EE432 Monetary Theory and Policy

Lecture 2 Present Value, Interest Rate and Risk

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Outline

- Future value and present value
- Relation between interest rate and bond price
- Risk defining and monitoring
- Source of risk and risk reducing

Chapter 4

Future Value, Present Value, and Interest Rates

Interest Rate

- **Credit** is one of the critical ***mechanisms*** we have ***for allocating resources***.
- Although **interest** has historically been unpopular, this comes from the failure to appreciate the ***opportunity cost of lending***.
- **Interest rates**
  - Link the ***present*** to the ***future***.
  - Tell the future reward for lending today.
  - Tell the cost of borrowing now and repaying later.

Valuing Monetary Payments Now and in the Future

- We must learn how to *calculate and compare rates* on different financial instruments.
- ***We need*** a set of tools:
  - **Future value**
  - **Present value**

Future Value and Compound Interest

- **Future value** is the ***value*** on some ***future date*** of an ***investment made today***.
  - *$100 invested today* at 5% interest *gives $105* in a year
  - So the *future value of $100 today* at 5% interest *is $105 one year from now*.
  - The *$100 yields $5*, which is why *interest rates* are sometimes called a ***yield***.

Future Value and Compound Interest

- If the ***present value*** is \$100 and the ***interest rate*** is 5%, then the ***future value*** one year from now is:

$$\$100 + \$100(0.05) = \$105$$

- This also shows that the ***higher the interest rate***, the ***higher the future value***.
- In general:

$$FV = PV + PV(i) = PV(1 + i)$$

Future Value and Compound Interest

- When using one-year interest rates to compute the ***value repaid <u>more than one year</u> from now***, we must consider **compound interest**.
  - **Compound** interest is the *interest on the interest*.

Future Value and Compound Interest

- What if you leave your $100 in the bank for *two years* at 5% yearly interest rate?
- The future value is:

$$\$100 + \$100(0.05) + \$100(0.05) + \$5(0.05) = \$110.25$$

$$\$100(1.05)(1.05) = \$100(1.05)^2$$

- In general

$$FV_n = PV(1 + i)^n$$

Future Value and Compound Interest

- Table 4.1: Computing the future value of $100 at 5% annual interest

Years into Future	Computation	Future Value
1	$\$100(1.05)$	$105.00
2	$\$100(1.05)^2$	$110.25
3	$\$100(1.05)^3$	$115.76
4	$\$100(1.05)^4$	$121.55
5	$\$100(1.05)^5$	$127.63
10	$\$100(1.05)^{10}$	$162.89

Present Value

- Financial instruments *promise* **future cash payments**, so we need to know *how to value those payments*.
- **Present value** is the **value today** (in the present) of a ***payment*** that is ***promised to be made in the future***.
- Or, **present value** is the amount that must be ***invested today*** in order to ***realize a specific amount*** on a ***given future date***.

Present Value

- Solve the Future Value Formula for PV:

$$FV = PV \times (1+i), \text{ so}$$

$$PV = \frac{FV}{(1+i)}$$

- This is just the *future value calculation inverted*.

Present Value

- We can generalize the process as we did for future value.
- Present Value of **payment received *n* years** in the future:

$$PV = \frac{FV_n}{(1+i)^n}$$

Relation between Interest Rate and Bond Price

Internal Rate of Return

- Imagine that you run a **tennis racket company** and that you are considering ***purchasing a new machine***.
  - *Machine costs $1 million* and can produce 3000 rackets per year.
  - You *sell the rackets* for $50, *generating $150,000* in revenue *per year*.
  - Assume the machine is only input, have certainty about the revenue, no maintenance and *a 10 year lifespan*.

Internal Rate of Return

- If you borrow $1 million, is the revenue enough to make the payments?
- We need to compare the internal rate of return to the cost of buying the machine.
  - The **interest rate** that **equates** the *present value of an investment with its cost*.

Internal Rate of Return

- Balance the cost of the machine against the revenue.
  - $1 million today versus $150,000 a year for ten years.
- To find the **internal rate of return**, we take the **cost of the machine** and ***equate*** it to the **sum of the present value of each of the yearly revenues**.
  - Solve for *i* - the internal rate of return.

