ExerciseIII (Solution)

1 $\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}1&2&0&1\\0&1&1&0\\0&0&0&0\end{bmatrix}$; u and v are basic variables (pivot variables); w and y are free variables.

The general solution is $x = \begin{bmatrix}2w-y\\-w\\w\\y\end{bmatrix} = w\begin{bmatrix}2\\-1\\1\\0\end{bmatrix} + y\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$; $r = 2$.

2. $U = \begin{bmatrix}1&2\\0&0\\0&0\\0&0\end{bmatrix}$; u is the basic (pivot) variables
v is free. the general solution to $\underline{A}\underline{x} = 0$ is
$\underline{x} = \begin{bmatrix}2v\\v\end{bmatrix}$; $\underline{A}\underline{x} = \underline{b}$ is consistent if $b_1 = 0$, $b_3 - 4b_2 = 0$
and $b_4 = 0$; $\underline{x} = v\begin{bmatrix}-2\\1\end{bmatrix} + \begin{bmatrix}b_2\\0\end{bmatrix}$; $r = 1$.

3 $\begin{bmatrix}u\\v\\w\end{bmatrix} = \begin{bmatrix}-2v-3\\v\\2\end{bmatrix} = v\begin{bmatrix}-2\\1\\0\end{bmatrix} + \begin{bmatrix}-3\\0\\2\end{bmatrix}.$

4 n

5 4 , pivots = 5, 2, 6 , NO.

6. $a = 4$ leads to a row exchange; $3b + 10a = 40$ leads to a singular matrix; $c = 0$ leads to a row exchange; $c = 3$ leads to a singular matrix.

7 a) $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&v_3/v_2&1&0\\0&v_4/v_2&0&1\end{bmatrix}\begin{bmatrix}0&v_1&0&0\\0&v_2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ b) $\begin{bmatrix}1&-v_1/v_2&0&0\\0&1/v_2&0&0\\0&-v_3/v_2&0&0\\0&-v_4/v_2&0&0\end{bmatrix}$

8. $A_1^{-1} = \begin{bmatrix}1&0&0\\-1&1&-1\\0&0&1\end{bmatrix}$; $A_2^{-1} = \begin{bmatrix}\frac{3}{4}&\frac{1}{2}&\frac{1}{4}\\\frac{1}{2}&1&\frac{1}{2}\\\frac{1}{4}&\frac{1}{2}&\frac{3}{4}\end{bmatrix}$; $A_3^{-1} = \begin{bmatrix}0&-1&1\\-1&1&0\\1&0&0\end{bmatrix}$.

9. Columns 2 and columns 3 are a basis for the column space, with the last column equal to 7 (col 2) − (col 3); rows 1 and 2 are a basis for the row space.

10. (a) The y axis in $\mathbf{R}^3$; (b) The yz plane in $\mathbf{R}^3$; (c) The yz plane in $\mathbf{R}^3$; (d) $\mathbf{R}^3$.

11. (a) $Ax = b$ has no solution, so b is not in the subspace. (b) The w's (with or without w_4) span $\mathbf{R}^3$.

12. a) $m = n = r$; b) $n > m = r$

13. $\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 3 & 6 & 12 \end{bmatrix}$

14. a) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} ; \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

15. a) $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$; $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$; $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

14b) $\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{bmatrix}$

14c) $\begin{bmatrix} -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

21. No.