GMM using Moment Equations

In the study of interest rate structure, the continuous time model can be specified as:

$$r_{t+\Delta t} - r_t = (\alpha + \beta r_t)\Delta t + \varepsilon_{t+\Delta t} \quad (1)$$

where: $E[\varepsilon_{t+\Delta t}] = 0$ and $E[\varepsilon^2_{t+\Delta t}] = \sigma^2 r_t^{2\gamma} \Delta t$

Then, the model can be transformed to be discrete time model by setting $\Delta t = 1$. The discrete time model can be stated as:

$$r_{t+1} - r_t = \Delta r_t = \alpha + \beta r_t + \varepsilon_{t+1} \quad (2)$$

where: $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon^2_{t+1}] = \sigma^2 r_t^{2\gamma}$

The above model consists of four parameters including $\alpha, \beta, \sigma^2, \gamma$. The model can be estimate using GMM. Four moment condition equations can be stated as:

Zero mean condition:	$E(\varepsilon_{t+1}) = 0$
Orthogonality condition:	$E(\varepsilon_{t+1} r_t) = 0$
Variance condition:	$E(\varepsilon^2_{t+1} - \sigma^2 r_t^{2\gamma}) = 0$
Zero covariance condition:	$E\left(\left(\varepsilon^2_{t+1} - \sigma^2 r_t^{2\gamma}\right) r_t\right) = 0$

The above model can be claimed as unrestricted model for other eight interest rate structure models which can be indicated as follows:

Model	α	β	σ^2	γ
Unrestricted				
Merton		0		0
Vasicek				0
CIR SR				0.5
Dothan	0	0		1
GBM	0			1
Brennan & Schwartz				1
CIR VR	0	0		1.5
CEV	0			

Unrestricted model has four unknown parameters to be estimated and four moment condition equations, thus, method of estimate will be method of moment since it is exact identify equations model. Other eight restricted models will be estimated using GMM.

GMM can be performed by using the following command:

Unrestricted model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r^(2*{gamma})) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r)
winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
warning: 1 missing value returned for equation 4 at initial values

Step 1
numerical derivatives are approximate
flat or discontinuous region encountered
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  .00005192  (backed up)
Iteration 2:   GMM criterion Q(b) =  .00004097
Iteration 3:   GMM criterion Q(b) =  3.562e-06
Iteration 4:   GMM criterion Q(b) =  2.174e-07
Iteration 5:   GMM criterion Q(b) =  1.644e-07

Step 2
Iteration 0:   GMM criterion Q(b) =  .01229538
Iteration 1:   GMM criterion Q(b) =   .0055188
Iteration 2:   GMM criterion Q(b) =  .00045234
Iteration 3:   GMM criterion Q(b) =  8.141e-07
Iteration 4:   GMM criterion Q(b) =  2.461e-12
Iteration 5:   GMM criterion Q(b) =  1.451e-23

GMM estimation

Number of parameters =   4
Number of moments    =   4
Initial weight matrix: Identity                      Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0009679   .0012837    -0.75   0.451    -.0034839    .0015481
       /beta |   .0014035   .0005181     2.71   0.007     .0003882    .0024189
     /sigma2 |   .0006419   .0001156     5.55   0.000     .0004153    .0008686
      /gamma |   .1273995   .0720773     1.77   0.077    -.0138695    .2686685
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(0) = 2.1e-20 (p =     .)

. est store Unrestricted
```

Merton model:

```
. gmm (dr-{alpha}) ((dr-{alpha})*r) ((dr-{alpha})^2-{sigma2}) (((dr-{alpha})^2-
{sigma2})*r) winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
...
warning: 1 missing value returned for equation 4 at initial values
```

```
Step 1
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  6.386e-07
Iteration 2:   GMM criterion Q(b) =  6.382e-07

Step 2
Iteration 0:   GMM criterion Q(b) =  .00760718
Iteration 1:   GMM criterion Q(b) =  .00652536
Iteration 2:   GMM criterion Q(b) =  .00652532

