Practice questions

1. Consider the standard overlapping generations model, where people have preferences defined over (c_1, c_2) and are endowed with earnings $(y_1, y_2) = (y, 0)$. There are no financial markets so that individuals cannot issue money or debt (i.e., $c_1 \leq y_1$). Let R denote the (gross) real return on money holdings; i.e., $R = p_t/p_{t+1}$, where p_t is the price-level at date t. For each of the utility functions below, derive the money demand function and explain it's properties.

   (a) $U = \ln c_1 + \beta \ln c_2$;

   (b) $U = \ln c_1 + \beta c_2$;

   (c) $U = 2c_1^{1/2} + 2c_2^{1/2}$;

   (d) $U = Ac_1^{\theta} c_2^{1-\theta}$, where $A > 0$ and $0 < \theta < 1$;

   (e) $U = c_1 + \beta \ln c_2$.

2. Suppose that individuals have preferences defined over (ℓ, c_2), where ℓ denotes current 'home production' (leisure) and c_2 denotes consumption of the 'market good' in the future period. Preferences are given by $U = \ln(\ell) + \beta c_2$. Individuals are endowed with T units of time in the current period, which they can allocate either to leisure or work (n); i.e., $n + \ell = T$. Current period output (per young person) is given by $y = \omega n$, where $\omega > 0$.

   (a) Write down the choice problem facing a young person (write out the budget constraint in terms of ℓ and c_2).

   (b) What happens to *real* GDP in this economy as inflation rises? Explain.

3. Consider two OLG economies, A and B, that are identical in every respect except for one: The citizens of country A are 'more patient' in the sense that $\beta_A > \beta_B$. The money supplies and populations in each country are constant.

   (a) Using the preferences of question 1a above, solve for the money demand function for each country. Derive the equilibrium inflation rate for each country. Derive the equilibrium value of money in each country. How do these objects differ (if at all) across countries? Explain.

Question 4.

Consider an economy consisting of two-period lived individuals with preferences given by $U = \ln c_1 + \beta \ln c_2$ (except for the initial old, who care only for c_2). Let N_t denote the population of young people at date t; the population grows at (gross) rate $n > 0$. Individuals are endowed with nonstorable output $y > 0$ when young and nothing when old. The initial old are endowed with an initial money stock M_0. Thereafter, the government expands the money stock at the (gross) rate $z > 1$. The new money is injected as a lump-sum transfer (with real value equal to a) to agents when they are old.

(a) Provide a mathematical characterization of the (stationary) general equilibrium (with valued fiat money) for this economy (it would be useful to write down the definition of an equilibrium).

(b) Solve for the equilibrium transfer level a as a function of the money growth parameter z. Plot the function (you may have to use *Maple* or some similar software). Is there a z that maximizes a?

(c) Compute the maximum utility function for a representative young person as a function of z (do this by inserting the equilibrium consumption allocations–which are functions of z – into the utility function). Compute the value function for an initial old person too. How do these value functions depend on the parameter z? Explain.

Question 5.

Consider the same environment as above except that $a = 0$; instead, new money creation is used to finance government spending g (where g denotes government purchases of output per young person). In addition, individuals have access to a private storage technology (e.g. capital) such that k units of investment today yields xk units of output in the future, where $x > 0$ is a parameter (the future marginal product of capital). Assume that there is a market for risk-free private debt that pays gross real interest R.

(a) Suppose that there is no fiat money. Compute the competitive equilibrium allocation and real interest rate (note that there will be no trade in this equilibrium).

(b) Determine the conditions under which the competitive equilibrium is Pareto optimal. Hint: Consider the choice problem of a planner who has the capacity to make intergenerational transfers; draw the feasible line and examine it's slope. Next, draw the equilibrium budget constraint and compare it's slope to the feasible line.

(c) Suppose that the competitive equilibrium is not Pareto optimal. Is there a role for fiat money? Can monetary policy alone achieve a Pareto optimal allocation (or will transfers be required to compensate potential losers)?

(d) Does the presence of an alternative asset place further restrictions on the government's ability to extract seigniorage revenues? Explain.

Question 6.

