Exercise 2 (Part 3)

1. Determine the truth value of each of these statements. Explain your answer.

   (a) $\forall n \in \mathbb{Z},\quad n^3 > n$ (b) $\exists n \in \mathbb{Z},\quad n^2 + n = 1$ (c) $\exists n \in \mathbb{R},\quad n^2 + n = 1$
   (d) $\forall n \in \mathbb{R}^+,\quad 2n > \frac{1}{n}$ (e) $\forall n \in \mathbb{R}, (n > 0) \to (3n > n)$

2. Let $\mathbb{R}$ be the domain of x. Determine the **truth set** for each of these statements.

   (a) $P(x)$: "$x < \frac{1}{x}$" (b) $P(x)$: "$2x + 1 < 0$ or $x \geq 1$"

3. Let the domain for variables x and y be the set of real numbers $\mathbb{R}$. Determine the truth values of the following statements. Explain you answer.
   (a) $\exists y \forall x,\quad xy = x$ (b) $\forall x \exists y,\quad xy = x$ (c) $\forall y \exists x,\quad y = x$ (d) $\exists x \forall y,\quad y = x$

4. Let $Q(x, y, z)$ be the statement "$xy = z$." If the domain for variables x, y, z is the set of all integers, determine the truth values of the following statements. Explain you answer.
   (a) $Q(1, 2, 2)$ (b) $Q(2, 0, 2)$ (c) $\exists y, Q(2, y, 1)$
   (d) $\forall x \forall y \exists z, Q(x, y, z)$ (e) $\exists z \forall x \forall y, Q(x, y, z)$

5. Let $\mathbb{Z}^+$ be the domain of x. Let $P(x)$ and $Q(x)$ be the predicates "x is not divisible by 3," and "x is divisible by 12," respectively. Determine whether the following statements are true or false. Give a counterexample for each false statement.
   (a) $Q(x) \Rightarrow P(x)$ (b) $P(x) \Rightarrow \sim Q(x)$

6. Write a negation for each statement without using *the negation symbol* "$\sim$."

   (a) $\exists z \forall x \forall y,\ xy = z$

   (b) $\forall x \forall y,\ (x < 0) \wedge (y \geq 0) \to (xy \leq 0)$

7. Show that each of the following arguments is valid by **universal modus ponens** , **universal modus tollens** and/or **universal transitivity**, or show that it is invalid from the **converse error** or the **inverse error**. In addition, use also the **diagram** to confirm that each argument is valid or invalid.

   (a)

   "Anyone who has a school email account has a school ID number."
   "Kevin has a school ID number."
   ∴ "Kevin has a school email account. "

   (b)

   "Anyone who has a school email account has a school ID number."
   "All students have school email accounts."
   "Kim does not have a school ID number."
   ∴ "Kim is not a student."