GMM estimation

Number of parameters =   2
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |   .0021651   .0007043     3.07   0.002     .0007847    .0035456
     /sigma2 |   .0008218   .0000853     9.63   0.000     .0006546     .000989
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 9.32468 (p = 0.0094)

. est store Merton
```

<u>Vasicek model</u>:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}) (((dr-{alpha}-{beta}*r)^2-{sigma2})*r) winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
...
warning: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  3.000e-09
Iteration 2:   GMM criterion Q(b) =  2.107e-09

Step 2
Iteration 0:   GMM criterion Q(b) =  .00263124
Iteration 1:   GMM criterion Q(b) =  .00226079
Iteration 2:   GMM criterion Q(b) =  .00226074

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0003515   .0012376    -0.28   0.776     -.002777    .0020741
       /beta |   .0012479   .0005106     2.44   0.015     .0002471    .0022487
     /sigma2 |   .0007951   .0000862     9.23   0.000     .0006262    .0009641
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons
```

```
. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 3.23059 (p = 0.0723)

. est store Vasicek
```

CIR SR model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r)*r) winitial(identity) nolog

warning: 1 missing value returned for equation 1 at initial values
...
warning: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) =  .0117512

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0031028   .0011644    -2.66   0.008     -.005385   -.0008207
       /beta |   .0018442   .0005066     3.64   0.000     .0008514    .0028371
     /sigma2 |   .0002351   .0000278     8.45   0.000     .0001805    .0002896
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 16.7924 (p = 0.0000)

. est store CIR_SR

. est table Unrestricted Merton Vasicek CIR_SR, star(0.1 0.05 0.01)

------------------------------------------------------------------------------
    Variable | Unrestricted      Merton          Vasicek         CIR_SR
-------------+----------------------------------------------------------------
alpha        |
       _cons | -.00096793      .00216513***   -.00035145      -.00310284***
-------------+----------------------------------------------------------------
beta         |
       _cons |  .00140354***                   .00124786**     .00184424***
-------------+----------------------------------------------------------------
sigma2       |
       _cons |  .00064191***   .00082182***    .00079513***    .00023507***
-------------+----------------------------------------------------------------
gamma        |
       _cons |  .12739949*
------------------------------------------------------------------------------
                                          legend: * p<.1; ** p<.05; *** p<.01

. *Test Unrestricted vs Merton

. test (_b[/beta]=0) (_b[/gamma]=0)

 ( 1)  [beta]_cons = 0
 ( 2)  [gamma]_cons = 0

           chi2(  2) =    8.96
         Prob > chi2 =    0.0113
```

GMM Examples Simulated Data

Example 1

```
set obs 500
capture drop y x z u
mat mCorr = (1, .7, .5\ .7, 1, 0 \ .5, 0, 1)
corr2data x z u, mean(10 10 0) sd(3 4 20) corr(mCorr) n(500) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z)
```

```
. set obs 500
number of observations (_N) was 500, now 500

. capture drop y x z u

. mat mCorr = (1, .7, .5\ .7, 1, 0 \ .5, 0, 1)

. corr2data x z u, mean(10 10 0) sd(3 4 20) corr(mCorr) n(500) clear
(obs 500)

. corr
(obs=500)

             |        x        z        u
-------------+---------------------------
           x |   1.0000
           z |   0.7000   1.0000
           u |   0.5000   0.0000   1.0000

. g y=1+2*x+u

. reg y x

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =    424.96
       Model |  127744.002         1  127744.002   Prob > F        =    0.0000
    Residual |  149700.001       498  300.602411   R-squared       =    0.4604
-------------+----------------------------------   Adj R-squared   =    0.4593
       Total |  277444.003       499  556.000006   Root MSE        =    17.338

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   5.333333   .2587168    20.61   0.000     4.825022    5.841644
       _cons |  -32.33333    2.70086   -11.97   0.000    -37.63982   -27.02685
------------------------------------------------------------------------------

. ivregress gmm y (x=z)

Instrumental variables (GMM) regression          Number of obs   =        500
                                                 Wald chi2(1)    =      19.64
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.2806
GMM weight matrix: Robust                        Root MSE        =      19.98

