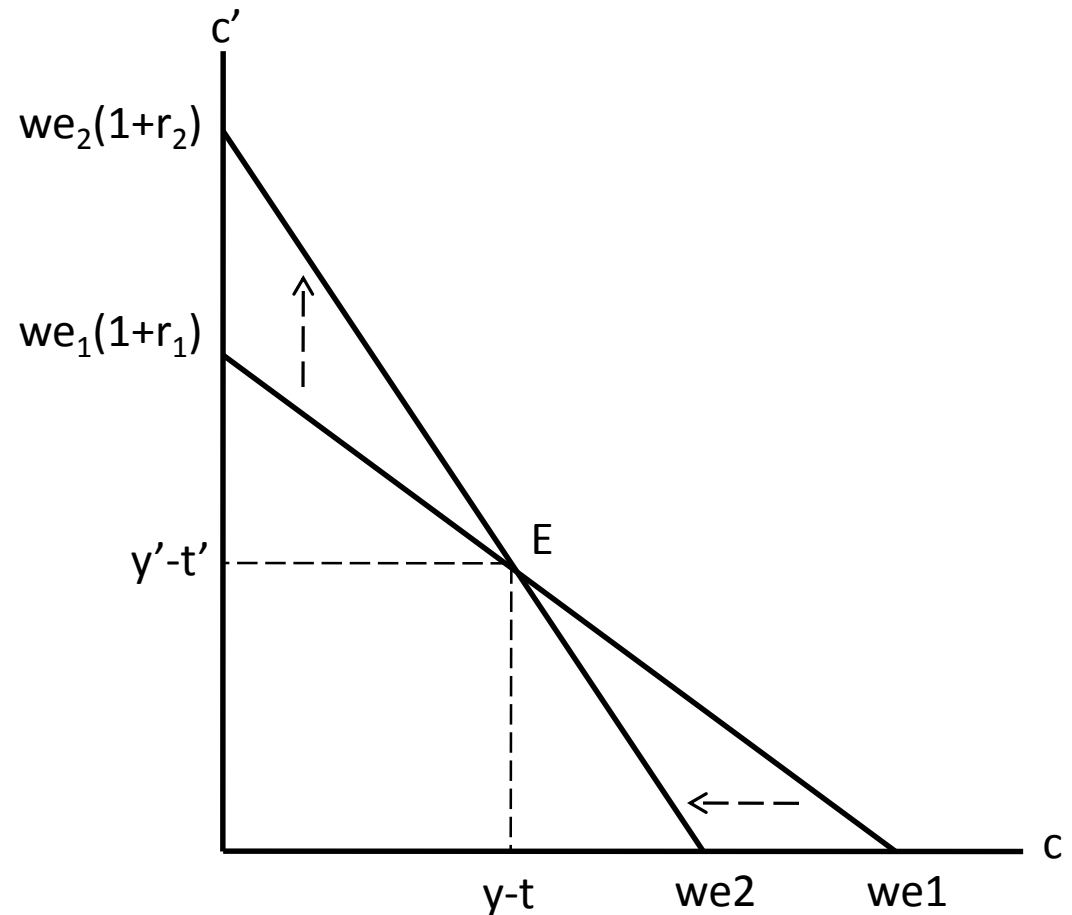
$$we = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$we(1+r) = (y-t)(1+r) + y' - t'$$

- The budget line rotates upwards as the real interest rate increases.
 - 'we' decreases (horizontal intercept).
 - $we(1+r)$ increases if r increases (vertical intercept).
 - The endowment point (E) remains the same (no change in the initial endowment of $y - t$, $y' - t'$).

The real interest rate rises.

- The increase in the real interest rate rotates the budget line upwards with the same E.



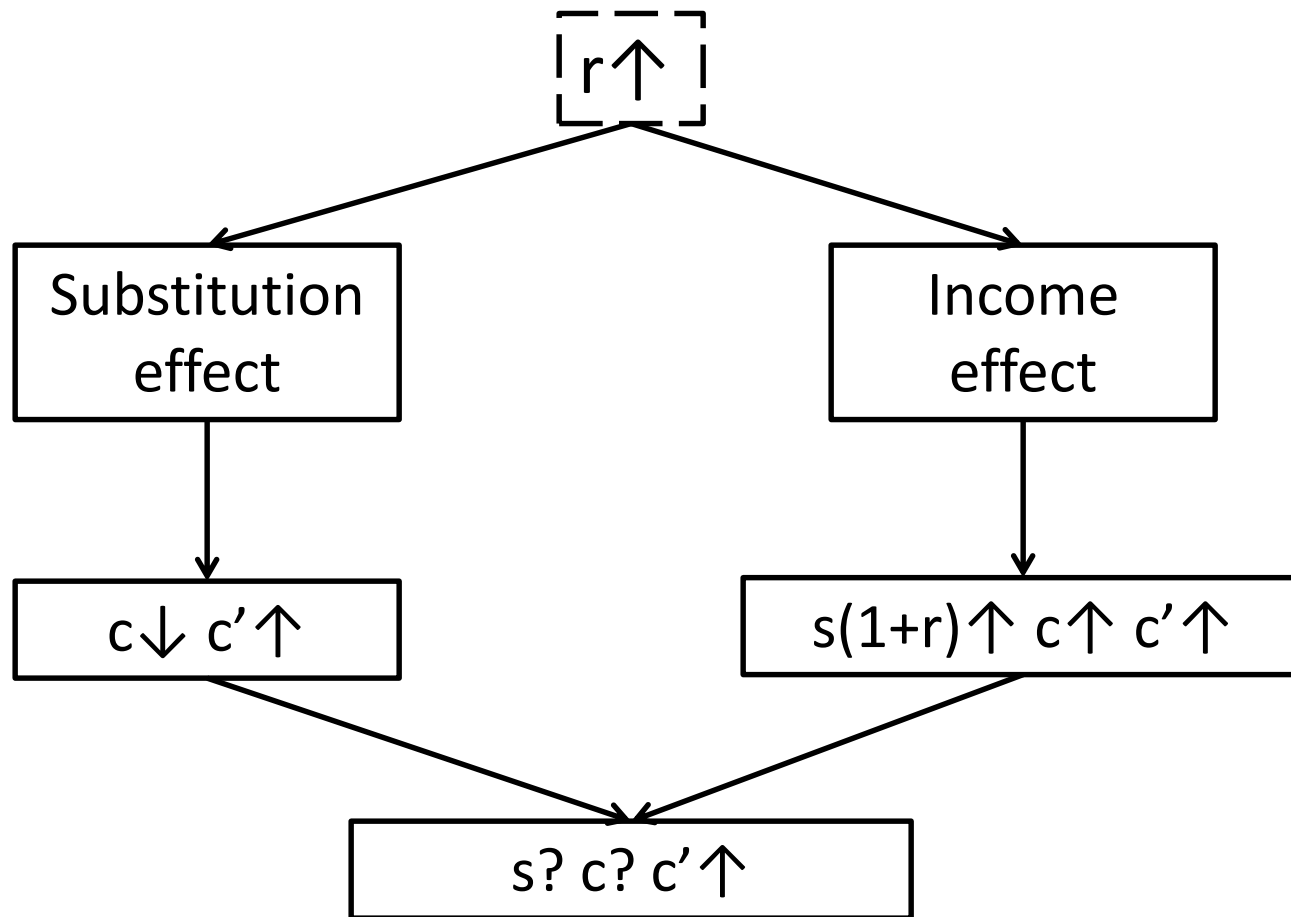
The dual effects of a higher r

- An increase in the real interest rate (r) causes a change in the relative price of current and future consumptions.
 - **Future consumption** becomes cheaper; **current consumption** more expensive.
 - Higher returns on savings for a lender; higher borrowing cost for a borrower.
 - **Lender**: cheaper future consumption.
 - **Borrower**: more expensive current consumption.

