Oct 8, 2020

→ Theory of Consumer Behavior

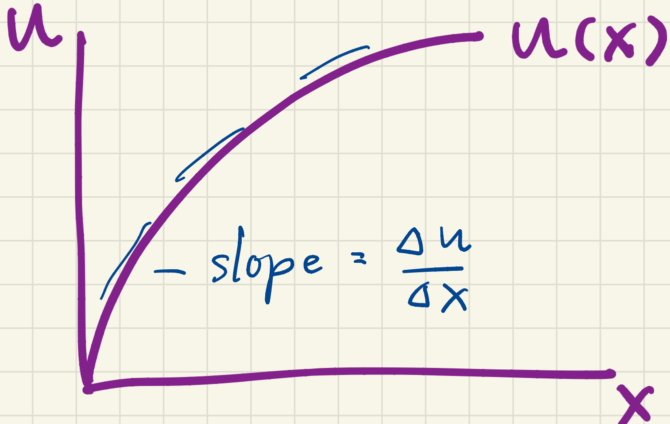
↳ Understand how the DEMAND curve is derived.

→ Key concept : UTILITY

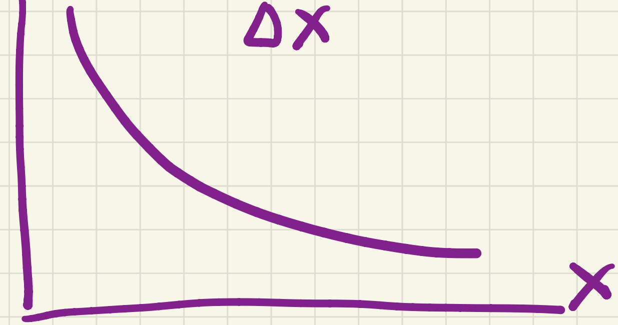
↳ $U(X, Y)$

↳ consumer's satisfaction from consumption

- 1- Commodity: $U = U(X)$



$$MU = \frac{\Delta U}{\Delta X}$$



- 2- commodities

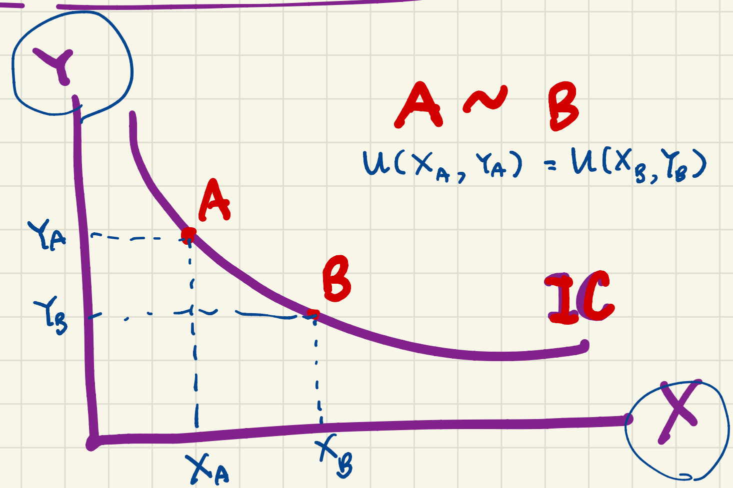
$U = U(X, Y)$ →

$$\frac{\Delta U}{\Delta X} = MU_X$$

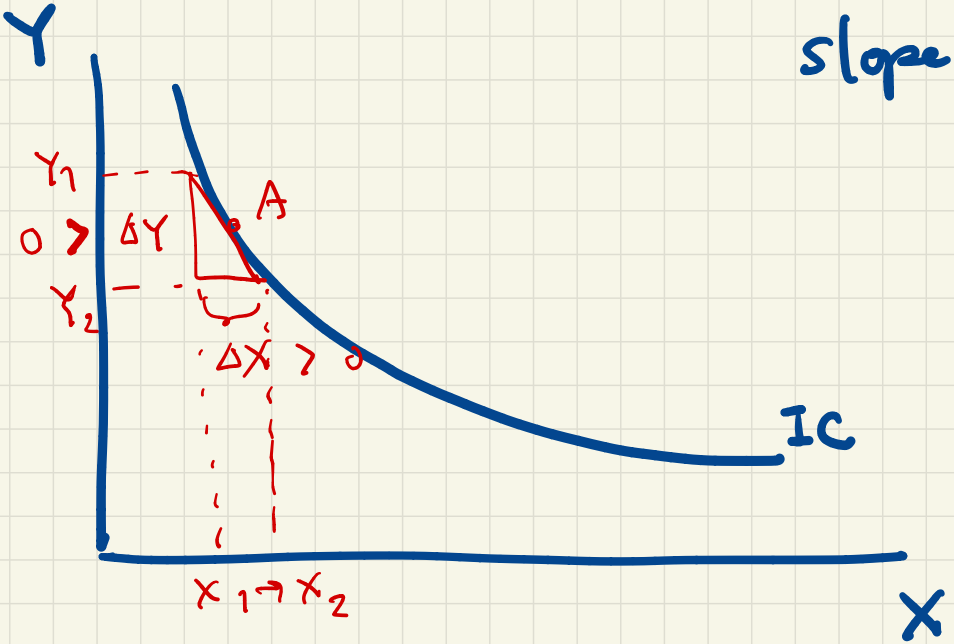
$$\frac{\Delta U}{\Delta Y} = MU_Y$$

$A \sim B$

$U(X_A, Y_A) = U(X_B, Y_B)$



Slope of IC



slope of IC = $\frac{\Delta Y}{\Delta X}$

$\frac{\Delta Y}{\Delta X} = MRS$

$MRS = -\frac{MU_x}{MU_Y}$

$\frac{\Delta Y}{\Delta X} = -\frac{MU_x}{MU_Y}$

MRS = marginal rate of substitution

(Additional)

$$U = U(X, Y)$$

Totally differentiate:

$$\Delta U \approx dU = \underbrace{\frac{dU}{dX}}_{MU_X} \cdot \Delta X + \underbrace{\frac{dU}{dY}}_{MU_Y} \cdot \Delta Y$$