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |          2   .4513481     4.43   0.000     1.115374    2.884626
       _cons |   .9999991    4.62114     0.22   0.829    -8.057269    10.05727
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z

. ivregress 2sls y (x=z)

Instrumental variables (2SLS) regression         Number of obs   =        500
                                                 Wald chi2(1)    =      22.05
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.2806
                                                 Root MSE        =      19.98

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |          2   .4259177     4.70   0.000     1.165217    2.834783
       _cons |   .9999991   4.351895     0.23   0.818    -7.529558    9.529556
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z
```

Example 2

```
set obs 500
capture drop y x z* u
mat mCorr = (1, .7, .7, .5\ .7, 1, .4, 0 \ .7, .4, 1, 0\ .5, 0, 0, 1)
corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(500) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z1)
ivregress gmm y (x=z1 z2)
ivregress 2sls y (x=z1 z2)
```

```
. set obs 500
number of observations (_N) was 0, now 500

. capture drop y x z* u

. mat mCorr = (1, .7, .7, .5\ .7, 1, .4, 0 \ .7, .4, 1, 0\ .5, 0, 0, 1)

. corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(500) clear
(obs 500)

. corr
(obs=500)

             |        x       z1       z2        u
-------------+------------------------------------
           x |   1.0000
          z1 |   0.7000   1.0000
          z2 |   0.7000   0.4000   1.0000
           u |   0.5000  -0.0000  -0.0000   1.0000


. g y=1+2*x+u

. reg y x

      Source |       SS           df       MS      Number of obs   =       500
-------------+----------------------------------   F(1, 498)       =    424.96
       Model |  127743.999         1  127743.999   Prob > F        =    0.0000
    Residual |      149700       498  300.602411   R-squared       =    0.4604
-------------+----------------------------------   Adj R-squared   =    0.4593
       Total |  277443.999       499  555.999998   Root MSE        =    17.338

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   5.333333   .2587168    20.61   0.000     4.825022    5.841644
       _cons |  -32.33333    2.70086   -11.97   0.000    -37.63982   -27.02685
------------------------------------------------------------------------------

. ivregress gmm y (x=z1)

Instrumental variables (GMM) regression          Number of obs   =       500
                                                 Wald chi2(1)    =     24.63
                                                 Prob > chi2     =    0.0000
                                                 R-squared       =    0.2806
GMM weight matrix: Robust                        Root MSE        =     19.98

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |          2   .4030215     4.96   0.000     1.210092    2.789908
       _cons |   .9999999   4.004532     0.25   0.803    -6.848738    8.848738
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1
```

```
. ivregress gmm y (x=z1 z2)

Instrumental variables (GMM) regression          Number of obs   =        500
                                                 Wald chi2(1)    =      35.10
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.2806
GMM weight matrix: Robust                        Root MSE        =      19.98

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |          2   .3375669     5.92   0.000     1.338381    2.661619
       _cons |          1   3.373732     0.30   0.767    -5.612393    7.612393
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z2

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 1.1e-14 (p = 1.0000)

. ivregress 2sls y (x=z1 z2)

Instrumental variables (2SLS) regression         Number of obs   =        500
                                                 Wald chi2(1)    =      31.50
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.2806
                                                 Root MSE        =      19.98

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |          2   .3563483     5.61   0.000      1.30157     2.69843
       _cons |          1   3.673801     0.27   0.785    -6.200517    8.200518
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z2
```

Example 3

```
set obs 500
capture drop y x z* u
mat mCorr = (1, .7, .3, .5\ .7, 1, .4, .2 \ .3, .4, 1, .3\ .5, .2, .3, 1)
corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(5000) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z1)
ivregress gmm y (x=z1 z2)
estat overid
ivregress 2sls y (x=z1 z2)
```

```
. set obs 500
number of observations (_N) was 0, now 500

. capture drop y x z* u

. mat mCorr = (1, .7, .3, .5\ .7, 1, .4, .2 \ .3, .4, 1, .3\ .5, .2, .3, 1)

. corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(5000) clear
(obs 5,000)

