

## EE431/438 Economics of Financial Markets and Institutions

## Exercise 3: Capital Asset Pricing Model (CAPM)

## Guidance and Solutions

Please submit at the BE office, 5th floor department of Economics building.

Deadline of submission : Friday, October 19th, 2012, before 15.00 hrs.

Late submission will not be accepted.

1. Let the expected rate of return on the market portfolio M be equal to 0.10. The standard deviation of the market portfolio M is equal to 0.20. The risk-free interest rate is equal to 0.02.
  - (a) Find the equation for the CML and interpret its meaning.

*Solutions.*

$$\begin{aligned} E(R_p) &= R_f + \left( \frac{E(R_M) - R_f}{\sigma_m} \right) \sigma_p \\ &= 0.02 + \left( \frac{0.10 - 0.02}{0.20} \right) \sigma_p \\ &= 0.02 + 0.4\sigma_p \end{aligned}$$

CML shows the relationship between market required rate of return and risk (measured by standard deviation) for the efficient portfolios (portfolios consisting of the risk-free asset and the market portfolio). As the level of risk ( $\sigma_p$ ) of the efficient portfolio increases, the market requires a higher rate of return in order to hold the portfolio.

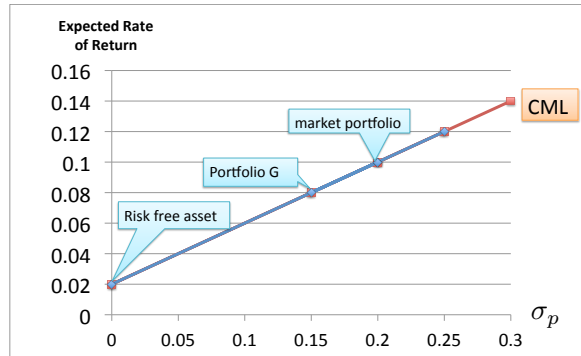
- (b) Let G be a portfolio. The expected rate of return on the portfolio G is equal to 0.08 and the variance of portfolio G is equal to 0.0225. Does the portfolio G lie on CML? Graphically illustrate and explain. If the portfolio G lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio G.

*Solutions.*

On the CML, a portfolio with  $\sigma_p = \sqrt{0.0225} = 0.15$  must have the expected rate of return equal to  $E(R_p) = 0.02 + 0.4 \times 0.15 = 0.08$ .

Portfolio G has 0.08 expected rate of return and 0.0225 variance. Therefore, portfolio G must lie on the CML.

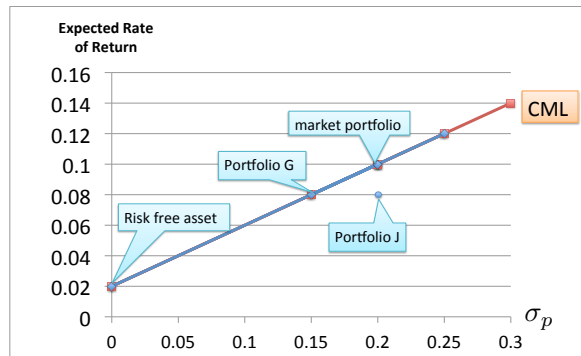
$R_G = aR_f + (1 - a)R_m = 0.08$ .  $0.08 = 0.02a + 0.10 - 0.10a$ .  $0.08a = 0.02$ . Therefore,  $a = 0.25$ . Portfolio G consists of 0.25 of risk-free asset and 0.75 of the market portfolio.



- (c) Let J be a portfolio. The expected rate of return on the portfolio J is equal to 0.08 and the variance of portfolio J is equal to 0.04. Does the portfolio J lie on CML? Graphically illustrate and explain. If the portfolio J lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio J.

*Solutions.*

On the CML, a portfolio with  $\sigma_p = \sqrt{0.04} = 0.2$  must have the expected rate of return equal to  $E(R_p) = 0.02 + 0.4 \times 0.2 = 0.10$ . On the CML, the portfolio with  $\sigma_p = \sigma_m$  must have the expected rate of return equal to the market portfolio. Portfolio J has the same variance as the market portfolio but the expected rate of return is different. Therefore, portfolio J does not lie on the CML.



- (d) According to James Tobin's investment decision process, which portfolio(s) investors might choose to hold? Describe James Tobin's investment decision process. Identify the portfolio(s) investors might choose to hold. Explain the reason.

*Solutions.*

According to James Tobin's investment decision process, each investor will have a utility-maximising portfolio that is a combination of the risk-free asset and a portfolio (or fund) of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investor's efficient set of risky assets. If investors have homogenous beliefs, they all have the same linear efficient set called the Capital Market Line (CML). Therefore, they will try to hold some combination of the risk-free asset,  $R_f$  and the tangency portfolio M. At equilibrium, all assets must be held. All prices will be adjusted so that the demand is equal

to supply. The tangency portfolio must be the market portfolio, M. [If  $V_i$  is the market value of the  $i$ th asset, then the percentage of wealth in each asset is equal to the ratio of the market value of the asset to the market value of all assets.  $w_i = \frac{V_i}{\sum_{i=1}^N V_i}$ .]

Investors will hold portfolios along the CML. Therefore, investors might hold portfolio M and G. No investor will choose to hold portfolio J.

- (e) According to James Tobin's investment decision process and the CML in (a), what are the market required rates of return on the portfolio M, G and J? Is the given information sufficient to determine the market required rate of returns of the three portfolios?