Internal Rate of Return: Example

- Solving for i, i = 0.0814 or 8.14%

$$\$1,000,000 = \frac{\$150,000}{(1+i)^1} + \frac{\$150,000}{(1+i)^2} + \frac{\$150,000}{(1+i)^3} + \ldots\ldots + \frac{\$150,000}{(1+i)^{10}}$$

- So long as your *interest rate* at which *you borrow* the money is less than 8.14%, then you should buy the machine.

Bond Basics

- A bond is a ***promise*** *to make a* ***series of payments*** on *specific* ***future dates***.
- Bonds create **obligations**, and are therefore thought of as *legal contracts* that:
  - Require the borrower to make payments to the lender, and
  - Specify *what happens if the borrower fails* to do so.

Bond Basics

- The most common type of bond is a **coupon bond**.
  - Issuer is required to *make* ***annual payments***, called **coupon payments**.
  - The ***annual interest*** the borrower pays (i_c), is the **coupon rate**.
  - The **date** on which the ***payments stop*** and the **loan is repaid** (n), is the **maturity date** or term to maturity.
  - The ***final payment*** is the **principal**, **face value**, or **par value** of the bond.

Valuing the Principal

- Assume a bond has a **principle payment of $100** and its **maturity date is *n* years** in the future.
- The ***present value*** of the bond principal is:
  - The higher the *n*, the lower the value of the payment.

$$P_{BP} = \frac{F}{(1+i)^n} = \frac{\$100}{(1+i)^n}$$

Valuing the Coupon Payments

- These resemble **loan payments**.
- The longer the payments go, the higher their total value.
- The higher the interest rate, the lower the present value.
- The present value expression gives us a general formula for the *string of yearly coupon payments* made over *n* years.

$$P_{CP} = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots\ldots + \frac{C}{(1+i)^n}$$

Valuing the Coupon Payments Plus Principal

- We can just combine the previous two equations to get:

$$P_{CB} = P_{CP} + P_{BP} = \left[\frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots\ldots + \frac{C}{(1+i)^n} \right] + \frac{F}{(1+i)^n}$$

- The value of the **coupon bond, *P_{CB}*,** *rises* when
  - The yearly **coupon payments**, ***C***, *rise* and
  - The **interest rate**, ***i***, *falls*.

Relationship between Bond Price and Interest Rate

- Bonds promise fixed payments on future dates, so the ***higher* the interest rate**, the ***lower* their present value**.
- *The value of a bond varies **inversely** with the interest rate used to calculate the present value of the promised payment.*

Real and Nominal Interest Rates

- **Borrowers** care about the ***resources required to repay***.
- **Lenders** care about the ***purchasing power of the payments*** they received.
- *Neither cares solely about the number of dollars, they care about* ***what the dollars buy****.*

Real and Nominal Interest Rates

- Nominal Interest Rates (*i*)
  - The interest rate expressed in current-dollar terms.
- Real Interest Rates (*r*)
  - The **inflation adjusted interest rate**.