Effect of higher r on **the lender**

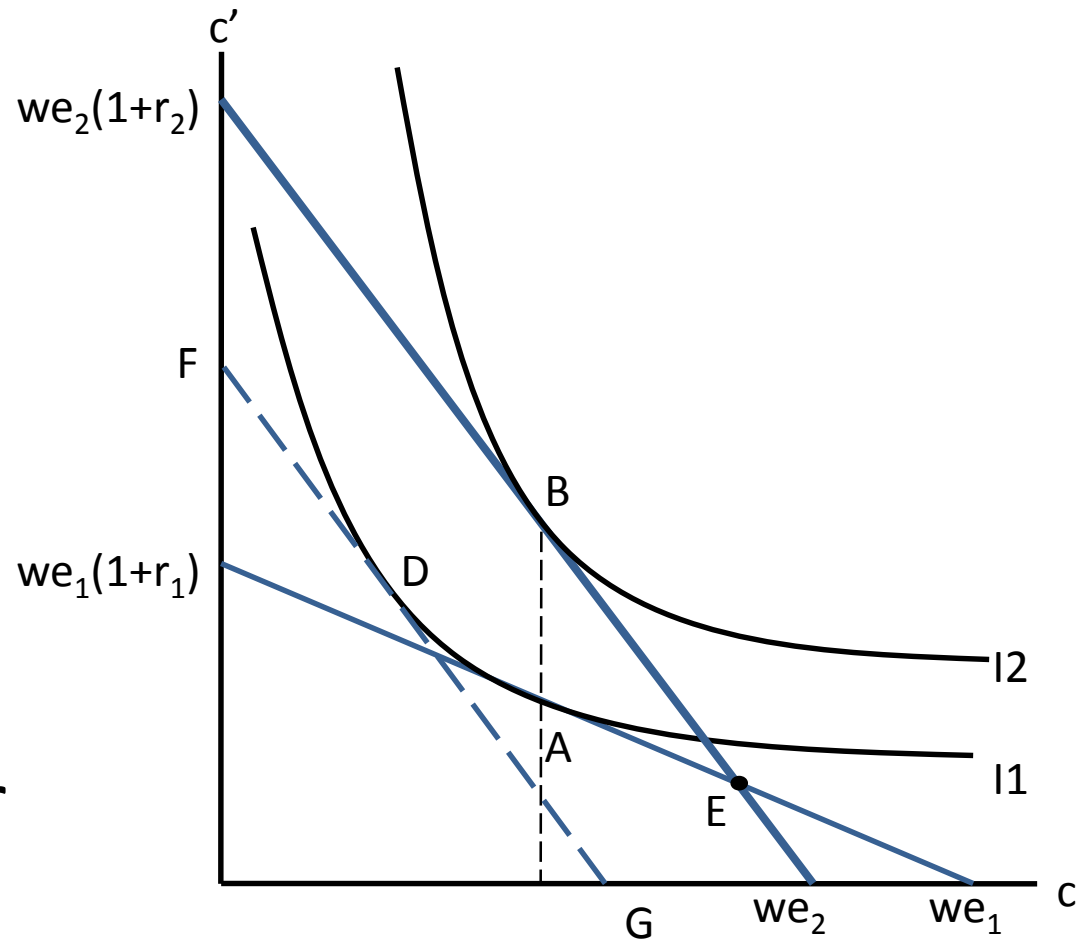
- Future consumption becomes cheaper in terms of current consumption.
 - **The substitution effect**: more future consumption for less current consumption.
 - **The income effect**: given savings yield more future income --- higher current and future consumptions.
- Higher future consumption; unclear current consumption and savings, given y and y' .

The consumer is a lender



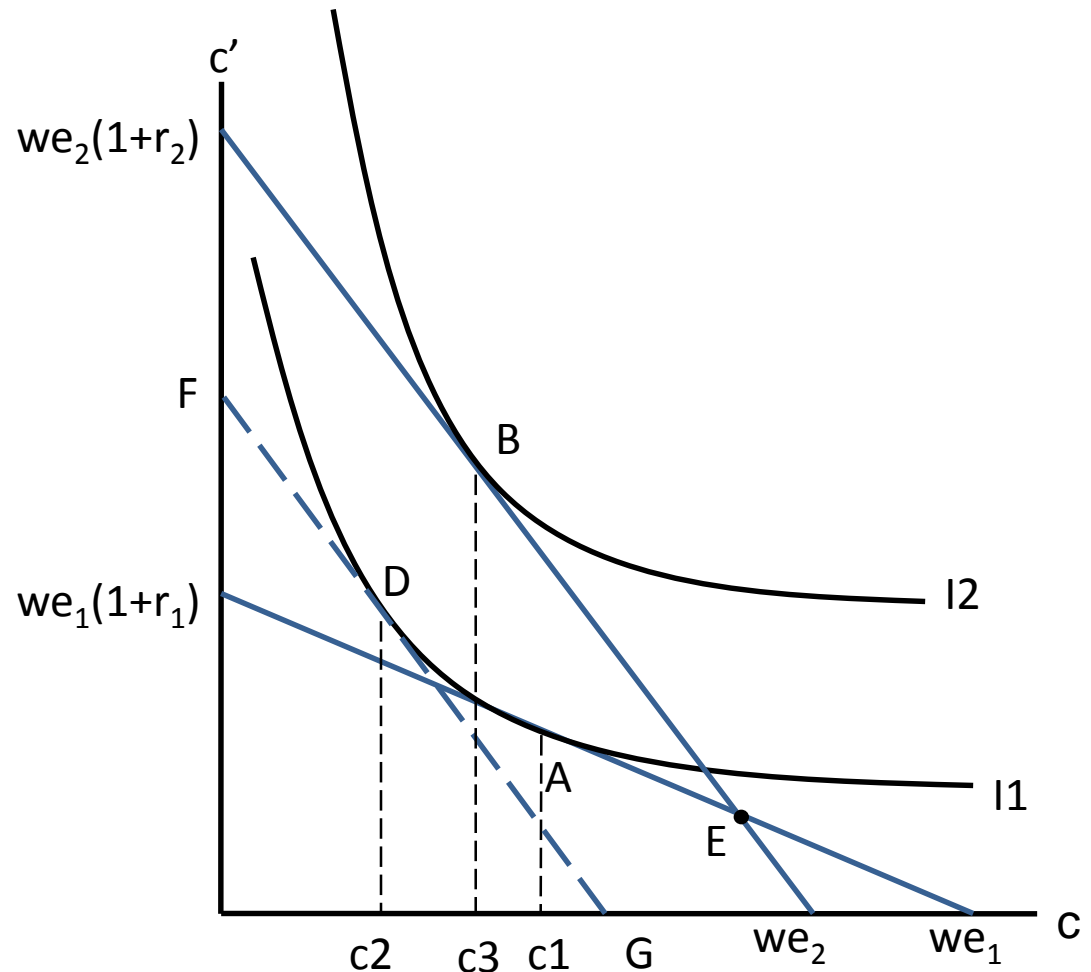
An increase in r for a lender.

- AD = **substitution effect**; lower c for higher c' .
- DB = **income effect**; higher c and c' .
- Net effect: higher c' ; unclear c .



Stronger substitution effect

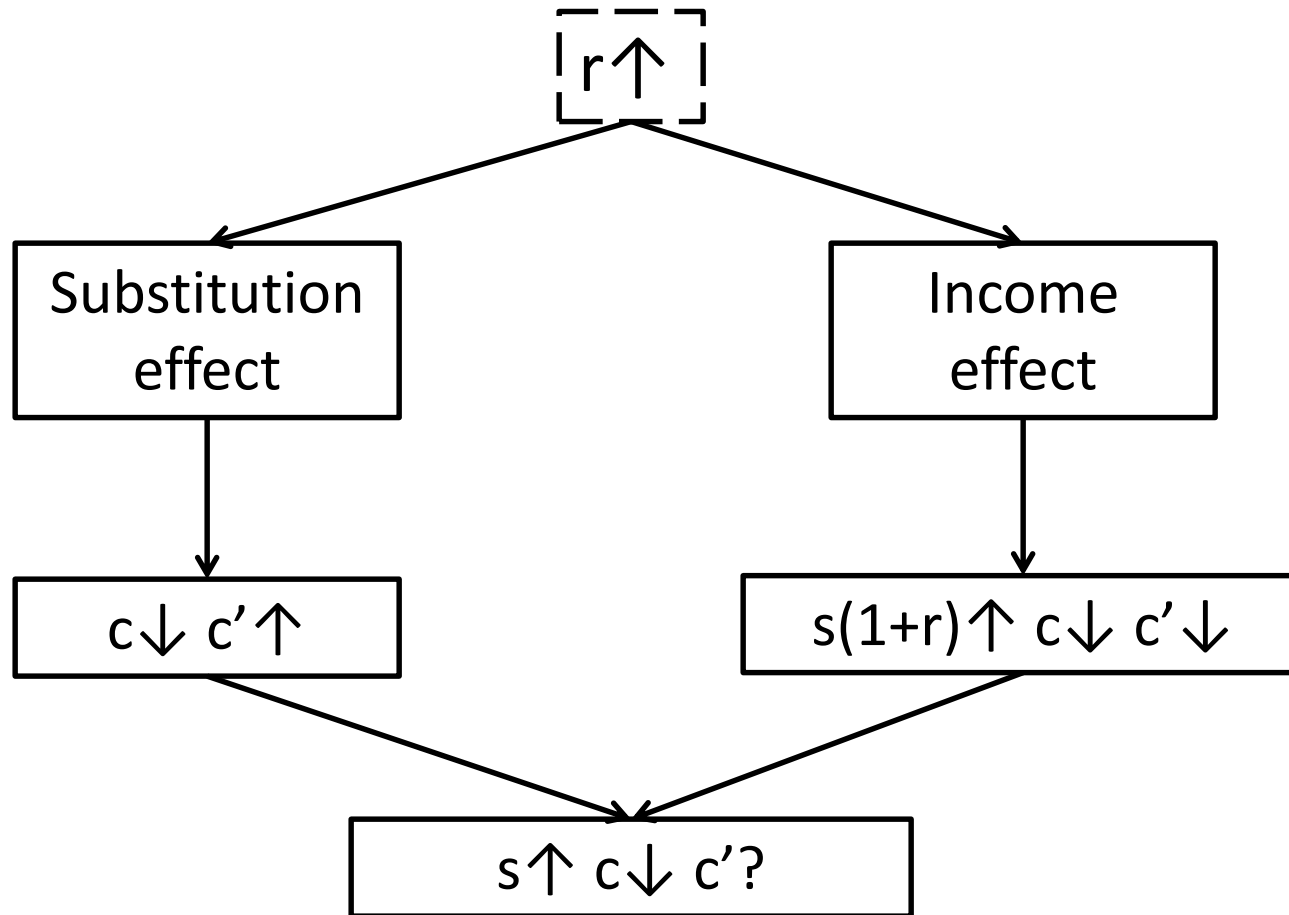
- AD = substitution effect; lower c .
- DB = income effect; higher c .
- AD > DB; net lower c at c_3 , **assuming a lender.**