$$\frac{dU}{dX} \approx \frac{\Delta U}{\Delta X}$$

$$\frac{dU}{dY} \approx \frac{\Delta U}{\Delta Y}$$

Since $\Delta U = 0$ along the IC,

$$\Delta U = \frac{dU}{dX} \cdot \Delta X + \frac{dU}{dY} \cdot \Delta Y = 0$$

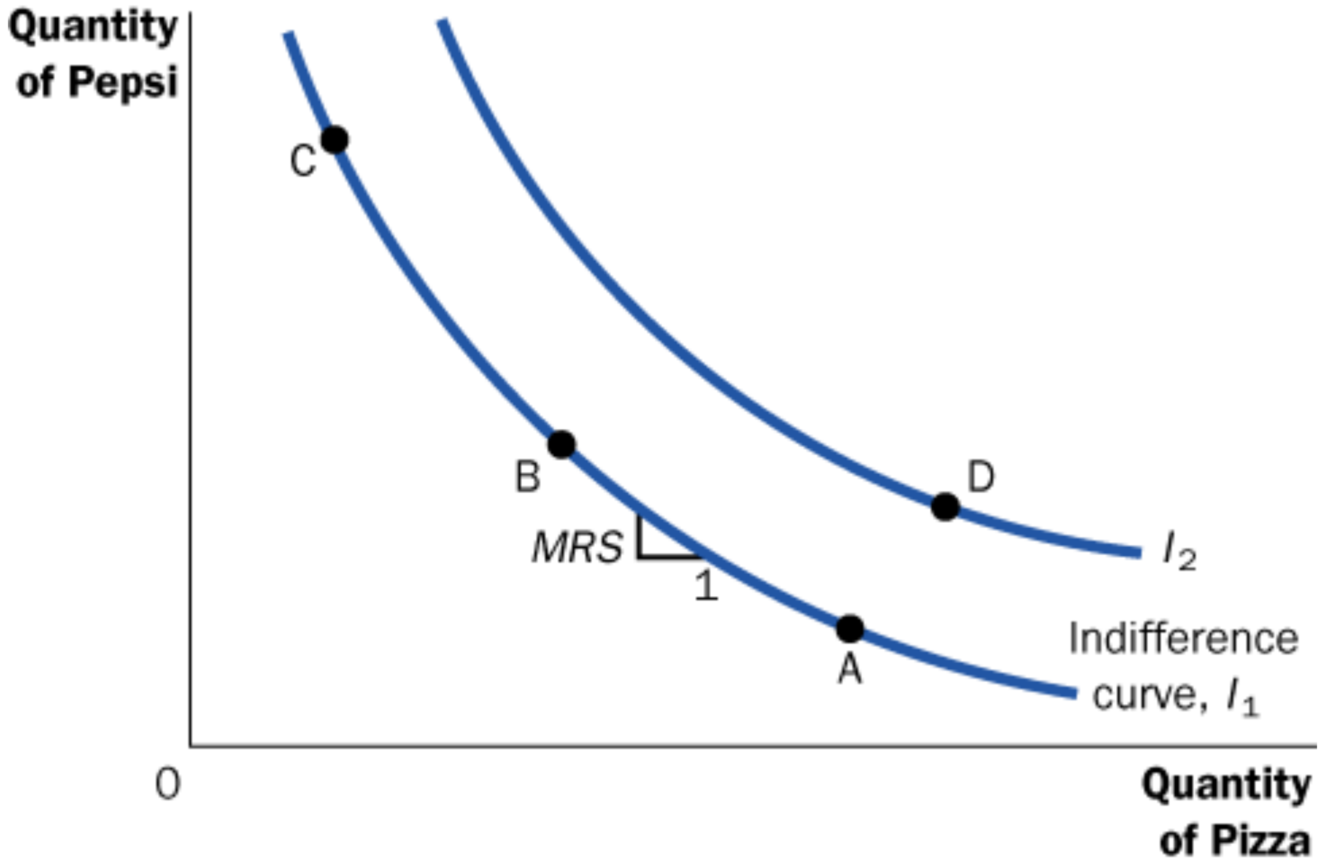
$$\frac{dU}{dX} \cdot \Delta X = -\frac{dU}{dY} \cdot \Delta Y$$

slope of IC

$$\Rightarrow \frac{\Delta Y}{\Delta X} = -\frac{dU/dX}{dU/dY} = -\frac{MU_X}{MU_Y} = MRS$$

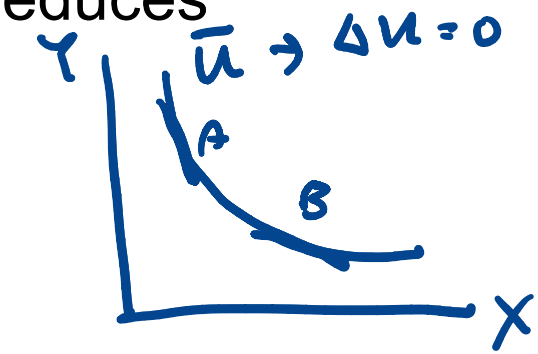
Part III. Consumer's Equilibrium

- Recall: **Indifference curve** shows consumption bundles that give the consumer the same level of satisfaction.



Extra Notes on Indifference Curve (1)

- MRS is the slope of the indifference curve, and MRS is diminishing (in absolute value) as X increases.
- At any point on indifference curve, the slope is $\frac{\Delta Y}{\Delta X}$.
- As the consumer consumes more X and reduces consumption of Y, her utility changes by:



$$\Delta X > 0 \rightarrow \Delta U_x \approx MU_x \times \Delta X$$ > 0

$$\Delta Y < 0 \rightarrow \Delta U_y \approx MU_y \times \Delta Y$$ < 0

- But on the an indifference curve, the utility is the same:

$$\Delta U = \rightarrow \Delta U_x + \Delta U_y = 0$$

Extra Notes on Indifference Curve (2)

- So,

$$\Delta U_x = -\Delta U_y$$

$$\mathrm{MU_x}\Delta X \approx -\mathrm{MU_y}\Delta Y$$

$$\frac{\Delta u}{\Delta x} \cdot \Delta x \approx -\frac{\Delta u}{\Delta Y} \cdot \Delta Y$$

or

$$\underbrace{\frac{du}{dx}}_{MU_x} \cdot \Delta X = \underbrace{-\frac{du}{dY}}_{MU_Y} \cdot \Delta Y$$