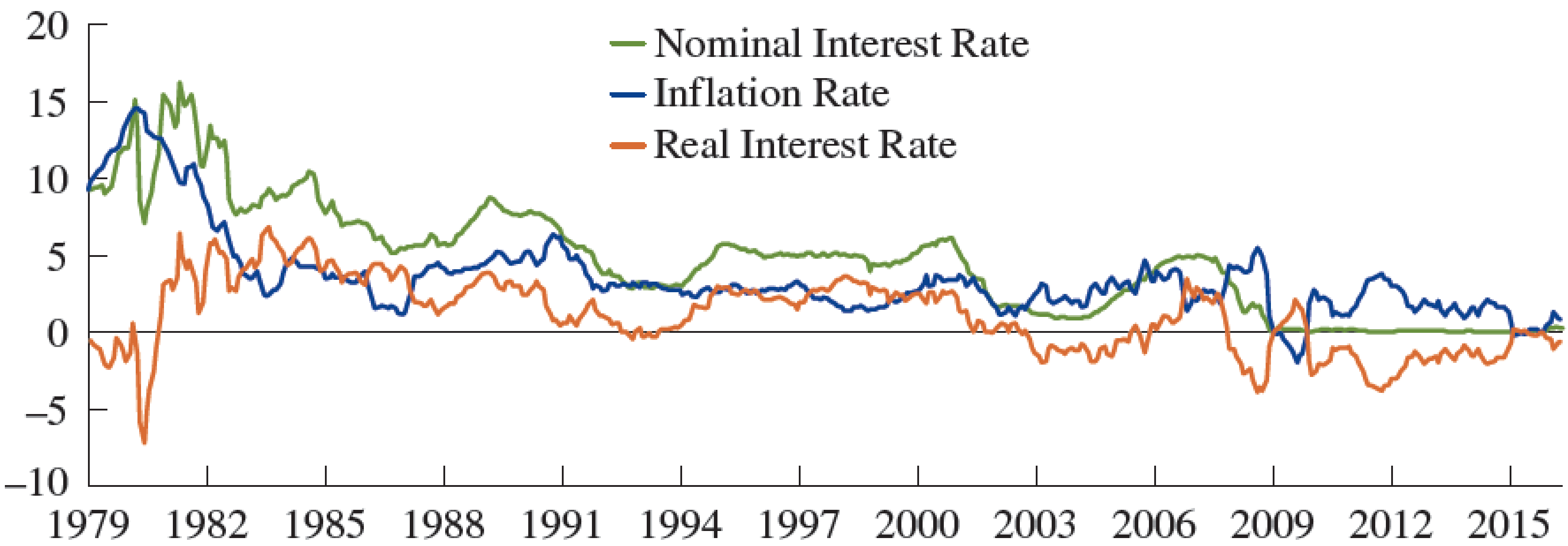
Real and Nominal Interest Rates

- The **nominal interest rate** you agree on (*i*) must be **based on *expected inflation* (π^e)** over the term of the loan **plus** the **real interest rate** you agree on (*r*).

$$i = r + \pi^e$$

- This is called the *Fisher Equation.*
- The higher expected inflation, the higher the nominal interest rate.

Figure 4.2: Nominal Interest Rate, Inflation Rate and Real Interest Rate



Real and Nominal Interest Rates

- Financial markets quote *nominal interest rates*.
- When people use the term interest rate, they are referring to the *nominal rate*.
- We cannot directly observe the *real interest rate*; we have to estimate it.

$$r = i - \pi^e$$

Chapter 5

Understanding Risk

Risk Defining and Measuring

Defining Risk

- Risk is "*the possibility of loss*."
- For outcomes of financial and economic decisions

<u>Risk</u> *is a measure of* ***uncertainty*** *about the* ***future payoff*** *to an investment, assessed over some time horizon and relative to a benchmark.*

Defining Risk

1. **Risk** is a *measure* that can be quantified.
   – The ***riskier*** the investment, the ***less desirable*** and the ***lower the price***.
2. **Risk** arises from ***uncertainty about the future***.
   – We do not know which of many possible outcomes will follow in the future.
3. **Risk** has to do with the **future *payoff*** *of an investment*.
   – We must imagine all the **possible payoffs** and the **likelihood** of each.

Defining Risk

4. Definition of **risk** refers to an *investment* or group of investments.
   – Investment described very broadly.
5. **Risk** must be **assessed over some *time horizon***.
   – In general, risk over shorter periods is lower.
6. **Risk** must be measured ***relative to some benchmark*** - not in isolation.
   – A good benchmark is the performance of a group of experienced investment advisors or money managers.

Measuring Risk

- In determining *expected inflation* or *expected return,* we need to understand **expected value**.
  - The **investments return** out of all possible values.

Possibilities, Probabilities, and Expected Value

- **Probability theory** states that considering uncertainty requires:
  - – Listing all the *possible outcomes*.
  - – Figuring out the *chance of each one occurring*.
- **Probability** is a ***measure of the likelihood*** that an event will occur.
  - It is always **between zero and one**.
  - Can also be *stated as **frequencies***.

Possibilities, Probabilities, and Expected Value

- We can construct a table of all **outcomes** and **probabilities** for an ***event***, like tossing a fair coin.