. corr
(obs=5,000)

             |        x       z1       z2        u
-------------+------------------------------------
           x |   1.0000
          z1 |   0.7000   1.0000
          z2 |   0.3000   0.4000   1.0000
           u |   0.5000   0.2000   0.3000   1.0000
```

```
. g y=1+2*x+u

. reg y x

      Source |       SS           df       MS      Number of obs   =     5,000
-------------+----------------------------------   F(1, 4998)      =   4264.96
       Model |  1279743.99         1  1279743.99   Prob > F        =    0.0000
    Residual |     1499700     4,998  300.060025   R-squared       =    0.4604
-------------+----------------------------------   Adj R-squared   =    0.4603
       Total |     2779444     4,999         556   Root MSE        =    17.322

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   5.333333    .081666    65.31   0.000     5.173232    5.493435
       _cons |  -32.33333    .852611   -37.92   0.000    -34.00483   -30.66184
------------------------------------------------------------------------------

. ivregress gmm y (x=z1)

Instrumental variables (GMM) regression          Number of obs   =      5,000
                                                 Wald chi2(1)    =    1044.34
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.4274
GMM weight matrix: Robust                        Root MSE        =     17.841

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   3.904762   .1208297    32.32   0.000      3.66794    4.141584
       _cons |  -18.04762   1.229494   -14.68   0.000    -20.45738   -15.63785
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1

. ivregress gmm y (x=z1 z2)

Instrumental variables (GMM) regression          Number of obs   =      5,000
                                                 Wald chi2(1)    =    1103.18
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.4315
GMM weight matrix: Robust                        Root MSE        =     17.776

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   3.997248   .1203476    33.21   0.000     3.761371    4.233125
       _cons |  -19.00297   1.224456   -15.52   0.000    -21.40286   -16.60308
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z2

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 293.132 (p = 0.0000)

. ivregress 2sls y (x=z1 z2)

Instrumental variables (2SLS) regression         Number of obs   =      5,000
                                                 Wald chi2(1)    =    1100.94
                                                 Prob > chi2     =     0.0000
                                                 R-squared       =     0.4305
                                                 Root MSE        =     17.792

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |    3.97411    .119773    33.18   0.000     3.739359    4.208861
       _cons |   -18.7411   1.223875   -15.31   0.000    -21.13985   -16.34235
------------------------------------------------------------------------------
Instrumented:  x
Instruments:   z1 z2
```

Example

Griliches (1976)'s wage model can be stated as:

$$LW = \alpha + \beta S + \gamma IQ + \delta' \mathrm{h} + \varepsilon \quad (2)$$

and $$\mathrm{h} \equiv (EXPR, TENURE, RNS, SMSA, YEAR\ DUMMIES)$$

where: LW = Log wages

S = Completed years of schooling

IQ = IQ Score

$EXPR$ = Experience in years

$TENURE$ = Tenure in years

RNS = Dummy for residency in the southern states

$SMSA$ = Dummy for residency in the metropolitan areas

$YEAR\ DUMMIES$ = Dummy for years

IQ score and years of completed schooling might be correlated with error term, thus, instrumental variables can be employed. Instrumental variables to be used include:

MED = Mother's education in years

KWW = Score on the "Knowledge of the World of Work" test

MRT = Dummy for marital status (1 if married)

AGE = Age of the individual

Create Dummy Variable

```
. xi i.year
i.year            _Iyear_66-73        (naturally coded; _Iyear_66 omitted)
```

OLS

```
. reg lw s iq expr tenure rns smsa _I*

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F( 12,   745) =   46.86
       Model |  59.9127611    12  4.99273009           Prob > F      =  0.0000
    Residual |  79.3733888   745  .106541461           R-squared     =  0.4301
-------------+------------------------------           Adj R-squared =  0.4210
       Total |   139.28615   757  .183997556           Root MSE      =  .32641