➔ $$\frac{\Delta Y}{\Delta X} \approx -\frac{MU_x}{MU_y}$$

➔ $$\frac{dY}{dX} = -\frac{MU_x}{MU_y}$$: MRS

$\frac{dY}{dX}$: slope of IC

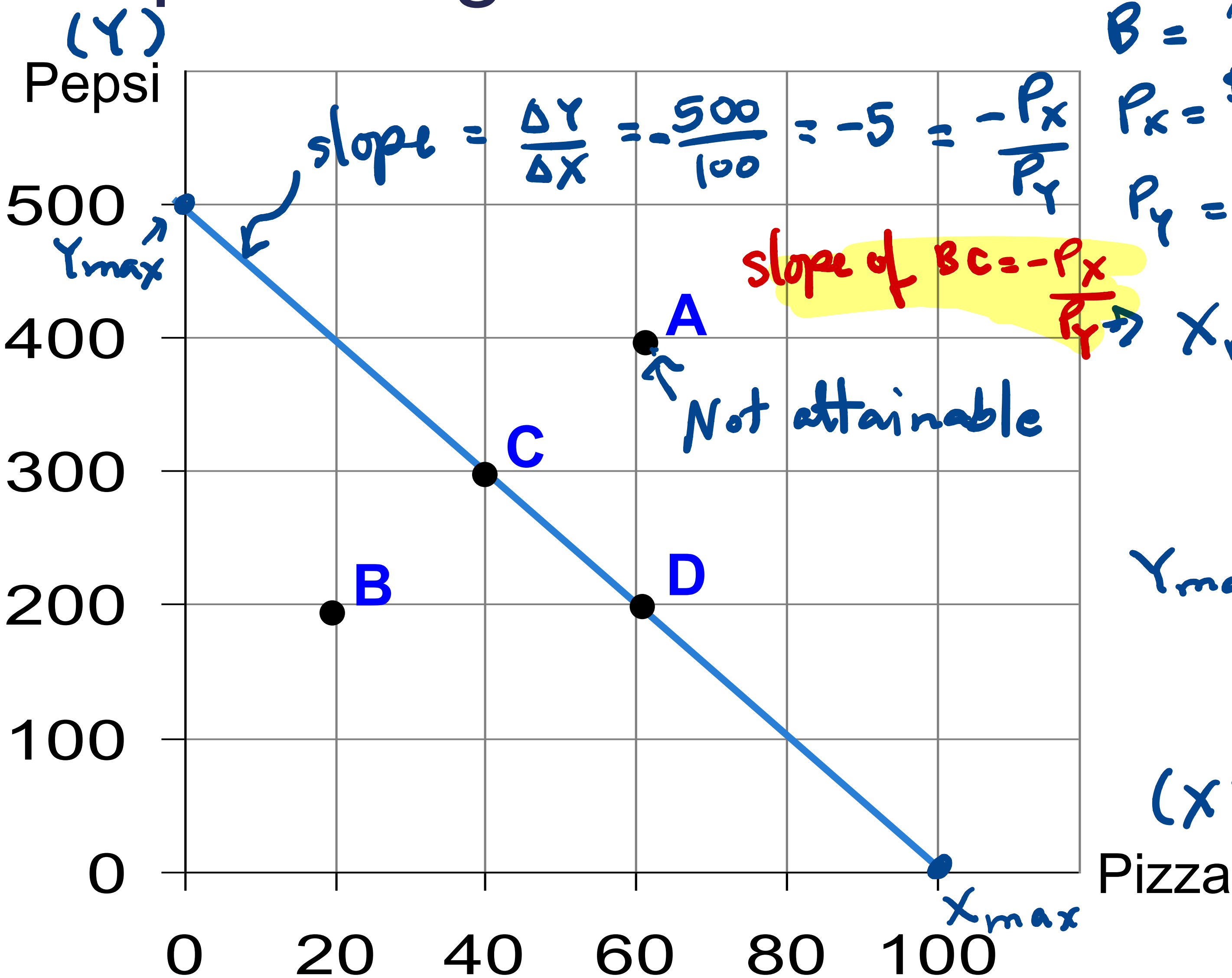
$$= \frac{du/dx}{du/dY}$$

Budget Constraint

- **Budget constraint (or budget line)**: the limit on the consumption bundles that a consumer can afford.
  - It shows all combinations (bundles) of the two goods that the consumer can afford to buy.
- Consider the case of 2 goods: Pizza (X) and Pepsi (Y). Suppose $P_x = \$10$, $P_y = \$2$, and budget (B) = \$1000.
- The budget line can be written as:

$$\underbrace{P_x X + P_y Y}_{\text{Total Expenditure}} = B \quad \leftarrow \text{total income}$$

Graph: Budget Constraint



$B = 1,000$

$P_X = \$10$

$P_Y = \$2$

$X_{max} = \frac{1000}{10} = 100$

$Y_{max} = \frac{1000}{2} = 500$

Slope of the Budget Constraint

- The slope of the budget constraint equals:
  - the rate at which the consumer can trade Pepsi for pizza
  - the opportunity cost of pizza in terms of Pepsi
  - the relative price of pizza.

That is:

$$Slope\ of\ Budget\ line = -\frac{P_x}{P_y}$$

Change in Budget Constraint: Higher Income

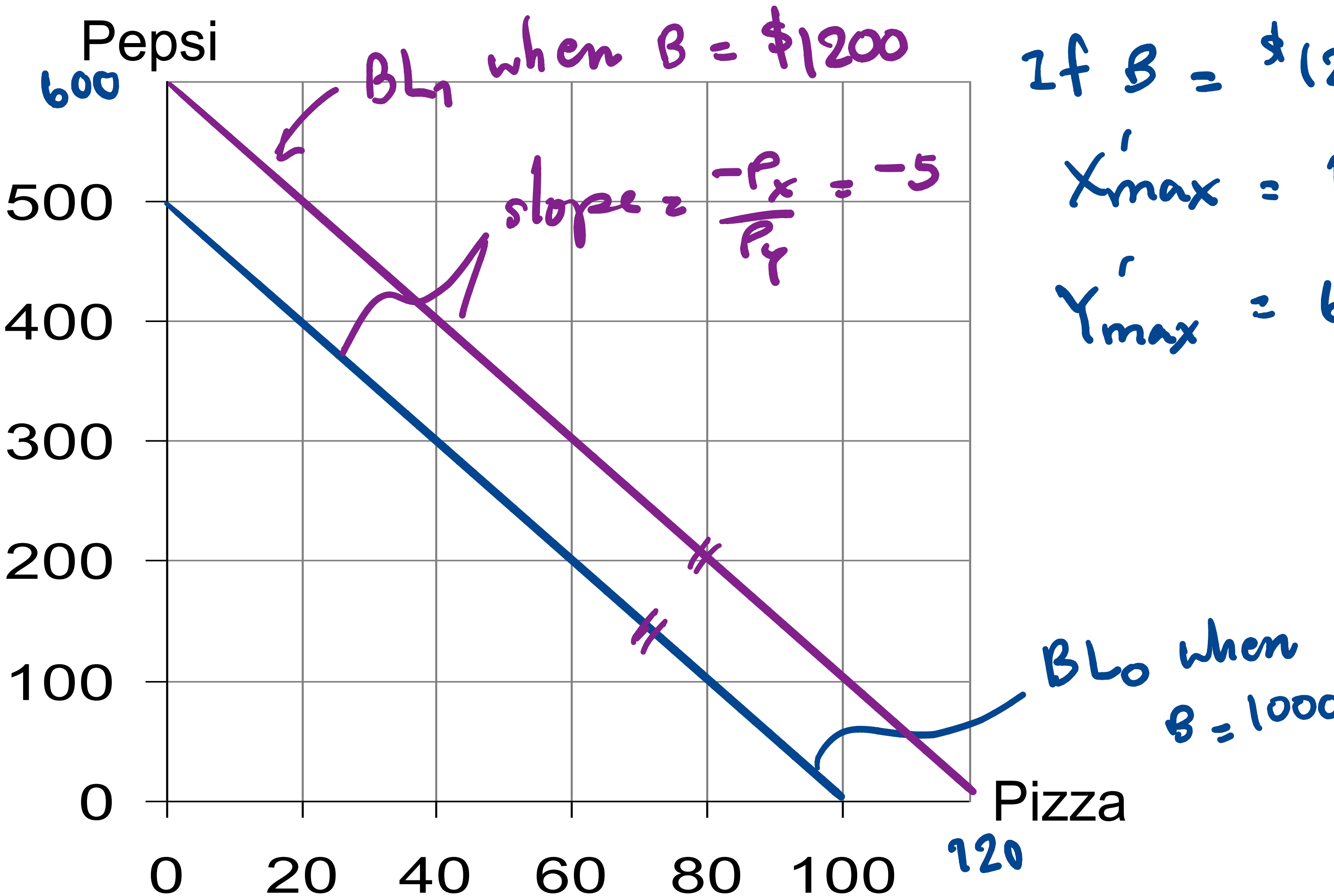
- Suppose budget increases to $1200.