Table 5.1 A Simple Example: All Possible Outcomes of a Single Coin Toss

Possibilities	Probability	Outcome
#1	$\frac{1}{2}$	Heads
#2	$\frac{1}{2}$	Tails

Possibilities, Probabilities, and Expected Value

- If constructed correctly, *the values in the* ***probabilities*** *column will* ***sum to one***.
- Assume instead we have an **investment** that ***can rise or fall*** in value.
  - *$1,000 stock* which can *rise to $1,400 or fall to $700*.
  - The amount you could get back is the *investment's payoff*.
  - We can construct a similar table and determine the *investment's* ***expected value*** *- the average or most likely outcome*.

Possibilities, Probabilities, and Expected Value

Table 5.2 Investing $1,000: Case 1

Possibilities	Probability	Payoff	Payoff × Probability
#1	$\frac{1}{2}$	$ 700	$350
#2	$\frac{1}{2}$	$1,400	$700
Expected value = Sum of (Probability × Payoff) = $1,050			

- **Expected value** is the ***mean*** - *the sum of their probabilities multiplied by their payoffs*.

Expected Value = 1/2($700) + 1/2($1,400) = $1,050

Possibilities, Probabilities, and Expected Value

Now, suppose that $1,000 Investment could

1. *Rise in value to $2,000*, with probability of 0.1
2. *Rise in value to $1,400*, with probability of 0.4
3. *Fall in value to $700*, with probability of 0.4
4. *Fall in value to $100*, with probability of 0.1

Possibilities, Probabilities, and Expected Value

Table 5.3 Investing $1,000: Case 2

Possibilities	Probability	Payoff	Payoff × Probability
#1	0.1	$ 100	$ 10
#2	0.4	$ 700	$280
#3	0.4	$1,400	$560
#4	0.1	$2,000	$200

Expected value = Sum of (Probability × Payoff) = $1,050

Expected Value =

$$0.1 \times (\$100) + 0.4 \times (\$700) + 0.4 \times (\$1{,}400) + 0.1 \times (\$2{,}000) = \$1{,}050$$

Possibilities, Probabilities, and Expected Value

- Using percentages allows ***comparison of returns*** regardless of the size of initial investment.
  - The **expected return** in *both cases* is $50 on a $1,000 investment, or *5 percent*.
- Are the two investments the same?
  - No - the ***second investment has a wider range of payoffs***.
- **Variability** equals **risk**.

Measures of Risk

- It seems intuitive that the *wider the range of outcomes*, the *greater the risk*.
- A risk free asset is *an investment whose future value is knows with certainty and whose return is the risk free* rate of return.
  - The payoff you receive is guaranteed and cannot vary.
- **Measuring the spread** allows us to **measure the risk**.

Variance and Standard Deviation

The **variance** is the ***average of the squared deviations of the possible outcomes*** from their *expected value*, weighted by their *probabilities*.

1. Compute **expected value**.
2. **Subtract expected value** from each of the **possible payoffs** and **square the result**.
3. **Multiply** each **result *times*** the **probability**.
4. **Add up** the results.

Variance and Standard Deviation

1. Compute the **expected value**:
   (\$1400 x ½) + (\$700 x ½) = \$1,050.

2. **Subtract** this **from each of the possible payoffs** and then **square** the results:
   \$1,400 – \$1,050 = $(\$350)^2$ = 122,500$(\text{dollars})^2$ and
   \$700 – \$1,050 = $(–\$350)^2$ =122,500$(\text{dollars})^2$

3. **Multiply each result** *times* its **probability,** and **add up the results**:
   ½ [122,500$(\text{dollars})^2$] + ½ [122,500$(\text{dollars})^2$] =122,500$(\text{dollars})^2$