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .0619548   .0072786     8.51   0.000     .0476658    .0762438
          iq |   .0027121   .0010314     2.63   0.009     .0006873    .0047369
        expr |   .0308395   .0065101     4.74   0.000     .0180592    .0436198
      tenure |   .0421631   .0074812     5.64   0.000     .0274763    .0568498
         rns |  -.0962935   .0275467    -3.50   0.001    -.1503719   -.0422151
        smsa |   .1328993   .0265758     5.00   0.000     .0807268    .1850717
   _Iyear_67 |  -.0542095   .0478522    -1.13   0.258    -.1481506    .0397317
   _Iyear_68 |   .0805808   .0448951     1.79   0.073    -.0075551    .1687168
   _Iyear_69 |   .2075915   .0438605     4.73   0.000     .1214867    .2936963
   _Iyear_70 |   .2282237   .0487994     4.68   0.000      .132423    .3240245
   _Iyear_71 |   .2226915   .0430952     5.17   0.000     .1380889     .307294
   _Iyear_73 |   .3228747   .0406574     7.94   0.000     .2430579    .4026915
       _cons |   4.235357   .1133489    37.37   0.000     4.012836    4.457878
------------------------------------------------------------------------------
```

2SLS with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress 2sls lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental variables (2SLS) regression               Number of obs =     758
                                                       Wald chi2(12) =  459.55
                                                       Prob > chi2   =  0.0000
                                                       R-squared     =  0.2280
                                                       Root MSE      =  .37665

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1724253   .0207381     8.31   0.000     .1317794    .2130712
          iq |  -.0090988   .0047044    -1.93   0.053    -.0183193    .0001216
        expr |   .0492895   .0081546     6.04   0.000     .0333068    .0652722
      tenure |   .0422171   .0088429     4.77   0.000     .0248854    .0595488
         rns |  -.1017935   .0341765    -2.98   0.003    -.1687781   -.0348088
        smsa |   .1261109   .0309275     4.08   0.000     .0654942    .1867277
   _Iyear_67 |  -.0596171   .0552955    -1.08   0.281    -.1679942      .04876
   _Iyear_68 |   .0486796   .0520161     0.94   0.349    -.0532701    .1506292
   _Iyear_69 |   .1528176    .051563     2.96   0.003      .051756    .2538792
   _Iyear_70 |   .1744361   .0597576     2.92   0.004     .0573133    .2915588
   _Iyear_71 |    .091666    .054144     1.69   0.090    -.0144543    .1977863
   _Iyear_73 |   .0932398   .0571819     1.63   0.103    -.0188347    .2053142
       _cons |    4.03351   .3154215    12.79   0.000     3.415295    4.651725
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age

. estat overid

  Tests of overidentifying restrictions:

  Sargan (score) chi2(2) =  13.2683  (p = 0.0013)
  Basmann chi2(2)        =  13.2375  (p = 0.0013)
```

GMM with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress gmm lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental variables (GMM) regression                Number of obs =     758
                                                       Wald chi2(12) =  506.92
                                                       Prob > chi2   =  0.0000
                                                       R-squared     =  0.2168
GMM weight matrix: Robust                              Root MSE      =  .37936

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1757958   .0208514     8.43   0.000     .1349279    .2166637
          iq |  -.0092862   .0049182    -1.89   0.059    -.0189256    .0003533
        expr |   .0502828    .008104     6.20   0.000     .0343992    .0661664
      tenure |   .0425214   .0095601     4.45   0.000     .0237838    .0612589
         rns |  -.1040931     .03373    -3.09   0.002    -.1702026   -.0379835
        smsa |   .1247512   .0309873     4.03   0.000     .0640172    .1854853
   _Iyear_67 |  -.0530432   .0517876    -1.02   0.306    -.1545449    .0484586
   _Iyear_68 |   .0459546   .0500069     0.92   0.358    -.0520572    .1439664
   _Iyear_69 |   .1554801   .0480126     3.24   0.001     .0613772     .249583
   _Iyear_70 |   .1669875   .0613341     2.72   0.006     .0467748    .2872001
   _Iyear_71 |   .0846485    .055817     1.52   0.129    -.0247508    .1940477
   _Iyear_73 |   .0996068   .0612394     1.63   0.104    -.0204201    .2196338
       _cons |   4.003924   .3364754    11.90   0.000     3.344445    4.663404
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 11.6015 (p = 0.0030)
```

Alternative Command ivreg2 and ivgmm0