Now, consider the same environment as in the question above, abstracting from the storage technology. In this model, however, there are two types of individuals who differ in terms of the timing of their earnings. In particular, type A agents have an endowment $(y, 0)$ while type B agents have an endowment $(0, y)$. Let $0 < \lambda < 1$ denote the fraction of type A agents in the economy.

(a) Suppose that there is no fiat money. Compute the competitive equilibrium allocations and real interest rate paid on private (risk-free) debt. Assume that $\lambda = (1 + \beta)^{-1}$. Observe that the allocations across types are identical.

(b) Now consider the choice problem of a planner who maximizes the utility of a representative young person (without harming the initial old); solve for this allocation. Show that for $n = 1$, this allocation corresponds to the equilibrium allocation derived above.

(c) Suppose that $n < 1$. Show the following: (i) the planner can make the representative young person better off relative to the equilibrium; (ii) that doing so will entail harming the initial old; and (iii) such an allocation cannot be implemented with fiat money.

(d) Suppose that $n > 1$. Show the following: (i) the planner can make the representative young person better off relative to the equilibrium; (ii) the planner can accomplish this while making the initial old better off; and (iii) while money can be valued in this environment, the monetary equilibrium will not implement the planner's solution (in particular, some people will benefit while others will be harmed).

Question 7

Suppose again that individuals within a generation are identical. However, the representative young person now has preferences given by $U = \ln c_1 + \beta \ln c_2 + \theta \ln g$, where $g < y$ is consumed by the young.

(a) Show that the solution to the planner's problem entails a level of government spending given by:

$$g^* = \left[\frac{\theta}{1+\beta+\theta}\right] y.$$

(b) Now, suppose that there is a government that purchases g but must do so by issuing new money (i.e., assume that direct taxation is not a possibility). Show that the optimal (gross) rate of money creation is given by:

$$\hat{z} = 1 + \frac{\theta}{\beta};$$

and that the optimal level of government spending is given by:

$$\hat{g} = \left(\frac{\theta}{1+\theta}\right)\left(\frac{\beta}{1+\beta}\right) y < g^*.$$

Explain what is going on here. Note: in order to answer this question, you'll need to compute the equilibrium allocations as a function of g (or z). Next, use these consumption allocations to formulate an individual's value function. Then maximize this function by choosing the appropriate g (or z)

Question 8

Consider an economy consisting of two-period-lived overlapping generations with constant population size equal to $2N$. The representative young person has preferences given by $U = \ln c_1 + \beta \ln c_2$. The initial old population cares only about their remaining consumption profile, c_2. There are two types of individuals in the economy, distinguished by the timing of their earnings: type A individuals have an endowment $(y_1, 0)$, while type B individuals have an endowment $(0, y_2)$. Let λ denote the fraction of type A individuals in the economy.

(a) Let $R > 0$ denote the real (gross) return on saving. Let's consider R to be exogenous for the moment. Solve for each person's indirect utility function (including the initial old). How is the welfare of each segment of the population affected by an increase in R? Explain.

(b) Solve for the competitive equilibrium allocation and real interest rate. Explain who is trading with whom and what sort of securities are being bought and sold. How does the real rate of interest respond to an exogenous increase in the level of y_1? Explain.

(c) Suppose that the equilibrium interest rate derived above is $R^* < 1$. If the government prints up M dollars of fiat money and distributes it to the initial old (holding M fixed thereafter) then there is an equilibrium with valued fiat money (i.e., the money will end up circulating). Derive the equilibrium consumption allocations for all agents (including the initial old) as well as the equilibrium rate of return on money and bonds. Describe who is trading what and with whom. Explain who gains and who loses in this equilibrium (relative to the equilibrium without fiat money). Hint: refer to part (a) of this question.

(d) Now, suppose that the government wishes to extract seigniorage revenue. Clearly, an upper bound on the rate of money expansion is given by $z \leq 1/R^*$ (why?). As it turns out, this is not binding (in this economy) for a government interested in maximizing seigniorage. Show that the revenue maximizing money growth rate is given by $z = (1/R^*)^{1/2}$. Explain why the upper bound does not bind.