4. The **Standard deviation** is the ***square root of the variance***:

$$= \sqrt{Variance} = \sqrt{122,500 \ \ dollars^2} = \$350$$

Variance and Standard Deviation

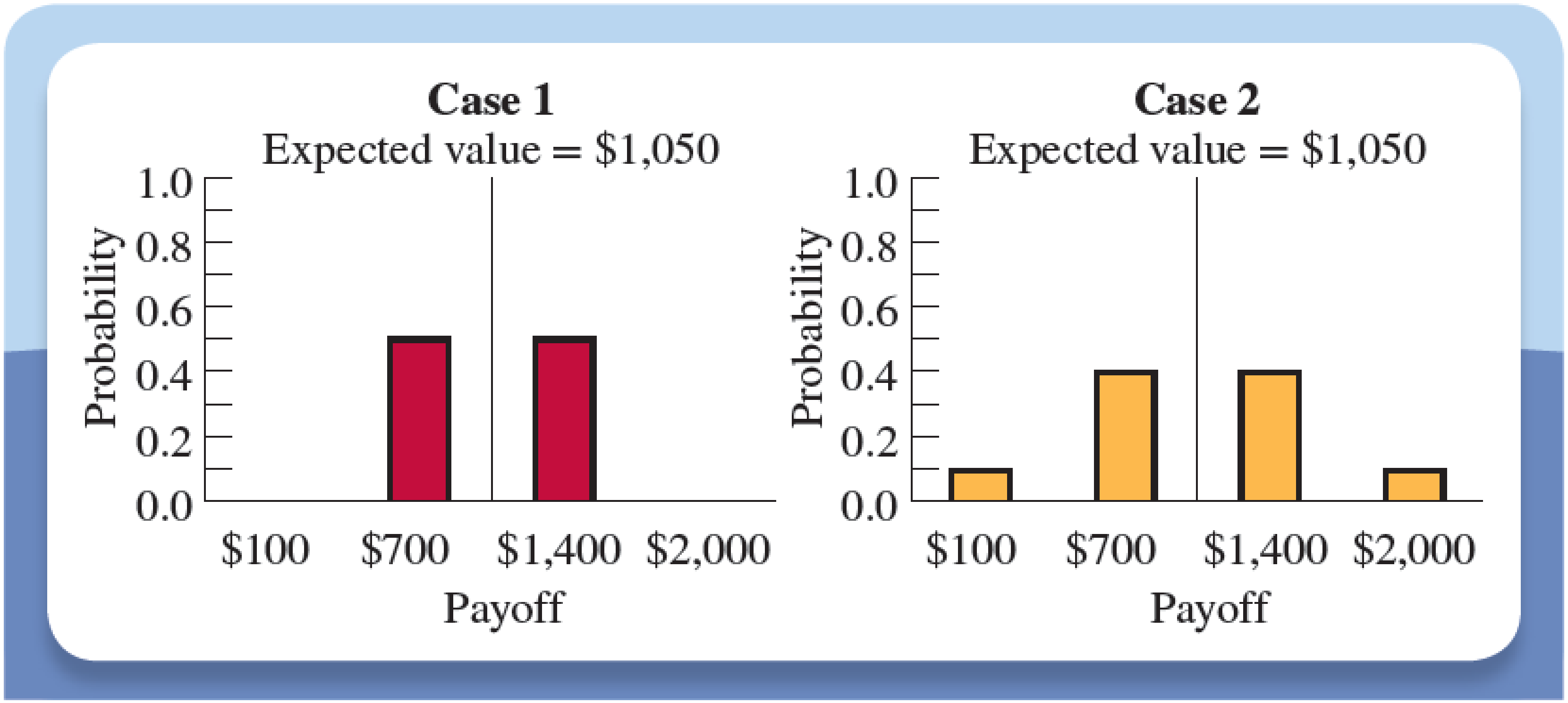
- **Standard deviation** is the (positive) *square root of the variance*

  standard deviation = $\sqrt{Variance}$

- The **standard deviation** is more useful because it deals in ***normal units***, not squared units (like dollars-squared).
- We can **calculate standard deviation** into a **percentage of the initial investmen**t.
- We can ***compare other investments*** to this one.
- *Given* a choice between *two investments with equal expected payoffs*, most will **choose the one with the <u>lower standard deviation</u>**.
  - The *greater the standard deviation*, the *higher the risk*.

Variance and Standard Deviation

Figure 5.1 Investing $1,000



- We can see **Case 2 is <u>more spread out</u>** - *higher standard deviation* - therefore it carries **more risk**.