$P_x = \$10$, $P_Y = \$2$

If $B = \$1200$,

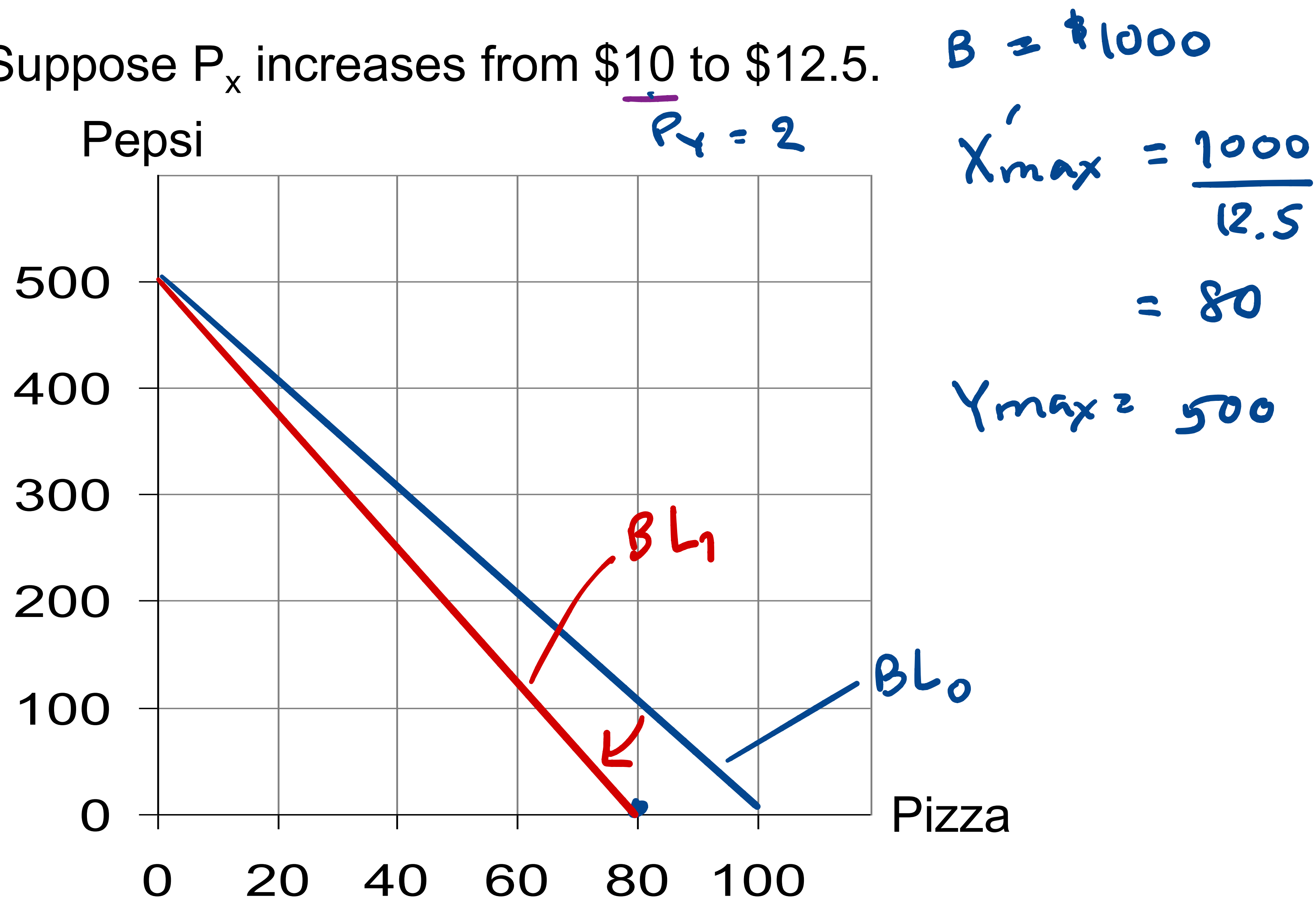
$X'_{max} = 120$

$Y'_{max} = 600$



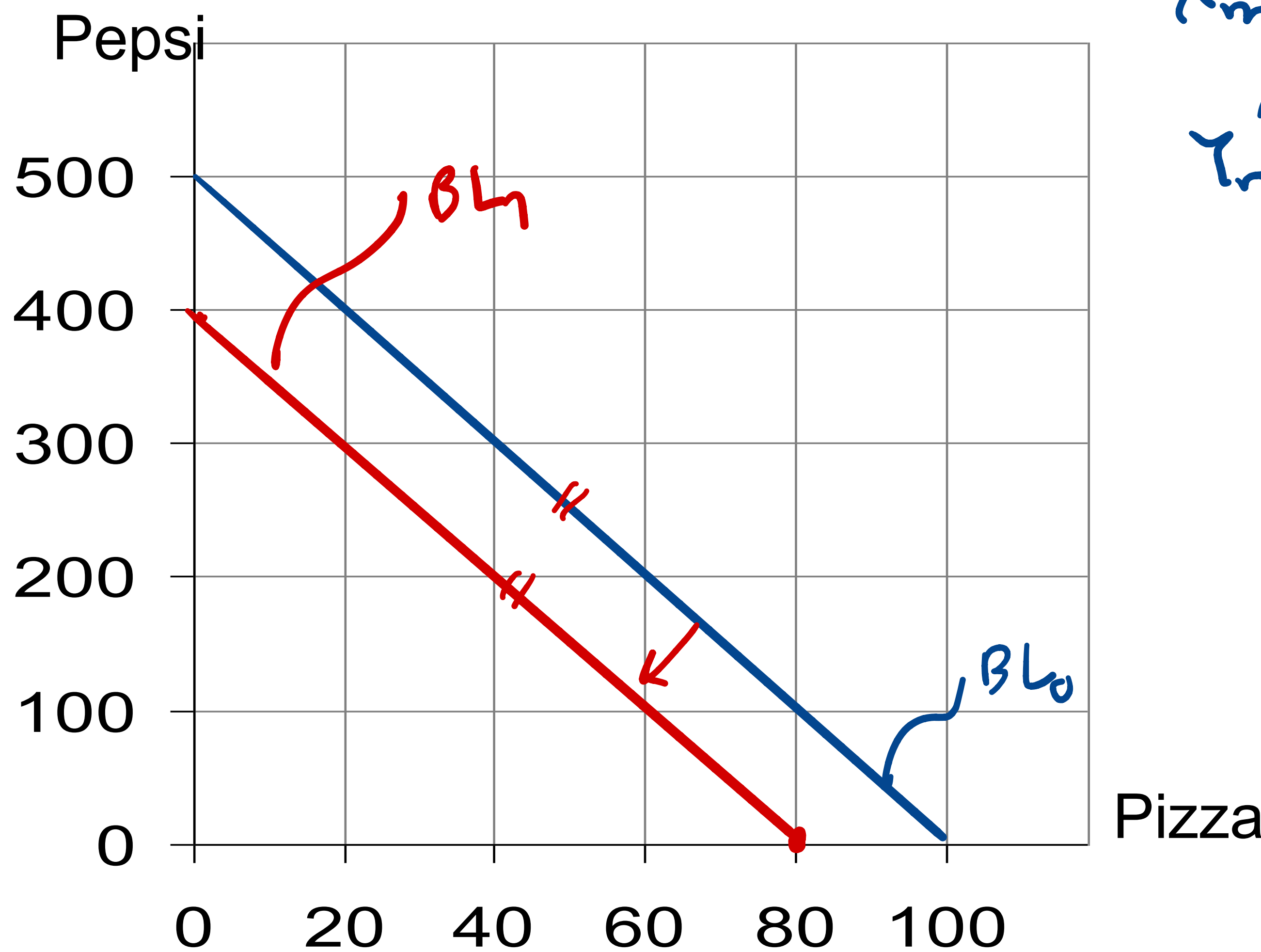
Change in Budget Constraint: Px changes.

- Suppose P_x increases from \$10 to \$12.5.



Change in Budget Constraint: Px & Py change by the same proportion.

- Suppose P'_x = \$12.5 and P'_y = \$2.5.