Effect of higher r on the borrower

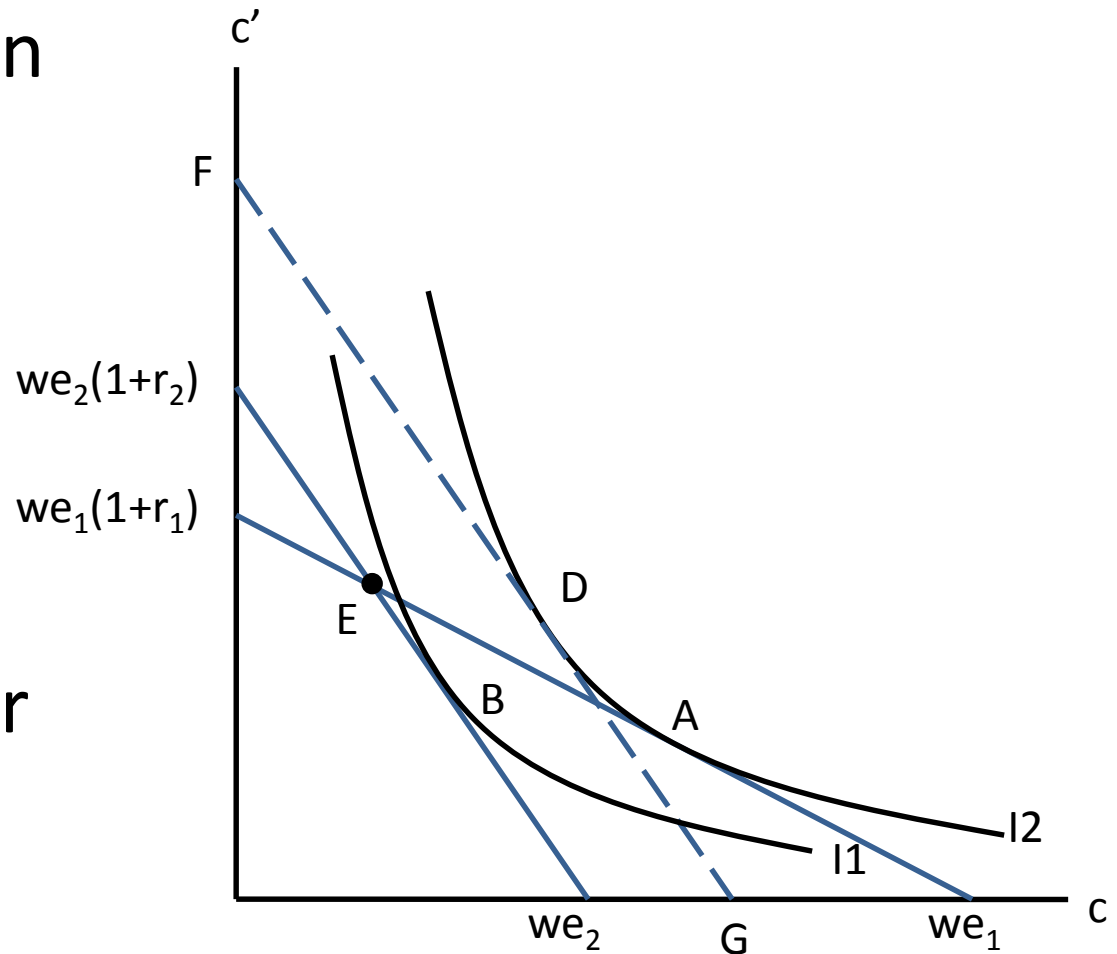
- Current consumption becomes more expensive in terms of future consumption.
 - **The substitution effect:** more future consumption and less current consumption.
 - **The income effect:** loans for current consumption become more expensive --- reduced current and future consumptions.
- Current consumption decreases while savings increase; unclear future consumption.

The consumer is a borrower



An increase in r for a borrower

- AD = substitution effect; lower c and higher c' .
- DB = income effect; lower c and c' .
- Net effect: lower c ; unclear c' .



Conclusions on effects of Δr

- A higher real interest rate (r) has an **intertemporal substitution effect**.
 - Future consumption is substituted for current consumption --- savings increase.
- Positive income effect for lenders but negative income effect for borrowers.
- No theoretical certainty that current consumption will fall if the real interest rate rises.

Summing up: Behavioral responses

Variable	Current consumption	Saving
Income (increase)		
• Temporary	Increase/Small <i>(consumption smoothing)</i>	Increase
Current Future		Decrease
• Permanent	Increase/Larger	Depends
Tax cut (more cut)		
• Temporary	Increase/Small	Increase
• Permanent	Increase/Large	Depends
Interest rate (increase)		
• Borrower	Decrease	Increases
• Saver	Depends	Depends

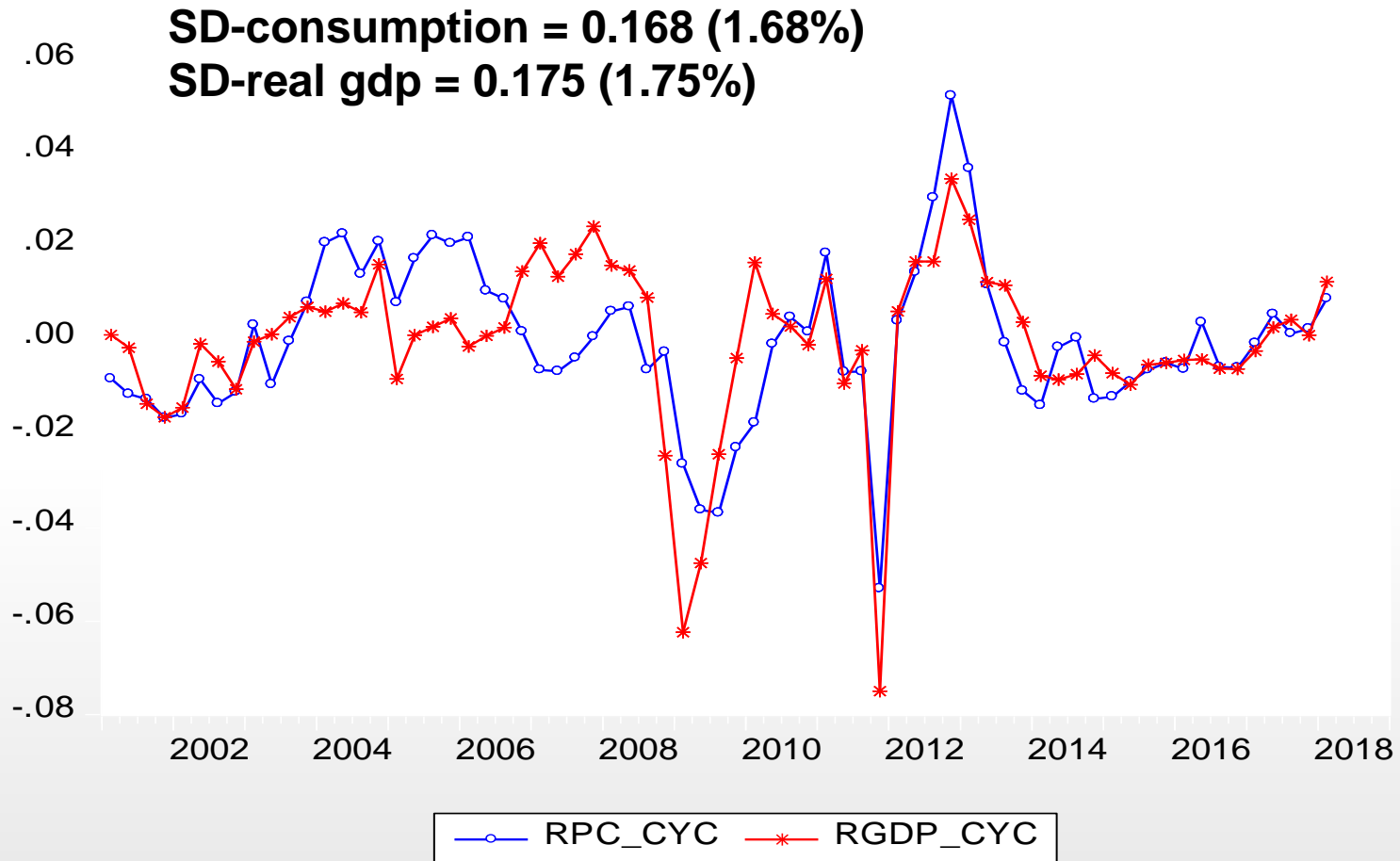
Empirical application: How well does the theory explain the pattern of consumption?