Value at Risk

- Sometimes we are *less concerned with spread* than with the **worst possible outcome**
  - Example: We don't want a bank to fail
- **Value at Risk (VaR):** The ***worst possible loss*** over a *specific horizon* at a *given probability*.
- For example, we can use this to assess whether a fixed or variable-rate mortgage is better.

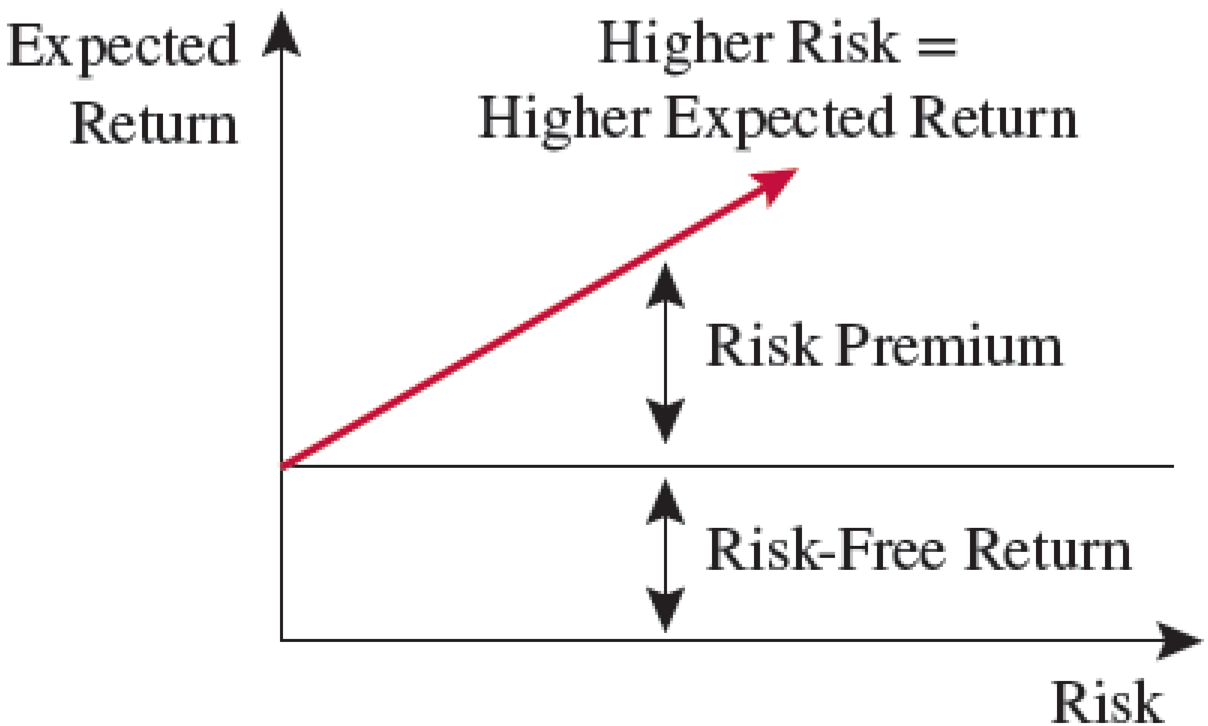
Value at Risk

- For a *mortgage*, the *worst case scenario* means you ***cannot afford your mortgage*** and will ***lose you home***.
  - Expected value and standard deviation do not really tell you the risk you face, in this case.
- **VaR** answers the question: ***how much will I lose*** if the *worst possible scenario* occurs?
  - Sometimes this is the most important question.

Risk Aversion, the Risk Premium, and the Risk-Return Tradeoff

- Most people do not like risk and will **pay to avoid** it because most of us are ***risk averse***.
  - *Insurance* is a good example of this.
- A risk averse investor will always *prefer an investment* with ***a certain return*** to one with the same expected return but *any amount of uncertainty*.
- Therefore, the riskier an investment, the *higher the* ***risk premium***.
  - The *compensation* investors *required to hold the risky asset*.

