

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^5 (a + bx_i)$
 $5a + bx_1 + bx_2 + bx_3 + bx_4 + bx_5$

b. $\sum_{y=0}^5 f(x+y)$
 $f(x) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

c. $\sum_{i=1}^{10} i^2$
 $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$
 $= 385$

d. $\sum_{x=1}^2 \sum_{y=2}^3 (2x+y)$
 $= \sum_{x=1}^2 (2x+2 + 2x+3)$
 $= \sum_{x=1}^2 (3(2x+y))$
 $= 2(1)+2 + 2(2)+3$
 $= 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$$

$$8b = 1$$

- b. Find the answer for $P(X \leq 2)$

$$1 - (0.5(0.125) + 0.25(0.125))$$

$$1 - 0.08 = 0.92$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$1 - 0.25(0.125) = 0.96875$$

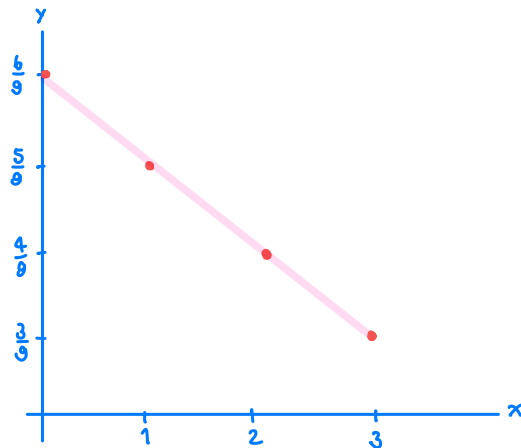
- d. Find the answer for $P(X \geq 1)$

$$1 - P(X \leq 0) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = 0.53125$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



if $x = 0$	if $x = 2$
$-\frac{1}{9}(0) + \frac{6}{9} = y$	$-\frac{1}{9}(2) + \frac{6}{9} = y$
$\frac{6}{9} = y$	$\frac{4}{9} = y$
if $x = 1$	if $x = 3$
$-\frac{1}{9}(1) + \frac{6}{9} = y$	$-\frac{1}{9}(3) + \frac{6}{9} = y$
$\frac{5}{9} = y$	$\frac{3}{9} = y$

- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned}
 P(1 \leq X \leq 3) &= \int_1^3 \left(-\frac{1}{9}x + \frac{6}{9}\right) dx \\
 &= \left[-\frac{1}{18}x^2 + \frac{2}{3}x\right]_1^3 \\
 &= \left(-\frac{1}{18}(3)^2 + \frac{2}{3}(3)\right) - \left(-\frac{1}{18}(1)^2 + \frac{2}{3}(1)\right) \\
 &= \left(-\frac{1}{2} + 2\right) - \left(-\frac{1}{18} + \frac{2}{3}\right) \\
 &= \frac{16}{18}
 \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned}
 P(X \geq 2) &= \int_2^3 \left(-\frac{1}{9}x + \frac{6}{9}\right) dx \\
 &= \left[-\frac{1}{18}x^2 + \frac{2}{3}x\right]_2^3 \\
 &= \left(-\frac{1}{18}(3)^2 + \frac{2}{3}(3)\right) - \left(-\frac{1}{18}(2)^2 + \frac{2}{3}(2)\right) \\
 &= \frac{3}{18} = \frac{1}{6}
 \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned}
 E(X) &= \int_0^3 x f(x) dx \\
 &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9}\right) dx \\
 &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{2}{3}x\right) dx \\
 &= \left(-\frac{1}{27}x^3 + \frac{1}{3}x^2\right) \Big|_0^3 \\
 &= \left(-\frac{1}{27}(3)^3 + \frac{1}{3}(3)^2\right) - 0 \\
 &= 2
 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0. x and y are independent so. $P(x) \cdot P(y) = P(x, y)$

- a. Construct the joint probability distribution function (PDF) table of X and Y

$Y \backslash X$	1	2	3	4	5	6
0	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