- Our theory of consumption generates two sets of testable implications
 - **Consumption smoothing**
 - **Behavioral responses to key determinants**

Consumption smoothing

- With the credit market, households could borrow and lend so that they can better timing of the consumption under a fluctuation in income
- In the data, we expect that cyclical variation of consumption should be less volatile than that of income (measured by GDP)

Consumption smoothing in Thailand



Consumption smoothing: where do we stand?

Less Consumption Smoothing in Poor and Emerging than in Rich Countries

Business-Cycle			
Statistic	Poor	Emerging	Rich
σ_c/σ_y	1.12	0.98	0.87

Relative Volatility of Consumption, σ_c/σ_y , Controlling for Country Size and Income

	σ_c/σ_y		
	Poor	Emerging	Rich
All	1.1	0.98	0.87
Small	1.4	0.97	0.92
Medium	1.1	0.93	0.93
Large	1.1	1.1	0.84

Poor Countries

-Benin, Bhutan, Burkina Faso, Burundi, Central African Republic, Comoros, Gambia, Guyana, Honduras, Lesotho, Malawi, Mali, Mauritania, Mongolia, Niger, Papua New Guinea, Rwanda, Senegal, Sierra Leone, Togo, Zambia, Zimbabwe.
 -Cameroon, Congo, Côte d'Ivoire, Ghana, Kenya, Madagascar, Mozambique, Nepal, Sri Lanka, Sudan, Uganda.
 -Bangladesh, China, Ethiopia, India, Indonesia, Pakistan, Philippines.

Emerging Countries

-Albania, Antigua and Barbuda, Bahrain, Barbados, Bolivia, Botswana, Bulgaria, Chile, Costa Rica, Cuba, Cyprus, Dominica, Dominican Republic, Ecuador, El Salvador, Fiji, Gabon, Greece, Grenada, Guatemala, Hungary, Israel, Jordan, Malta, Mauritius, Namibia, New Zealand, Panama, Paraguay, Portugal, Puerto Rico, Seychelles, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Suriname, Swaziland, Tonga, Trinidad and Tobago, Tunisia, Uruguay.
 -Algeria, Argentina, Colombia, Iran, Malaysia, Morocco, Peru, South Africa, South Korea, Spain, Syria, Thailand, Turkey, Venezuela.
 -Brazil, Egypt, Mexico.

Rich Countries

-Austria, Belgium, Denmark, Finland, Hong Kong, Iceland, Ireland, Luxembourg, Macao, Netherlands, Norway, Singapore, Sweden, Switzerland.
 -Australia, Canada, France, Italy, United Kingdom.
 -Germany, Japan, United States.

Consumption smoothing: What's wrong?

- In many countries, **consumption is more volatile than output**. Why so?
- Wrong theory?
 - Predictions are always correct under assumptions made
 - No, assumptions do not match with the reality!
- Economists generally account for this phenomena (excess variability of consumption) to the **imperfection in the credit market**

Behavioral responses

- Let's bring in some regression analyses
- To answer this, I estimated a simple consumption function using the aggregate (quarterly) data between 2001 and 2017.
- I used growth estimation; the estimated parameters represent the “**elasticity**”

Thailand (aggregate) consumption function

Dependent Variable: D(LOG(RPC),0,4)

Method: Least Squares

Date: 03/16/18 Time: 00:18

Sample: 2001Q1 2017Q3

Included observations: 67

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.010056	0.005742	1.751204	0.0847
4-quarter change in real borrowing cost	-2.12E-05	1.34E-05	-1.588959	0.1170
Y-o-Y growth in disposable income	0.644946	0.159053	4.054921	0.0001

R-squared	0.563797	Mean dependent var	0.035161
Adjusted R-squared	0.550166	S.D. dependent var	0.030145
S.E. of regression	0.020218	Akaike info criterion	-4.920725
Sum squared resid	0.026162	Schwarz criterion	-4.822008
Log likelihood	167.8443	Hannan-Quinn criter.	-4.881663
F-statistic	41.36037	Durbin-Watson stat	0.607689
Prob(F-statistic)	0.000000	Wald F-statistic	12.77705
Prob(Wald F-statistic)	0.000021		

Implications

- Real Interest rate has a **small effect** on consumption growth; **statistically, it's not significant.**
 - Hard to tell the reason. It's aggregate data.
 - Individual behaviors are ***heterogeneity***; ***aggregation might*** have already **washed out all individual details.**
 - Need to look at micro-level data; cohort analysis
 - Household surveys data; take EE461.

Implications

- Sensitivity to the growth in disposable income is 0.64.
 - Statistically significant! Good.
- The value seems higher than that obtained from the estimation using US data
 - **Possible reason 1:** Permanent income rises!
 - **Possible reason 2:** if most of the increase in income represent a temporary increase, the effect should have been smaller. (**Excess sensitivity puzzle**)

Agenda

- Two-period model set-up and optimality condition
- Implications for behavioral responses of consumption
- **Credit market equilibrium**

The government sector

- G = current government purchase of goods.
- G' = future government purchase of goods.
- T = current taxes collected by the government.
 - N = number of consumers, each paying the current tax of t ; so $T = Nt$.
- T' = future taxes; and $T' = Nt'$.
- Government borrows by issuing bonds (B) at the real interest rate of r .

Government's current budget constraint

- Government spending is financed by taxes and bond issue in each period.
- Government's current budget constraint:
- $G = T + B$ or $G - T = B$.
 - $B > 0$ government is a borrower;
 - $B < 0$ government is a lender.
- Government's future budget constraint:
- $G' + (1+r)B = T'$

Government present-value budget constraint

- The present value of government purchases must equal the present value of taxes.

$$G = T + B$$

$$G' + (1+r)B = T'$$

$$B = \frac{(T' - G')}{1+r}$$

$$\text{then } G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Competitive equilibrium

- Consumers and government interact in the credit market.
 - Trading of future consumption goods for current consumption goods through the **real interest rate**.
- **Equilibrium conditions:**
 - Each consumer optimizes current and future consumptions and savings, given r .
 - Government budget constraint holds.
 - The credit market clears.

The credit market clears.

- Aggregate private savings (S^p) equals government borrowing (B) or $S^p = B$.
 - National savings is the sum of aggregate private savings and government savings; $S = S^p + S^g$.

$$S = S^p + S^g$$

$$S^p = B; S^g = -B$$

$$S = B - B = 0$$

The income-expenditure identity

- The credit-market clearing implies that the income-expenditure identity holds.

$$S^p = Y - C - T$$

$$B = G - T$$

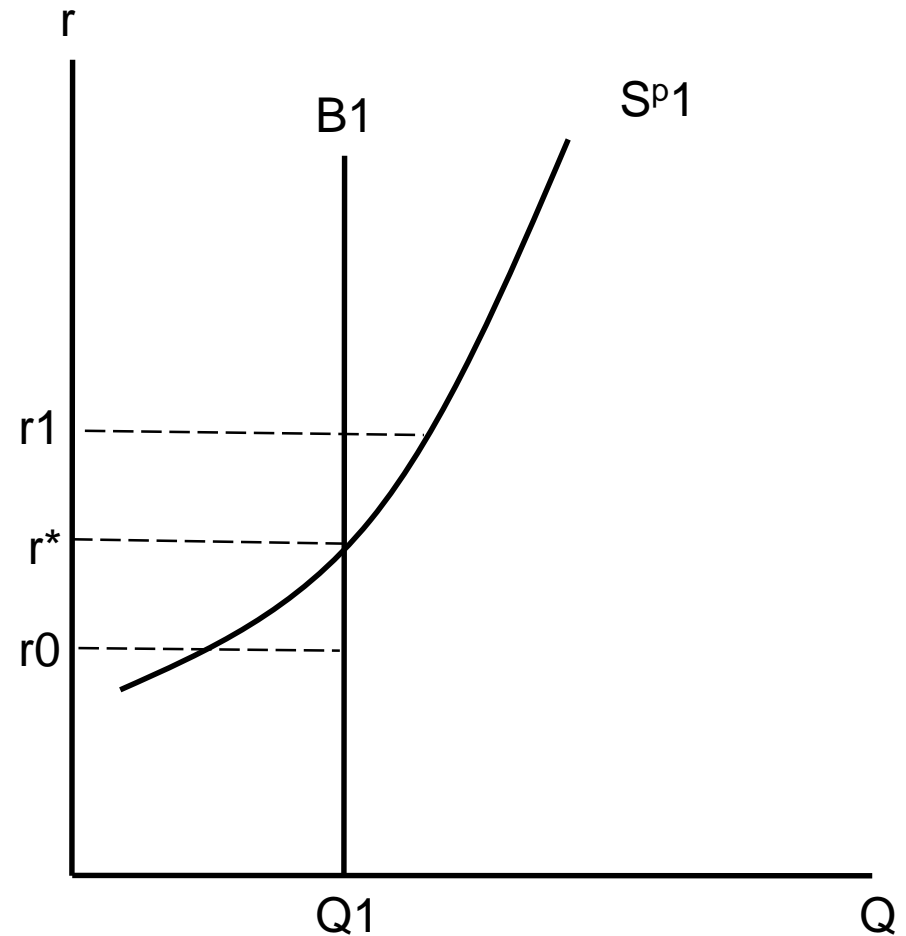
$$S^p = B$$

$$Y - C - T = G - T$$

$$Y = C + G$$

Equilibrium representation

- Suppose a given budget deficit equal to $G - T = B1$
 - $B1$ is demand for credit
- r^* is the equilibrium interest rate
 - $r1$: excess supply
 - $r0$: excess demand