Risk Aversion, the Risk Premium, and the Risk-Return Tradeoff



Source of Risk and Risk Reducing

Sources of Risk: Idiosyncratic and Systematic Risk

All **risks** can be <u>*classified*</u> *into two groups*:

1. Those *affecting a **small number of people*** but no one else:

   Idiosyncratic or *unique* risks

2. Those *affecting **everyone***:

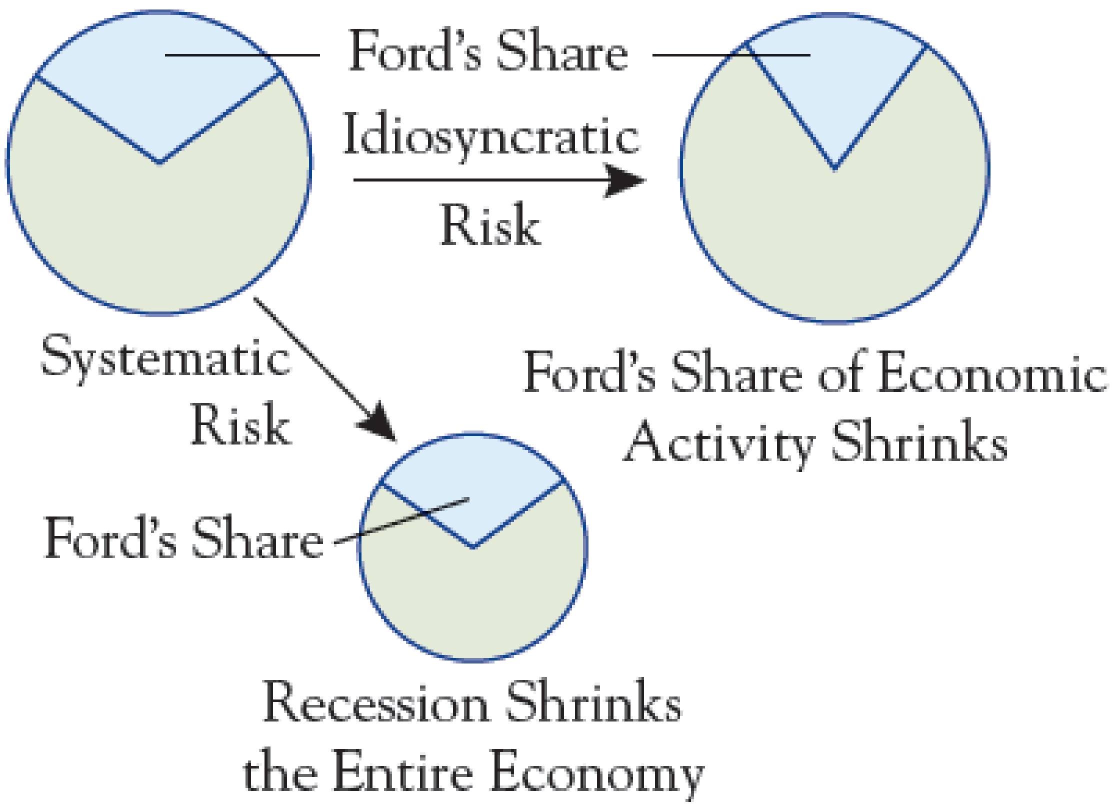
   Systematic or *economy-wide* risks

Sources of Risk: Idiosyncratic and Systematic Risk

Idiosyncratic risks can be classified into two types:

1. A *risk is <u>bad for one sector</u> of the economy* but good for another.
   - A rise in oil prices is bad for car industry but good for the energy industry.
2. *Unique risks <u>specific to one</u> person or company* and no one else.

Sources of Risk: Idiosyncratic and Systematic Risk



Reducing Risk through Diversification

- Some people take on so much risk that a **<u>single big loss</u> can wipe them out**.
  - Traders call this **"blowing up."**
- Risk can be *reduced through* **diversification**, the principle of ***<u>holding more than one risk</u>*** at a time.
  - This **reduces the idiosyncratic risk** an investor bears.
- One can ***hedge*** risks or ***spread*** them among many investments.

Hedging Risk

- **Hedging** is the strategy of reducing idiosyncratic risk by *making two investments with* ***opposing risks***.
  - If one industry is volatile, the payoffs are stable.
- Let’s compare three strategies for investing $100:
  - Invest $100 in GE.
  - Invest $100 in Texaco.
  - Invest half in each company.

