Exercise 9

Applications of Integration: Area

1. Find the area of the region bounded by the given graphs.

   (a) $y = x, \quad y = x^2$ Ans: 1/6

   (b) $y = 5x - x^2, \quad y = x$ Ans: 32/3

   (c) $x = y^2 - 2$, $x = e^y$, $y = 1$, $y = -1$ Ans: $e - \frac{1}{e} + \frac{10}{3}$

   (d) $x = y^2 - 4y, \quad x = 2y - y^2$ Ans: 9

   (e) $y = 1 + \sqrt{x}, \quad y = (3 + x)/3$ Ans: 9/2

   (f) $y^2 = x, \quad x - 2y = 3$ Ans: 32/3

   (g) $y = |x|, \quad y = x^2 - 2$ Ans: 20/3

2. The average value of a continuous function f on $[a, b]$ is defined as

$$f_{avg} := \frac{1}{b-a} \int_a^b f(x)dx.$$

   Consider the function

$$f(x) = 4 - x^2, \qquad x \in [0, 2].$$

   (a) Let f_{avg} be the average value of $f(x)$ on the interval $[0, 2]$. Find f_{avg}. Ans: 8/3

   (b) Find a number $c \in [0, 2]$ such that $f(c) = f_{avg}$. Ans: $c = 2/\sqrt{3}$

3. For each of the problems below, find the area under the graph of the given function on the indicated interval (Hint: Use improper integrals).

   (a) $f(x) = \dfrac{1}{(2x+1)^2}$, $[1, \infty)$ Ans: $\frac{1}{6}$

   (b) $f(x) = e^{-|x|}$, $(-\infty, \infty)$ Ans: 2