Serial Correlation and Heteroskedasticity in Time Series Regressions

1 The Nature of Time Series Data

TABLE 10.1

Partial Listing of Data on U.S. Inflation and Unemployment Rates, 1948–2003

Year	Inflation	Unemployment
1948	8.1	3.8
1949	−1.2	5.9
1950	1.3	5.3
1951	7.9	3.3
⋮	⋮	⋮
1998	1.6	4.5
1999	2.2	4.2
2000	3.4	4.0
2001	2.8	4.7
2002	1.6	5.8
2003	2.3	6.0

2 Examples of Time Series Regression Models

There are many time series regression models. Different models would be suitable for different types of relationship we want to estimate. Some examples of time series models are Static Model, AR (Autoregressive), ADL (Autoregressive Distributed Lag), FDL (Finite Distributed Lag), ARMA (Autoregressive Moving Average), ARCH (Autoregressive Conditional Heteroskedasticity) etc.

In this class we will talk about 2 examples 1) Static Models and 2) FDL.

2.1 Static Models

Studies a contemperaneous (occurring in the same period of time) relationship of variables.

For example:

2.2 Finite Distributed Lag Models

For example,

In general,

$$y_t = \alpha_0 + \delta_0 x_t + \delta_1 x_{t-1} + \delta_2 x_{t-2} + u_t$$

$\delta_0 = \frac{dy_t}{dx_t}$

$\delta_1 = \frac{dy_t}{dx_{t-1}}$

3 Properties of OLS under classical assumptions

Assumption TS1. Linear in Parameter – Y is linear in X.

Assumption TS2. No Perfect Collinearity
Assumption TS3. Zero Conditional Mean

***** Under Assumptions TS1 to TS3, $\hat{\beta}_{OLS}$ would be unbiased *****

Assumption TS4. Homoskedasticity

Assumption TS5. No Serial Correlation

***** Under Assumptions TS1 to TS5, $\hat{\beta}_{OLS}$ would be BLUE (best linear unbiased estimators)*****

The variance of $\hat{\beta}_{OLS}$ (under ass.TS1-TS5)

4 Properties of OLS with Serially Correlated Errors

5 Unbiasedness and Consistency

6 Efficiency and Inference

With serial correlation, $\hat{\beta}_{OLS}$ would not be BLUE ($var(\hat{\beta}_{OLS})$ would not be minimized). Consider

$$u_t = \rho u_{t-1} + e_t \ ; \ t = 1, 2, ..., n \quad \text{and} \quad |\rho| < 1$$

where u_t is from a regression model

$$y_t = \beta_0 + \beta_1 x_t + u_t.$$

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7 Testing for Serial Correlation

Given the model

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$$

7.1 A "t-test" for AR(1) serial correlation with strictly exogeneous regressors

The most common type of serial correlation or autocorrelation is the AR(1) type:

To perform the test:

1. Estimate $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$

2. Obtain $\hat{u}_t$,$\hat{u}_{t-1}$; $\forall\ t = 1, 2, ..., n$

3. Estimate $\hat{u}_t = \rho\ \hat{u}_{t-1} + error$

4. Perform the $t - test$ for

7.2 The Durbin-Watson Test (DW test)

This implies

$$\begin{aligned}
\hat{\rho} &= 0 \quad \Rightarrow \quad DW = 2 \\
\hat{\rho} &> 0 \quad \Rightarrow \quad DW < 2 \\
\hat{\rho} &< 0 \quad \Rightarrow \quad DW > 2
\end{aligned}$$

$$\begin{aligned}
H_o &: \text{ no positive autocorrelation, serial-correlation} \\
H_a &: \text{ no negative serial correlation}
\end{aligned}$$

To perform the test:

1. Estimate $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$
2. Obtain $\hat{u}_t$,$\hat{u}_{t-1}$; $\forall\ t = 1, 2, ..., n$
3. Calculate DW from eq.(2)
4. Find the critical d_L and d_u values (say, at the 5% level of significance) for the given sample size and # of regressors.
5. Follow the decision rule in the picture.

Example:
Suppose the calculated value of $DW = 0.80$, $n = 45$, $k = 4$.
From this, we get $d_L =$ _______________ and $d_u =$ ________________

7.3 Testing for AR(1) serial correlation "without" strictly exogenous regressors

7.4 Testing for AR(q) serial correlation "without" strictly exogenous regressors

8 Correcting for serial correlation

8.1 Passive way

Use the type of standard error that is robust to the serial correlation, autocorrelation problem

8.2 Active way –