Hedging Risk

Table 5.6 Results of Possible Investment Strategies: Hedging Risk

Initial Investment = $100

Investment Strategy	Expected Payoff	Standard Deviation
GE only	$110	$10
Texaco only	$110	$10
$\frac{1}{2}$ and $\frac{1}{2}$	$110	$0

- Investing $50 in each stock to ensure your payoff.
- Hedging has *eliminated your risk entirely*.

Spreading Risk

- You can't always hedge as *investments don't always move in a predictable fashion*.
- The alternative is to **spread risk** around.
  - Find investments ***whose payoffs are <u>unrelated</u>***.
- We need to look at the possibilities, probabilities and associated payoffs of different investments.

Spreading Risk

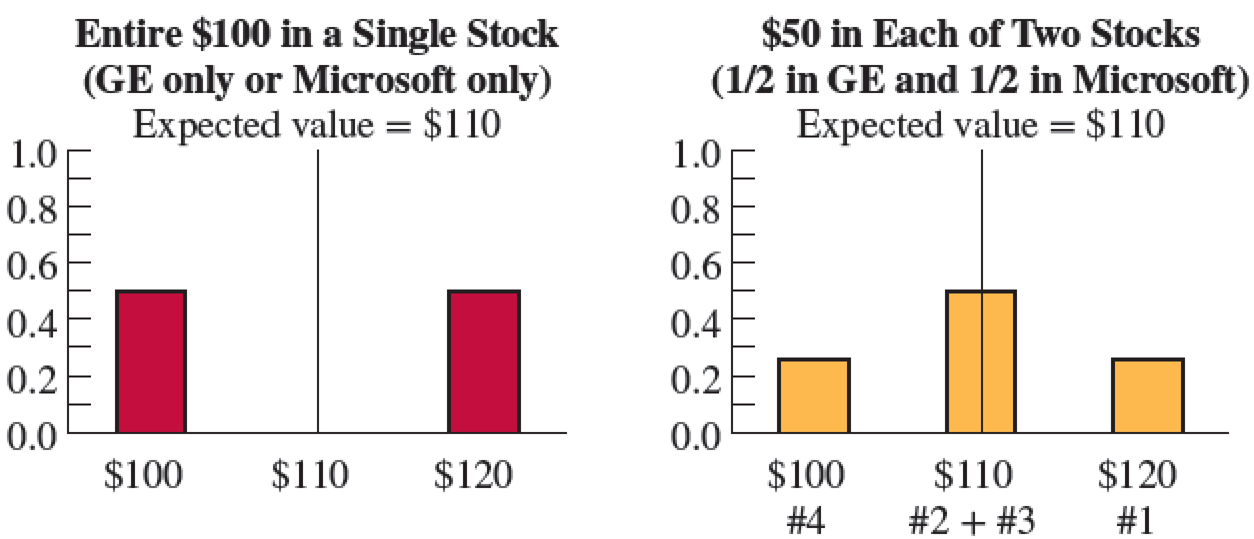
- Let's again compare three strategies for investing $1,000:
  - Invest $1,000 in GE.
  - Invest $1,000 in Microsoft.
  - Invest half in each company.

Table 5.7 Payoffs from Investing $50 in Each of Two Stocks
Initial Investment = $100

Possibilities	GE	Microsoft	Total Payoff	Probability
#1	$60	$60	$120	$\frac{1}{4}$
#2	$60	$50	$110	$\frac{1}{4}$
#3	$50	$60	$110	$\frac{1}{4}$
#4	$50	$50	$100	$\frac{1}{4}$

Spreading Risk

- We can see the distribution of outcomes from the possible investment strategies.
- This figure clearly shows **spreading risk** *lowers the spread of outcome* and *lowers the risk*.



Spreading Risk

- The **more independent sources of risk** you hold in your portfolio, the **lower your overall risk**.
- As we add more and more independent sources of risk, the ***standard deviation becomes negligible***.
- Diversification through the spreading of risk is the basis for the insurance business.

End of lecture