*Solutions.*

The given information is sufficient to determine the market required rate of return for portfolio M and G. Portfolio M and G lie on the CML. They are efficient. Their market required rate of returns are determined by the relationship between the market required rate of return and standard deviation along the CML. Therefore, the market required rate of return on portfolio M and G are equal to 0.10 and 0.08, respectively. The given information is insufficient to determine the market required rate of return for portfolio J. This is because portfolio J does not lie on the CML. Portfolio J is inefficient. Eventhough we know the variance and the rate of return on portfolio J, we cannot be sure at what rate of return that the market will require from asset J because it is not on the CML. Therefore, we cannot use our knowledge of the mean and the variance of asset J to determine the rate of return that the market will require from asset J in order to hold it in equilibrium. The variance may not be the correct measure of riskiness for an individual asset. Only the portion of total variance that is correlated with the economy(covariance risk) is relevant. Any portion of total risk that is not correlated with the economy is irrelevant and can be avoided at zero cost through diversification. We need the information on how the rate of return on asset J is correlated with the economy(covariance risk, systematic risk) to determine the market required rate of return on asset J.

2. The following information is provided for a stock market.

	$ER_i$	$\sigma_i$	$r_{im}$
Asset 1	0.07	0.30	0.2
Asset 2	0.08	0.20	0.4
Asset 3	0.10	0.15	0.8
Market Portfolio (M)	0.09	0.10	1

$ER_i$  is the expected rate of return on asset  $i$ .  $\sigma_i$  is the standard deviation of the rate of return on asset  $i$ .  $r_{im}$  is **the correlation coefficient** between the rate of return on asset  $i$  and the rate of return on the market portfolio M. At equilibrium, the expected rate of return on the market portfolio is equal to 0.09 and the standard deviation of the market portfolio is equal to 0.10. The risk free interest rate is 4%. (Hint:  $\sigma_{xy} = r_{xy} \times \sigma_x \times \sigma_y$ ,  $r_{xy} = \frac{COV(X,Y)}{\sigma_x \sigma_y}$ )

- (a) In the context of CAPM, find the beta-coefficient for each asset.

*Solutions.*

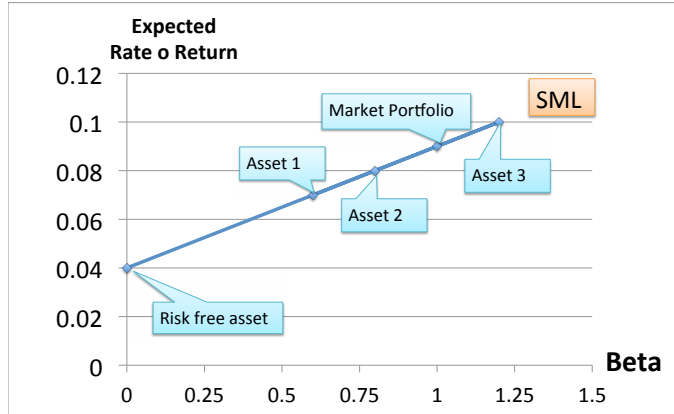
	$\sigma_i$	$r_{im}$	$\beta_i = \frac{r_{im} \times \sigma_i \times \sigma_m}{\sigma_m^2}$
Asset 1	0.30	0.2	0.6
Asset 2	0.20	0.4	0.8
Asset 3	0.15	0.8	1.2
Market Portfolio (M)	0.10	1	1.0

- (b) Construct SML from the given information and interpret its meaning. Graphically illustrate SML and show the points where each asset lie in the graph.

*Solutions.*

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f).$$

$$E(R_i) = 0.04 + 0.05\beta_i.$$



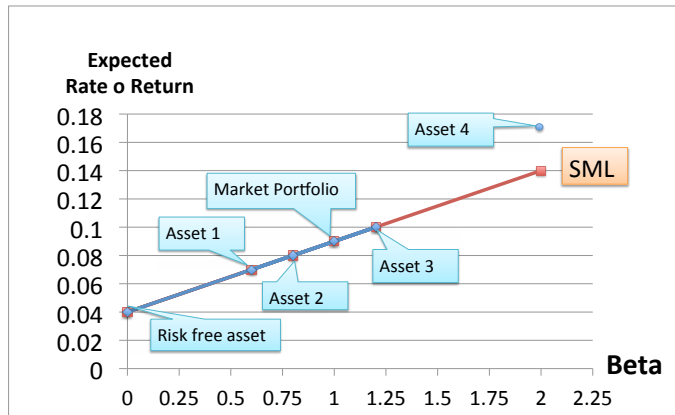
- (c) In the context of CAPM, determine whether each asset is overpriced, underpriced, or correctly priced. Explain the price adjustment process that might happen (if any).

*Solutions.* All assets are correctly priced. They lie on the SML.

- (d) You are informed that the fourth asset, with the beta-coefficient is equal to 2 is available. Empirical evidence reveals that its expected rate of return is 17%. Determine whether this asset is overpriced, underpriced, or correctly priced. In the context of CAPM, explain the price adjustment process that might happen (if any).

*Solutions.*

Asset 4 lies above the SML. Asset 4 is underpriced. Investors will buy asset 4. The price of asset 4 will go up and the rate of return on asset 4 will go down. At equilibrium asset 4 must lie on the SML.



3. A portfolio K consists of 0.10 of asset 1 and 0.90 of asset 2. What is the beta-coefficient for the portfolio K? In the context of CAPM, what is the equilibrium market required rate of return on the portfolio K?

*Solutions.*

$$\beta_K = 0.1\beta_1 + 0.90\beta_2 = 0.1(0.6) + 0.90(0.80) = 0.78.$$

$$E(R_K) = R_f + \beta_K(E(R_m) - R_f) = 0.04 + 0.78(0.05) = 0.079 = 7.9\%$$