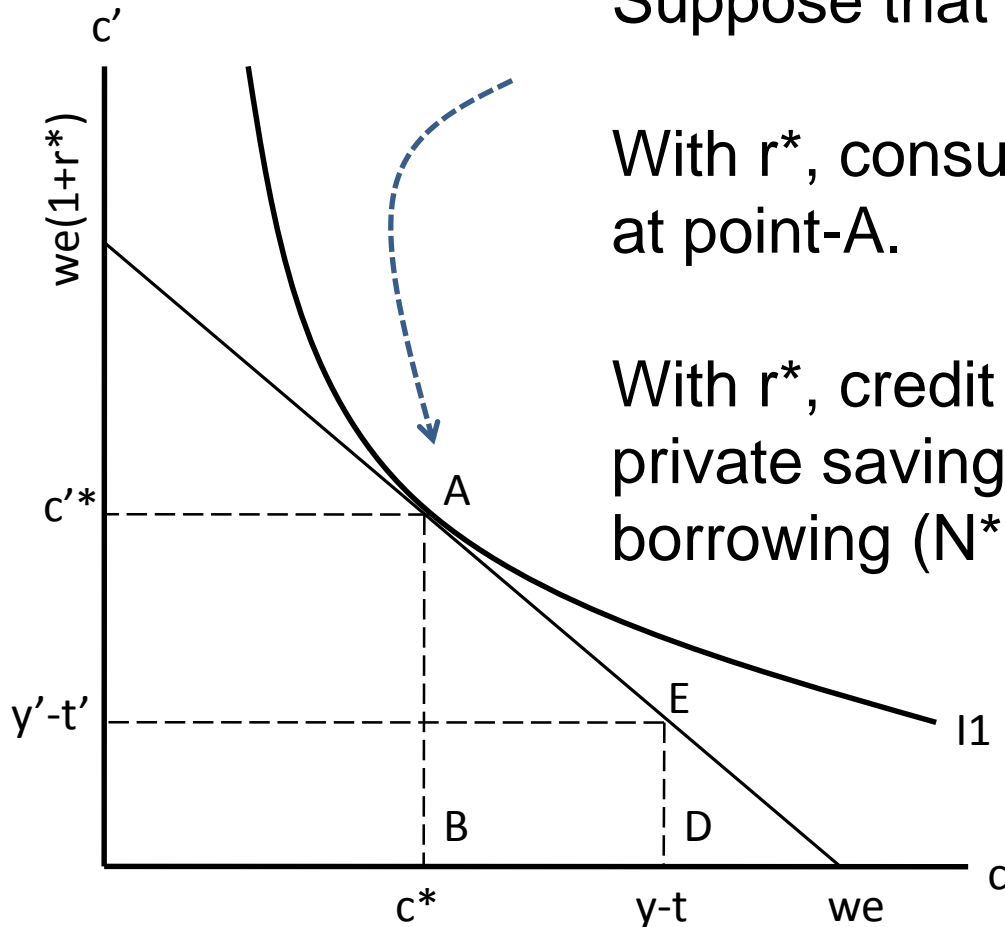


Equilibrium representation

Suppose that $N(y-t-c^*) = G - T$

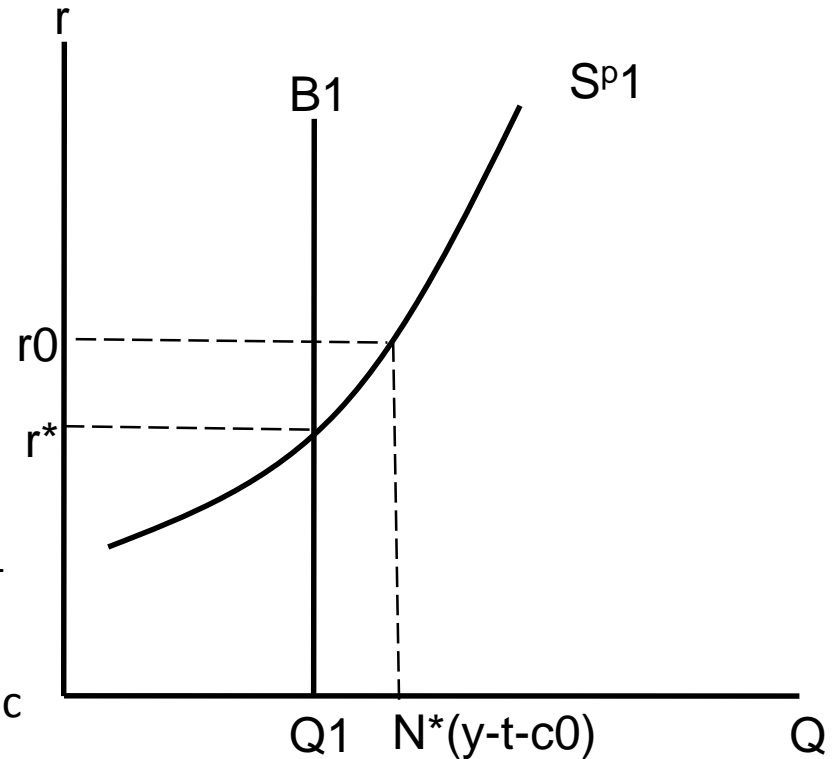
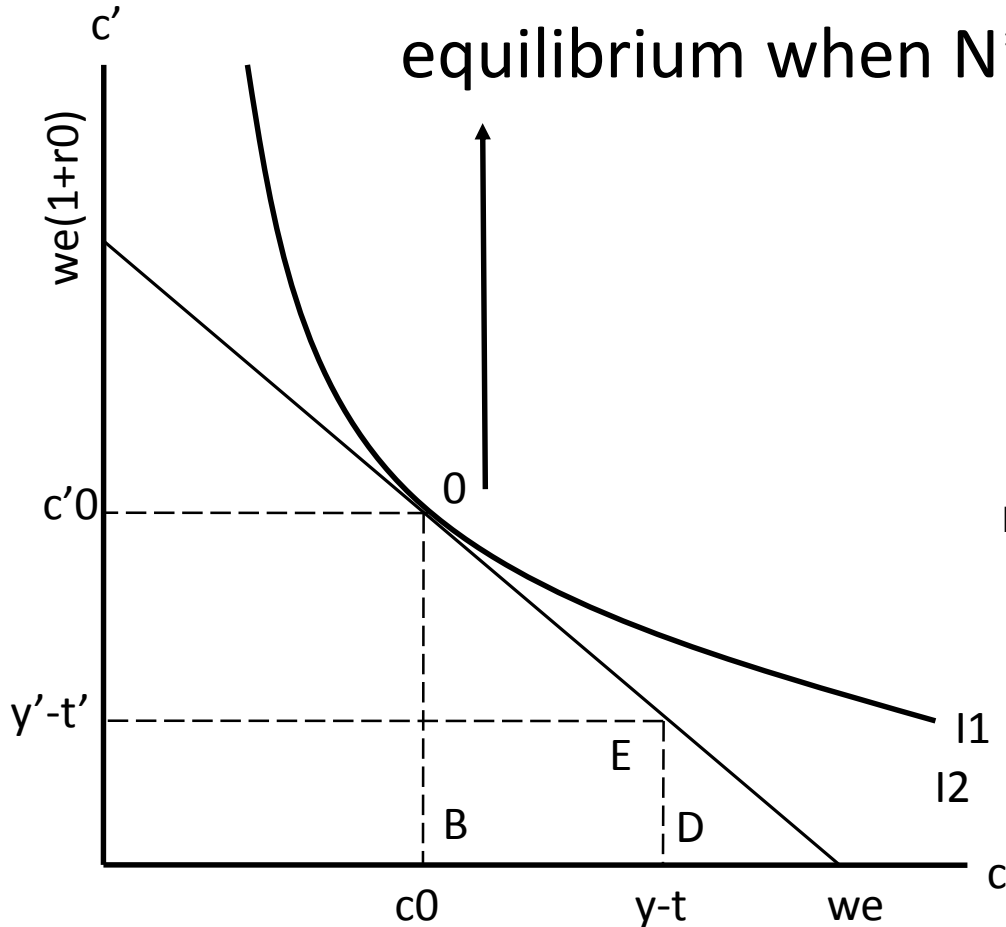
With r^* , consumers are maximizing utility at point-A.