```
. ivreg2 lw expr tenure rns smsa _I* (s iq=med kww age mrt)

IV (2SLS) estimation
--------------------

                                                      Number of obs =      758
                                                      F( 12,   745) =    37.64
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.2280
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9956
Residual SS             =  107.5313411                Root MSE      =    .3766

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1724253   .0207381     8.31   0.000     .1317794    .2130712
          iq |  -.0090988   .0047044    -1.93   0.053    -.0183193    .0001216
        expr |   .0492895   .0081546     6.04   0.000     .0333068    .0652722
      tenure |   .0422171   .0088429     4.77   0.000     .0248854    .0595488
         rns |  -.1017935   .0341765    -2.98   0.003    -.1687781   -.0348088
        smsa |   .1261109   .0309275     4.08   0.000     .0654942    .1867277
   _Iyear_67 |  -.0596171   .0552955    -1.08   0.281    -.1679942      .04876
   _Iyear_68 |   .0486796   .0520161     0.94   0.349    -.0532701    .1506292
   _Iyear_69 |   .1528176    .051563     2.96   0.003      .051756    .2538792
   _Iyear_70 |   .1744361   .0597576     2.92   0.004     .0573133    .2915588
   _Iyear_71 |    .091666    .054144     1.69   0.090    -.0144543    .1977863
   _Iyear_73 |   .0932398   .0571819     1.63   0.103    -.0188347    .2053142
       _cons |    4.03351   .3154215    12.79   0.000     3.415295    4.651725
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          49.564
                                                   Chi-sq(3) P-val =    0.0000
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    12.552
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    11.04
                                         10% maximal IV relative bias     7.56
                                         20% maximal IV relative bias     5.57
                                         30% maximal IV relative bias     4.73
                                         10% maximal IV size             16.87
                                         15% maximal IV size              9.93
                                         20% maximal IV size              7.54
                                         25% maximal IV size              6.28
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Sargan statistic (overidentification test of all instruments):          13.268
                                                   Chi-sq(2) P-val =    0.0013
------------------------------------------------------------------------------
Instrumented:         s iq
Included instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69
                      _Iyear_70 _Iyear_71 _Iyear_73
Excluded instruments: med kww age mrt
------------------------------------------------------------------------------

. ivreg2 lw expr tenure rns smsa _I* (s iq=med kww age mrt), gmm

2-Step GMM estimation
---------------------

Statistics robust to heteroskedasticity

                                                      Number of obs =      758
                                                      F( 12,   745) =    41.98
                                                      Prob > F      =   0.0000
Total (centered) SS     =  139.2861498                Centered R2   =   0.2168
Total (uncentered) SS   =  24652.24662                Uncentered R2 =   0.9956
Residual SS             =  109.0846511                Root MSE      =    .3794

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1757958   .0206766     8.50   0.000     .1352703    .2163212
          iq |  -.0092862   .0048824    -1.90   0.057    -.0188555    .0002832
        expr |   .0502828   .0080438     6.25   0.000     .0345171    .0660484
      tenure |   .0425214   .0094549     4.50   0.000     .0239901    .0610526
         rns |  -.1040931   .0335239    -3.11   0.002    -.1697986   -.0383875
```