$B = 1000$

$X'_{max} = 80$

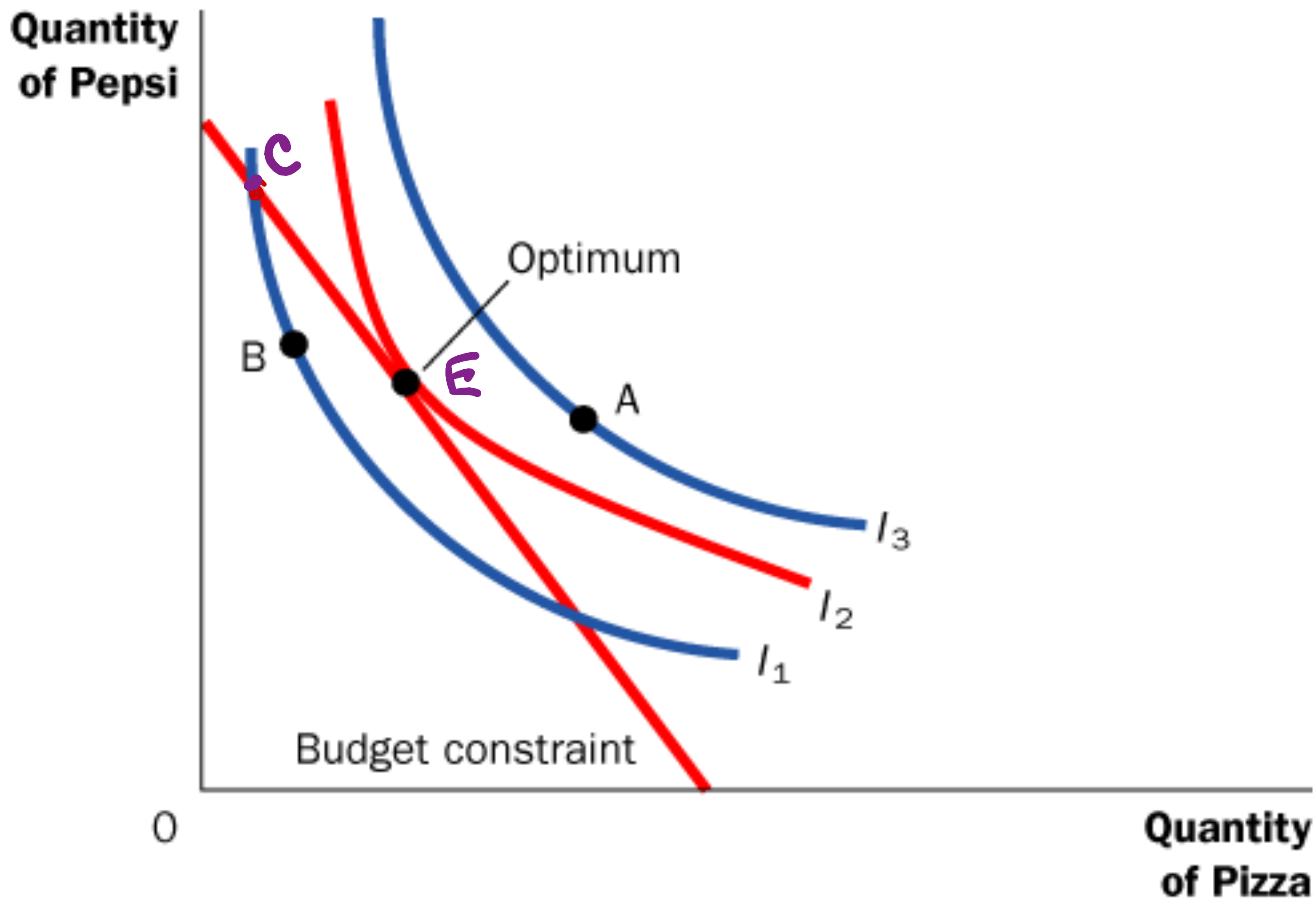
$Y'_{max} = 400$

Consumer's Problem: Optimization

- Consumer's problem is to maximize his/her utility (i.e. satisfaction) under the budget constraint.
- The **optimal bundle** (X^*, Y^*) is at the point where the budget constraint touches the highest indifference curve.
  - i.e., the indifference curve and budget constraint have the same slope.
- Since the slope of IC is the MRS and the slope of the budget constraint is the relative price, the optimal bundle is where:

$$\frac{P_x}{P_y} = \frac{MU_x}{MU_y}$$

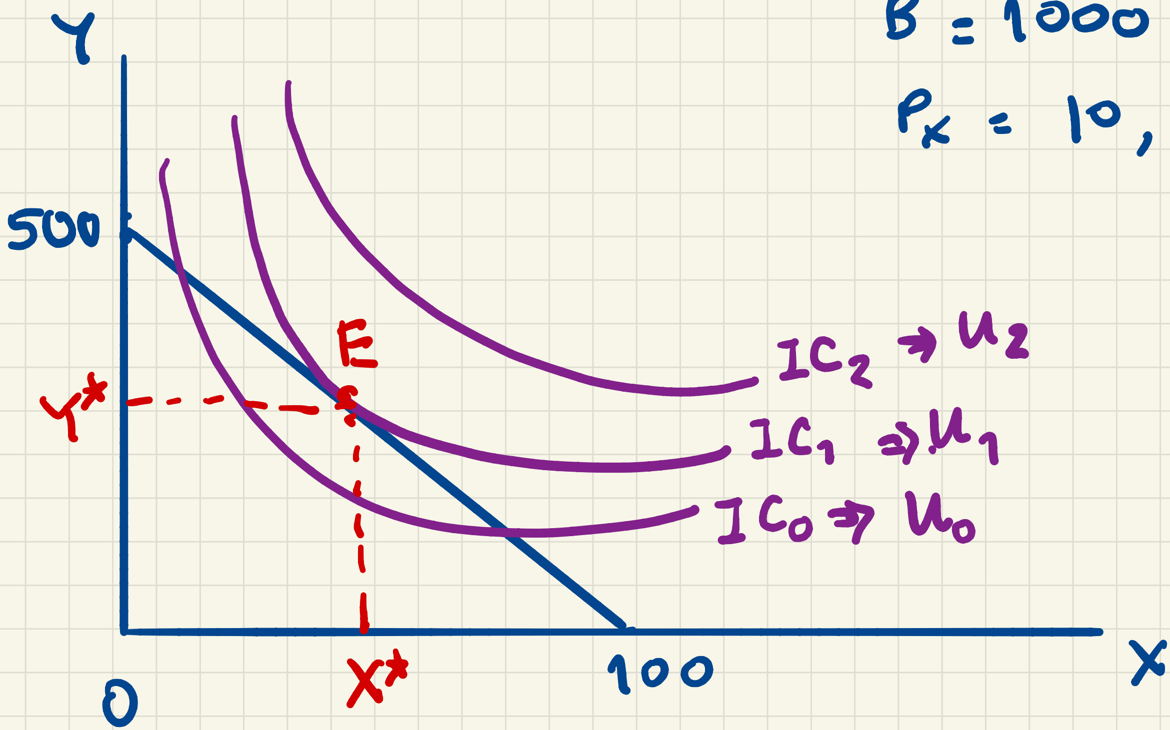
Graph: Consumer's Optimal Choice



Consumer's Optimization

$B = 1000$

$P_X = 10, \; P_Y = 2$



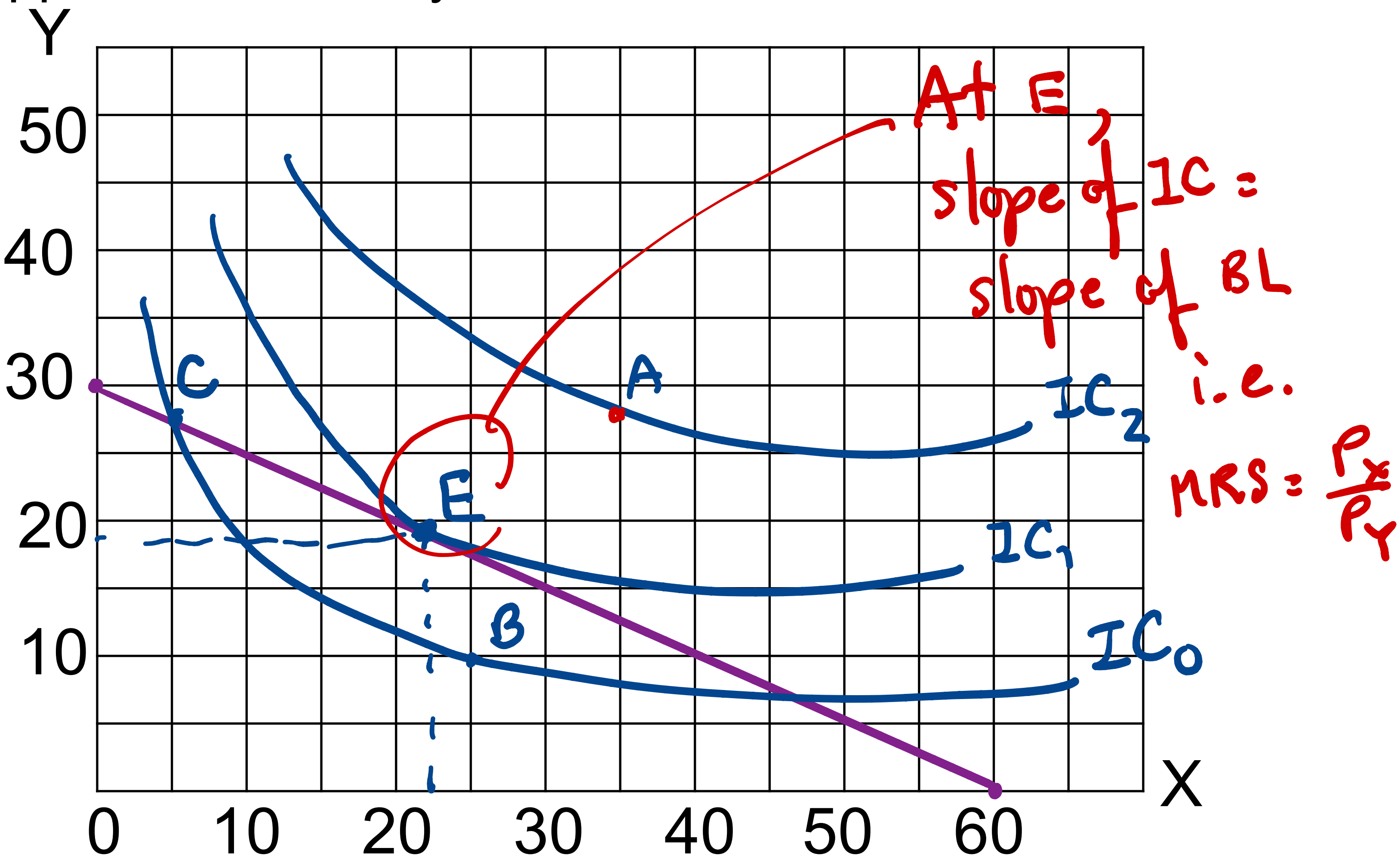
At E, $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