With r^* , credit market is cleared; net private saving is equal to net government borrowing ($N^*BD = G - T = B1$)



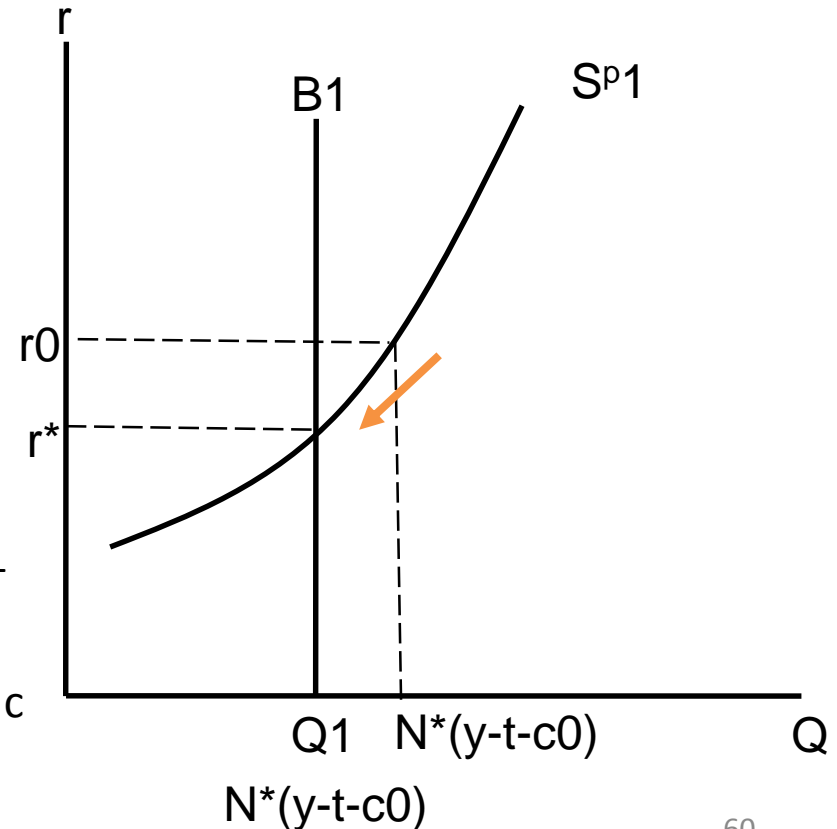
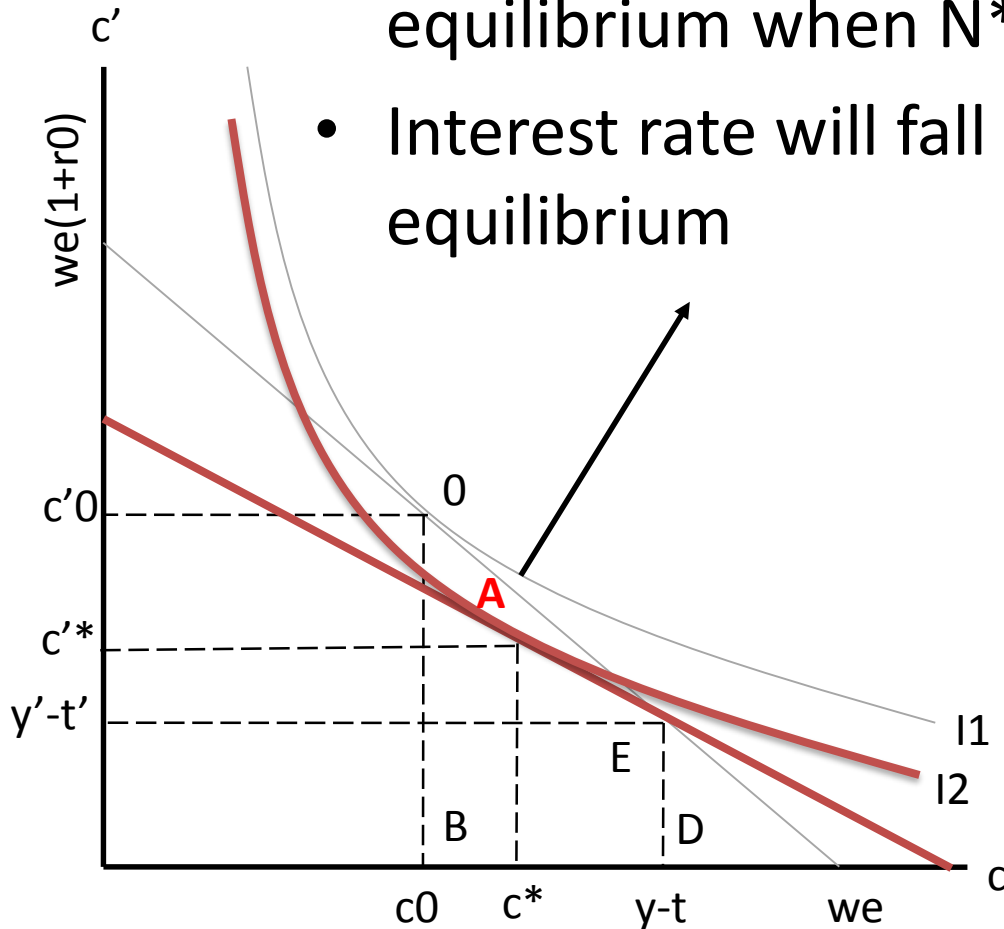
Disequilibrium and adjustment

- **Question:** Given r_0 , does point-0 represent the equilibrium when $N^*(y-t-c_0) > B_1$.



Disequilibrium and adjustment

- **Question:** Given r_0 , does point-0 represent the equilibrium when $N^*(y-t-c_0) > B_1$.
- Interest rate will fall to r^* ; point-A is the equilibrium



Impact of exogenous variables on equilibrium variables

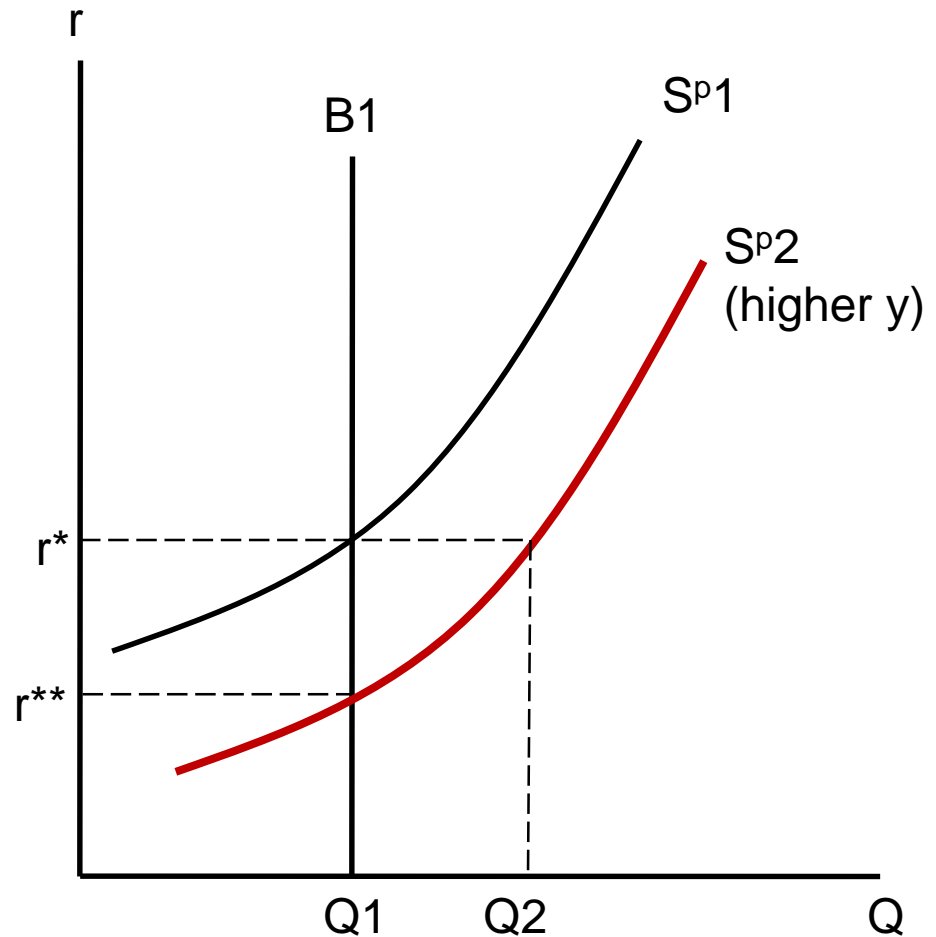
- Taking variables $\{Y, Y', T, T', G, G'\}$ as given, we can determine an equilibrium allocation and price, i.e. C^* (current consumption), $C^{*'} (Future consumption), r^* (real interest rate)$
- **Question:** what happens to the equilibrium if the given exogenous variables are varied!