- b. Find the marginal probability distribution function (PDF) of X

$$P(X=1) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} \approx 0.167$$

X_i	$P(X=X_i)$
1	0.167
2	0.167
3	0.167
4	0.167
5	0.167
6	0.167

- c. Find the marginal probability distribution function (PDF) of Y

$$P(Y=1) = \frac{1}{12} \cdot 6 = \frac{1}{2}$$

Y_i	$P(Y=Y_i)$
0	$\frac{1}{2}$
1	$\frac{1}{2}$

- d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X=X_i | Y=1) = \frac{P(X=X_i, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

$$P(X=X_i | Y=1) = \frac{P(X=2, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

$$P(X=X_i | Y=1) = \frac{P(X=3, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

$$P(X=X_i | Y=1) = \frac{P(X=4, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

$$P(X=X_i | Y=1) = \frac{P(X=5, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

$$P(X=X_i | Y=1) = \frac{P(X=6, Y=1)}{P(Y=1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

- e. Find the expected value of X given Y is equal to 1

$$E(X | Y=1) = \frac{\sum_i X_i P(X=X_i, Y=1)}{P(Y=1)}$$

$$= \frac{1}{\frac{1}{2}} \sum_i X_i P(X=X_i, Y=1)$$

$$= \frac{1}{\frac{1}{2}} \left[(1) \frac{1}{12} + (2) \frac{1}{12} + (3) \frac{1}{12} + (4) \frac{1}{12} + (5) \frac{1}{12} + (6) \frac{1}{12} \right]$$

$$= 3.5$$

- f. Find the variance of X given Y is equal to 1

$$\text{Var}(X | Y=y_i)$$

$$= \sum_x [x - E(X | Y=y)]^2 f(x | Y=y_i)$$

$$\text{Var}(X | Y=1) = \sum_x [x - E(X | Y=1)]^2 f(x | Y=1)$$

$$= [(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2] \left(\frac{1}{6}\right)$$

$$= 2.91667$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left[\frac{1}{3}(X_1 + X_2 + X_3)\right] \\ &= \frac{1}{3}E(X_1 + X_2 + X_3) \end{aligned}$$

$$E(\bar{X}) = \frac{1}{3}(3\mu)$$

$$E(\bar{X}) = \mu$$

$$\text{var}(\bar{X}) = \text{var}\left[\frac{1}{3}(X_1 + X_2 + X_3)\right]$$

$$= \frac{1}{9}\text{var}(X_1 + X_2 + X_3)$$

$$= \frac{1}{9}[\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) + 2\text{cov}(X_1, X_2) + 2\text{cov}(X_1, X_3) + 2\text{cov}(X_2, X_3)]$$

$$= \frac{1}{9}\left[3\sigma^2 + \frac{3}{4}\sigma^2\right]$$

$$\text{var}(\bar{X}) = \frac{\sigma^2}{2}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right] \\ &= \frac{1}{4}\left[\underbrace{E(X_1)}_{\mu} + \underbrace{E(X_2)}_{\mu} + \underbrace{E(X_3)}_{\mu} + \underbrace{E(X_4)}_{\mu}\right] \\ &= \frac{1}{4}[4\mu] \\ &= \mu \end{aligned}$$

$$\begin{aligned} \text{var}(\bar{X}) &= \text{var}\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right] \\ &= \frac{1}{16}[\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) + \text{var}(X_4)] \\ &= \frac{1}{16}[4\sigma^2] \\ &= \frac{1}{4}\sigma^2 \end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$E(\tilde{X}) = E\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right]$$

$$E(\tilde{X}) = \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$E(\tilde{X}) = \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu$$

$$E(\tilde{X}) = \mu$$

$E(\tilde{X}) = \mu$, so $\tilde{X}$ is an unbiased estimator of μ .

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$$\text{var}(\tilde{X}) = \text{var}\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right]$$

$$\text{var}(\tilde{X}) = \frac{1}{64}\text{var}(X_1) + \frac{1}{16}\text{var}(X_2) + \frac{1}{64}\text{var}(X_3) + \frac{1}{4}\text{var}(X_4)$$

$$\text{var}(\tilde{X}) = \frac{11}{32}\sigma^2$$

$\text{var}(\bar{X}) < \text{var}(\tilde{X})$, so $\bar{X}$ is the better estimator.