Example: Optimization

$X_{max} = 60$

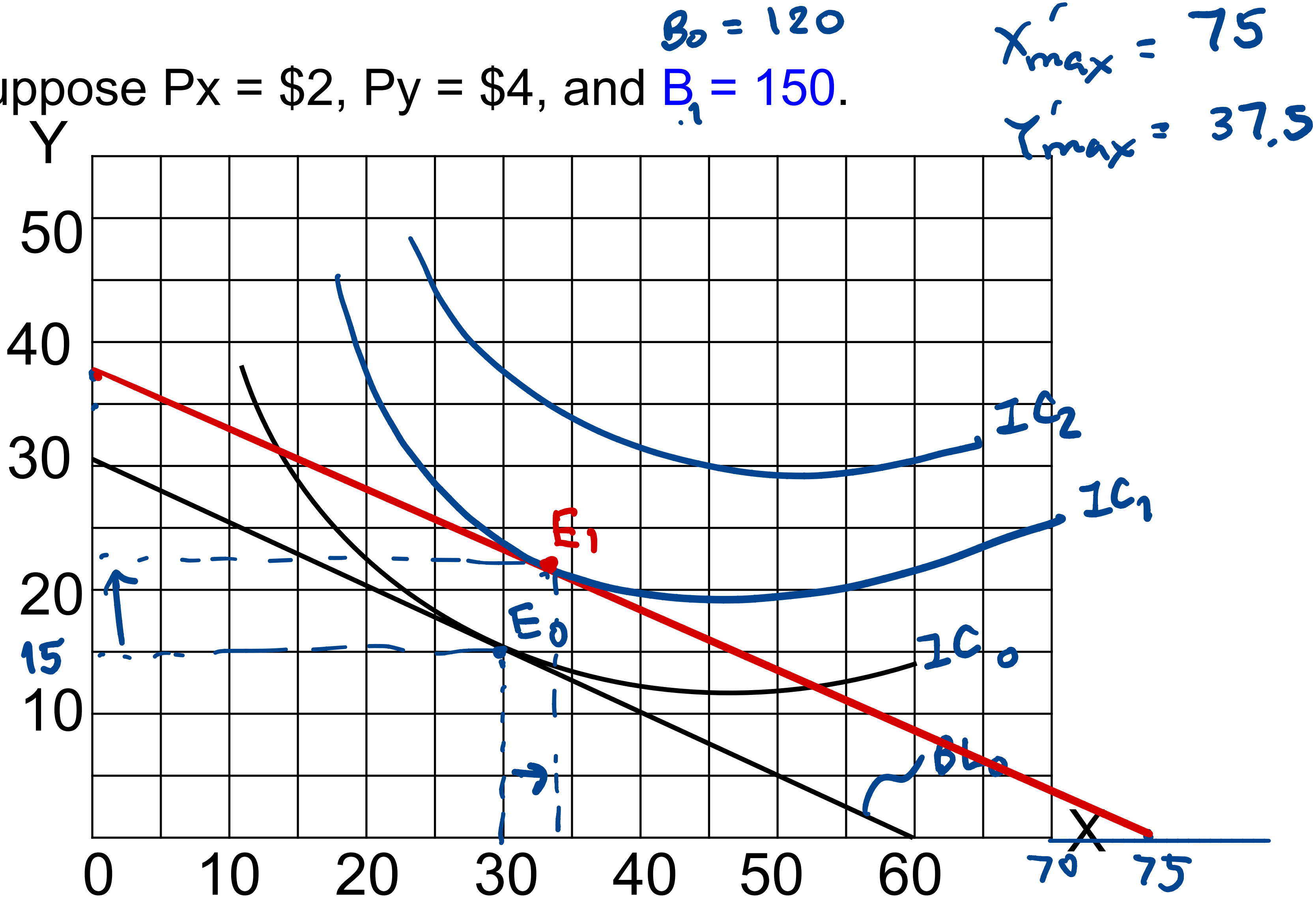
$Y_{max} = 30$

- Suppose Px = \$2, Py = \$4, and B = 120.



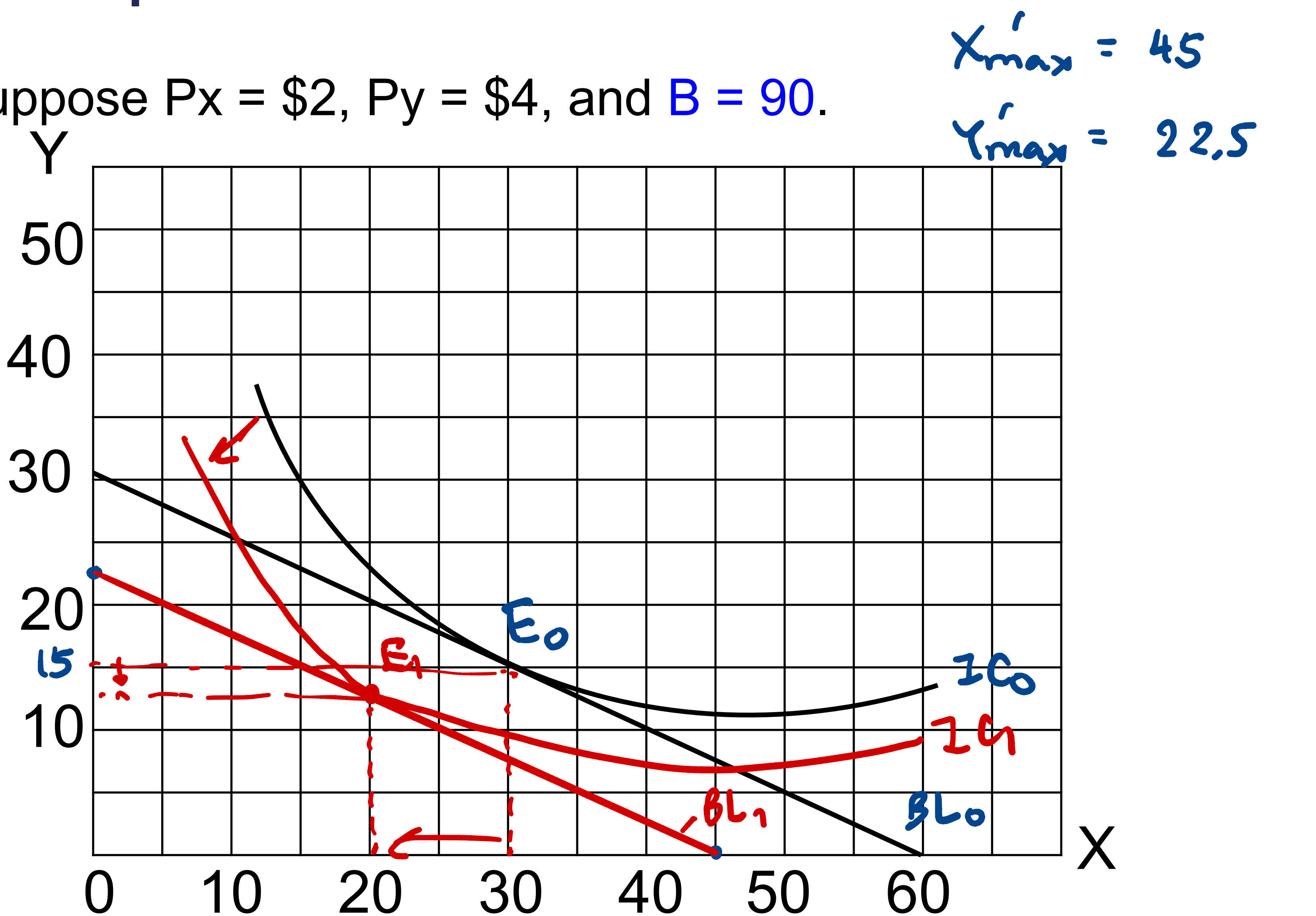
Example: Effect of Income Increase

- Suppose Px = $2, Py = $4, and B = 150.



Example: Effect of Income Reduction

- Suppose Px = \$2, Py = \$4, and B = 90.



Income Consumption Curve (ICC)

- A change in income, *ceteris paribus*, will shift the consumer's budget constraint.
- For each level of income, there will be a utility maximizing points where IC is tangent to the relevant budget line.
- **Income Consumption Curve (ICC)** is the line that connects all the utility-maximizing points for different levels of income, given prices Px and Py constant.
- I.e. , ICC shows how the consumer's purchases react to a change in money income with relative prices being held constant.

Graph: ICC

- Suppose Px = \$2, Py = \$4, and B = 90, 120, 150.