Questions

- How does an increase in current income affect (equilibrium) real interest rate etc.?
- How does an increase in future income affect (equilibrium) real interest rate etc.?
- How does an increase in permanent income affect (equilibrium) real interest rate etc.?
- **How does a tax cut affect (equilibrium) real interest rate etc.?**

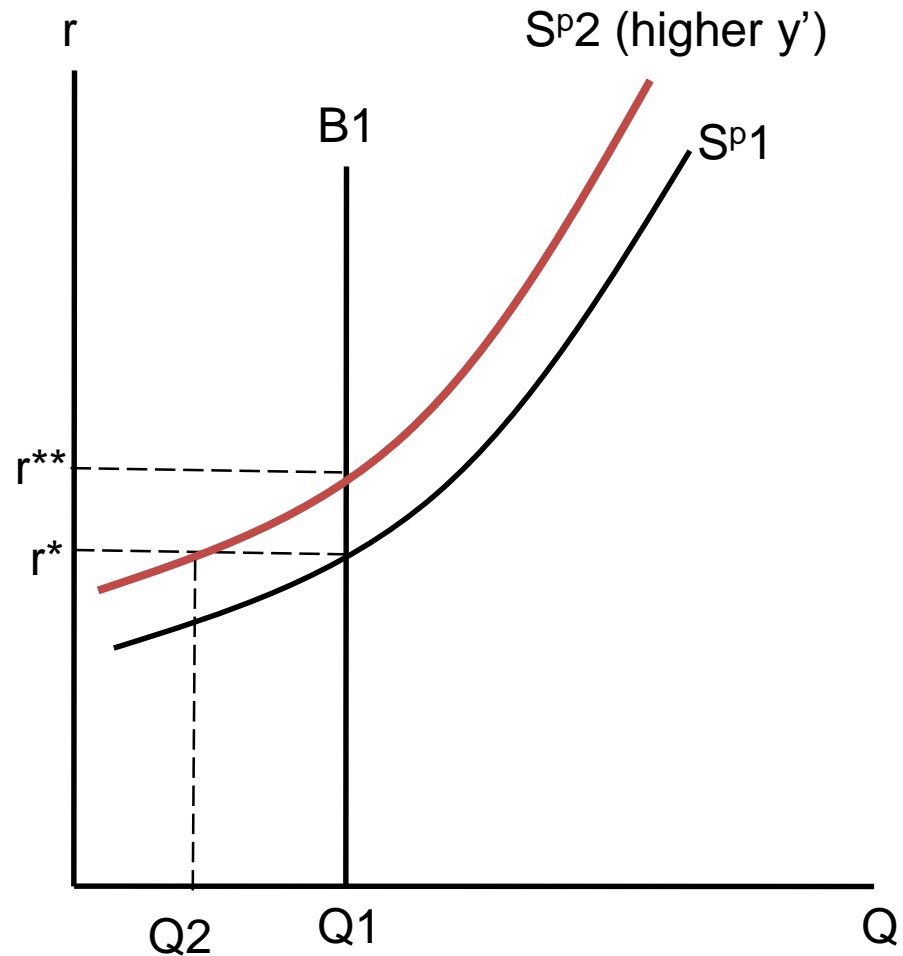
How does an increase in current income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “ r ”.



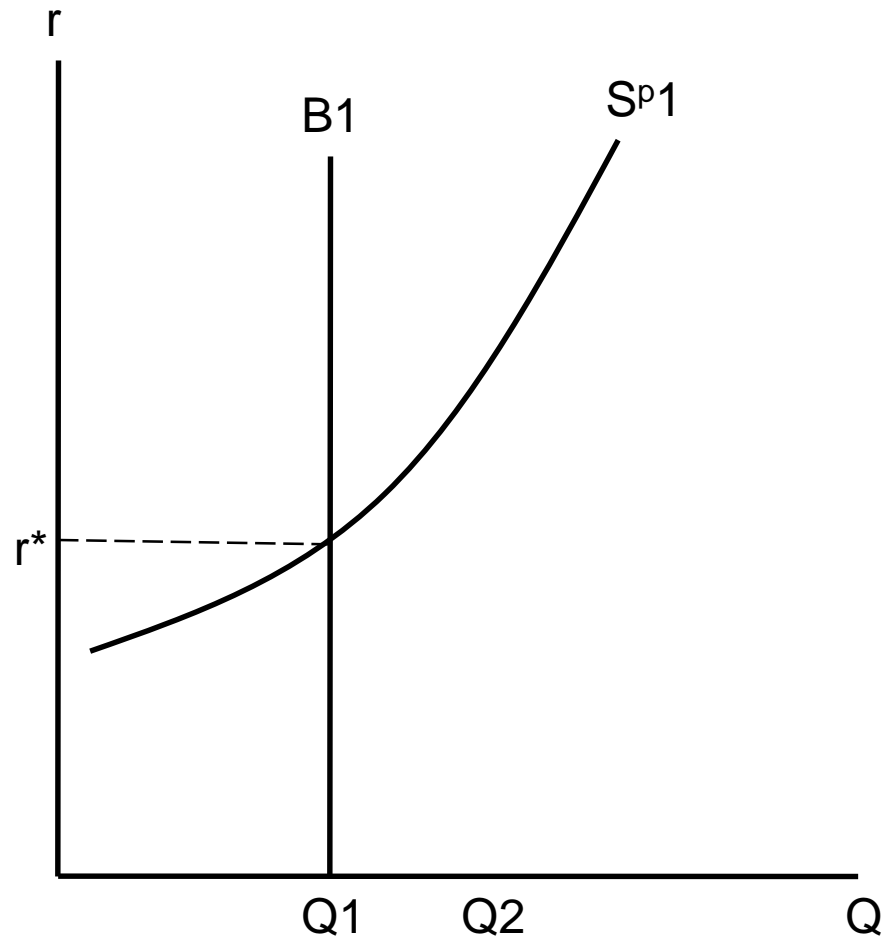
How does an increase in future income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “ r ”.



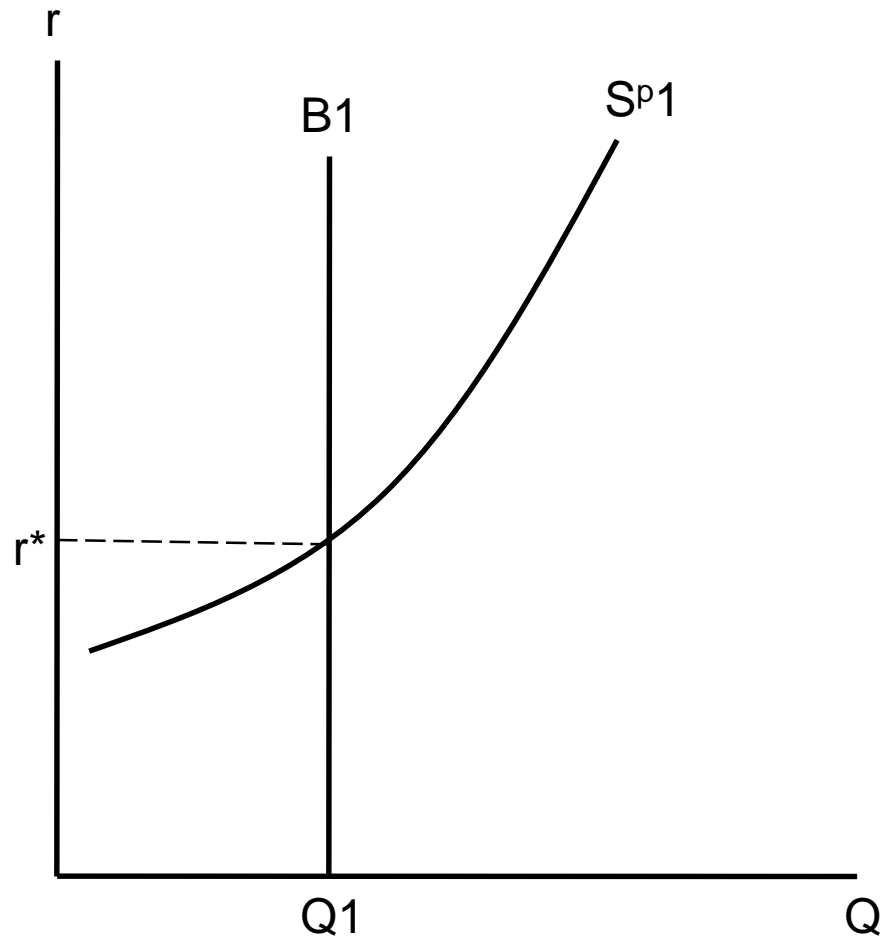
How does an increase in permanent income affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “ r ”.
- **Exercise!**



How does a tax cut affect (equilibrium) real interest rate etc.?