```
        smsa |   .1247512   .0307747     4.05   0.000     .0644338    .1850686
    _Iyear_67 |  -.0530432   .0514609    -1.03   0.303    -.1539047    .0478184
    _Iyear_68 |   .0459546   .0495735     0.93   0.354    -.0512077    .1431169
    _Iyear_69 |   .1554801   .0476311     3.26   0.001     .0621249    .2488352
    _Iyear_70 |   .1669875   .0610006     2.74   0.006     .0474285    .2865464
    _Iyear_71 |   .0846485   .0554035     1.53   0.127    -.0239404    .1932373
    _Iyear_73 |   .0996068   .0607034     1.64   0.101    -.0193696    .2185833
        _cons |   4.003924   .3348423    11.96   0.000     3.347645    4.660203
------------------------------------------------------------------------------
Anderson canon. corr. LR statistic (underidentification test):          49.564
                                                   Chi-sq(3) P-val =    0.0000
Test statistic(s) not robust
------------------------------------------------------------------------------
Cragg-Donald F statistic (weak identification test):                    12.552
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    11.04
                                         10% maximal IV relative bias     7.56
                                         20% maximal IV relative bias     5.57
                                         30% maximal IV relative bias     4.73
                                         10% maximal IV size             16.87
                                         15% maximal IV size              9.93
                                         20% maximal IV size              7.54
                                         25% maximal IV size              6.28
Test statistic(s) not robust
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Hansen J statistic (overidentification test of all instruments):        11.601
                                                   Chi-sq(2) P-val =    0.0030
------------------------------------------------------------------------------
Instrumented:         s iq
Included instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69
                      _Iyear_70 _Iyear_71 _Iyear_73
Excluded instruments: med kww age mrt
------------------------------------------------------------------------------

. ivgmm0 lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental Variables Estimation via GMM          Number of obs  =       758
                                                    Root MSE       =    0.3794
                                                    Hansen J       =   11.6015
                                                    Chi-sq( 2) P-val = 0.00303
------------------------------------------------------------------------------
             |               GMM
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1757958   .0206766     8.50   0.000     .1352703    .2163212
          iq |  -.0092862   .0048824    -1.90   0.057    -.0188555    .0002832
        expr |   .0502828   .0080438     6.25   0.000     .0345171    .0660484
      tenure |   .0425214   .0094549     4.50   0.000     .0239901    .0610526
         rns |  -.1040931   .0335239    -3.11   0.002    -.1697986   -.0383875
        smsa |   .1247512   .0307747     4.05   0.000     .0644338    .1850686
    _Iyear_67 |  -.0530432   .0514609    -1.03   0.303    -.1539047    .0478184
    _Iyear_68 |   .0459546   .0495735     0.93   0.354    -.0512077    .1431169
    _Iyear_69 |   .1554801   .0476311     3.26   0.001     .0621249    .2488352
    _Iyear_70 |   .1669875   .0610006     2.74   0.006     .0474285    .2865464
    _Iyear_71 |   .0846485   .0554035     1.53   0.127    -.0239404    .1932373
    _Iyear_73 |   .0996068   .0607034     1.64   0.101    -.0193696    .2185833
        _cons |   4.003924   .3348423    11.96   0.000     3.347645    4.660203
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age
------------------------------------------------------------------------------

. ivgmm0 lw expr tenure rns smsa _I* (s iq = med kww mrt age), gres

Instrumental Variables Estimation via GMM          Number of obs  =       758
                                                    Root MSE       =    0.3794
                                                    Hansen J       =   11.6015
                                                    Chi-sq( 2) P-val = 0.00303
------------------------------------------------------------------------------
             |               GMM
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1757958   .0208513     8.43   0.000     .1349279    .2166637
          iq |  -.0092862   .0049182    -1.89   0.059    -.0189256    .0003533
        expr |   .0502828    .008104     6.20   0.000     .0343992    .0661663
      tenure |   .0425214   .0095601     4.45   0.000     .0237839    .0612589
```

```
         rns |  -.1040931   .0337299    -3.09   0.002    -.1702025   -.0379836
        smsa |   .1247512   .0309873     4.03   0.000     .0640172    .1854852
   _Iyear_67 |  -.0530432   .0517876    -1.02   0.306    -.1545449    .0484586
   _Iyear_68 |   .0459546   .0500068     0.92   0.358    -.0520569    .1439661
   _Iyear_69 |   .1554801   .0480124     3.24   0.001     .0613775    .2495827
   _Iyear_70 |   .1669875   .0613341     2.72   0.006     .0467749       .2872
   _Iyear_71 |   .0846485   .0558168     1.52   0.129    -.0247505    .1940474
   _Iyear_73 |   .0996068   .0612394     1.63   0.104    -.0204201    .2196338
       _cons |   4.003924   .3364752    11.90   0.000     3.344445    4.663404
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age
------------------------------------------------------------------------------
```