- Assuming dominant substitution effect
- Saving is upward sloping in “ r ”.



The Ricardian Equivalence

- A change in current taxes with *an equal and opposite change* in the present value of future taxes has **no effect** on the *real interest rate* and the *consumption of individual consumers*.
 - Assume equilibrium in the credit market, given r .
 - Current and future government spending are held constant.
 - Consumers' life-time budget constraint and government's present-value budget constraint.

Algebraic formulation

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

$$G + \frac{G'}{1+r} = Nt + \frac{Nt'}{1+r}$$

$$t + \frac{t'}{1+r} = \frac{1}{N} \left(G + \frac{G'}{1+r} \right) \quad (\text{eq.8.24})$$

$$\text{but } c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - t - \frac{t'}{1+r}$$

$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N} \left(G + \frac{G'}{1+r} \right) \quad (\text{eq.8.26})$$

$$t + \frac{t'}{1+r} = \frac{1}{N} \left(G + \frac{G'}{1+r} \right) \quad (\text{eq.8.24})$$

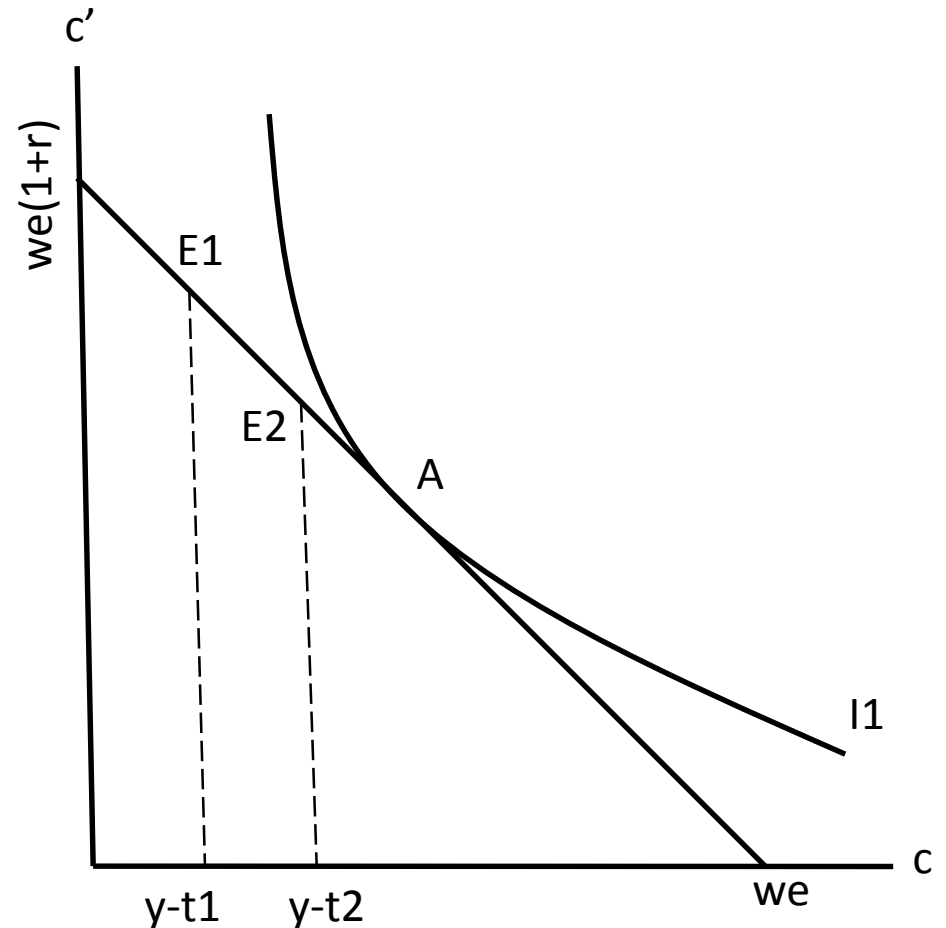
$$c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N} \left(G + \frac{G'}{1+r} \right) \quad (\text{eq.8.26})$$

- The change in current taxes (Δt) is matched by **$\Delta t' = -\Delta t * (1+r)$** so that equation 8.24 holds.
 - Equation 8.26 remains unchanged, given r (as y , y' , G , G' and N are the same).
 - And $Y = C + G$; the credit market clears.
 - **No welfare change for consumers.**

- But private and government savings do change due to the **different timings of taxes.**
 - A decrease in current taxes increases private savings and reduces government savings by the same amount ($S^p = Y - C - T$ and $S^g = T - G$).
- **Consumers respond to a tax cut by increasing private savings by the same amount.**
 - Private savings increase to pay for higher future taxes.
 - The consumption bundle remains the same.
 - $\Delta S^p = \Delta B = \Delta T$ so the credit market equilibrium remains.

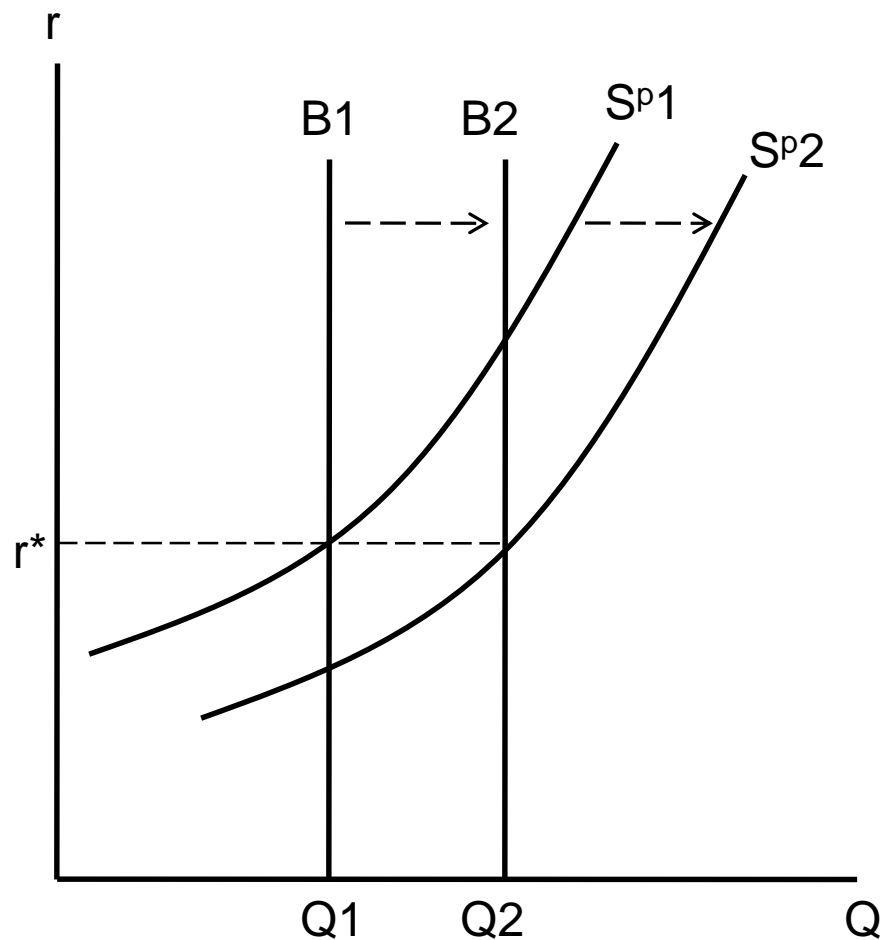
A current tax cut for a borrower

- **A current tax cut equals a future tax increase.**
- Lifetime wealth and consumption bundle (A) are the same.
- Only the endowment point changes from E1 to E2.



Unchanged credit market

- Private savings and government borrowing increase by the same amount.
- Equilibrium r is the same.



A tax cut is not a free lunch!

- A current tax cut gives all consumers higher current disposable income.
- But consumers must bear higher future taxes by the same amount.
- **No welfare gain for consumers!**

Ricardian equivalence assumptions

- A tax change affects every consumer by the same amount so that the present-value tax burden is unchanged.
 - If some consumers receive higher tax cuts, then their lifetime wealth and consumption choices (and the real interest rate) change.
 - Future tax burdens may be shared unequally.
- Government can redistribute wealth among income classes through tax policy!

- Government debt is paid off during the lifetimes of current consumers.
 - But the government can postpone debt payment (and future taxes) to next generations.
 - The old receive tax cuts and higher disposable incomes; the young pay higher future taxes.
- The government can redistribute wealth between generations!
 - The effect of the social security programs.

- The tax is lump-sum.
 - All taxes cause distortions in the relative prices and consumption choices.
 - Welfare loss is greater than tax revenues.
- **Perfect credit markets:** consumers can borrow and lend as much at the same interest rate.
 - But consumers have limits on borrowing.
 - The borrowing rate is higher than the lending rate.
 - Government borrows at a lower rate.
 - *Credit-constrained consumers* benefit from a tax cut.