commodity or a dollar today, costs and benefits occurring at different times cannot simply be added together to determine whether to undertake the project.

For example, for a project that involves the expenditure of \$1.00 this year and results in \$1.50 return next year, we cannot simply add the $-\$1.00$ of cost this year to the \$1.50 return next year and say that the net value of the project is \$0.50. The reason is that \$1.50 next year is worth less than \$1.50 today.[6] Specifically, if the rate of interest is 20%, \$1.50 next year is worth $\$1.50/(1.2) = \$1.25$ today. The reason is that \$1.25 today will grow to \$1.50 next year. That is, $\$1.25(1 + 0.2) = \$1.50$. Thus, to determine the net return of an investment we must compare the cost incurred today with the value of the benefits *today*.

The **net present value (*NPV*)** of an investment is the value today of all the *net* cash flows of the investment. Expenditures or outflows are subtracted from revenues or inflows in each year to find the net cash flow. For a two-period time horizon, NPV is given by

$$NPV = -C + \frac{R_1}{1+r} \qquad [15.5]$$

where C is the capital investment cost in the current year when the investment is made, R_1 is the net cash flow next year, and r is the rate of interest. For example, suppose that a firm purchases a machine this year for \$100, and this increases the firm's net income by \$120 next year. Suppose also that the rate of interest is 10%, and the machine has no salvage or scrap value at the end of the next year. The net present value of the machine (NPV) is

$$NPV = -\$100 + \frac{\$120}{(1+0.1)} = -\$100 + \$109.09 = \$9.09$$

This means that the purchase of the machine will increase the wealth of the firm by \$9.09.

Suppose the firm had to decide between the above project (with net present value of \$9.09) and another project that costs \$150 and generates a net income of \$180 next year. The firm should choose the second project because its net present value of $-\$150 + \$180/(1 + 0.1) = \$13.64$ exceeds the net present value of \$9.09 for the previous project. Such a choice arises because firms do not usually have or cannot usually borrow all of the resources required to undertake all of the projects that have a positive net present value.

We will see in the next section that the rule to undertake a project if its net present value is positive or to choose the project with the highest net present value is a general rule and applies to all projects, regardless of the number of periods or years over which the costs and returns of the project are spread. Furthermore, this rule is independent of the tastes or time preference of the investor. That is, regardless of the shape and location of the indifference curves

[6] We assume throughout this discussion that there is no price inflation. This assumption is relaxed in section 15.4.

of investors, the general investment or capital budgeting rule is to maximize the wealth of the firm. This is achieved by investing in projects with the highest (positive) net present value. The tastes of investors will then determine whether they will borrow or lend and how they will choose to use their (maximized) wealth. This means that if capital markets are perfect and costless (i.e., if borrowers and lenders are too small individually to affect the rate of interest and can borrow or lend at the same rate), then individuals' production and consumption decisions can be kept completely separate. This is sometimes called the **separation theorem.**

Net Present Value Rule for Investment Decisions: The Multiperiod Case

Most investment projects last longer than (i.e., give rise to net cash flows over more than) two periods. Thus, the investment rule given above must be extended to consider many periods (years). This can easily be done by "stretching" equation [15.5] to deal with many (n) years. This is given by

$$NPV = -C + \frac{R_1}{1+r} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} \qquad [15.6]$$

where NPV is the net present value of the investment, C is the capital investment cost incurred this year when the investment is made, R_1 is the net cash flow from the investment next year, R_2 is the net cash flow in two years, R_n is the net cash flow in n years, and r is the rate of interest. Net cash flows refer to the revenue of the firm resulting from the investment during any given year minus the expenses or costs of the project during the same year. Thus, equation [15.5] is a special case of equation [15.6], applicable when there is no net cash flow after the first year.[7]

For example, suppose that a firm purchases a machine this year for \$150, and this increases the firm's net income by \$100 in each of the next two years. If the rate of interest is 10% and the machine has no salvage value after two years, the net present value of the machine (NPV) is

$$NPV = -C + \frac{R_1}{1+r} + \frac{R_2}{(1+r)^2} = -\$150 + \frac{\$100}{1+0.1} + \frac{\$100}{(1+0.1)^2}$$
$$= -\$150 + \$90.91 + \$82.64 = \$23.55$$

[7]If we simply wanted to find the present discounted value (PDV) of a given sum (R) to be received in each of n years, starting with the next year, the equation would simply be

$$PDV = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n}$$

If the given sum (R) were to be received in each year indefinitely or in perpetuity (an example of this is the British "consols"), then $PDV = R/r$ (see section A.15 of the Mathematical Appendix).

This means that the purchase of the machine will increase the wealth of the firm by \$23.55. Specifically, \$100 received next year is worth $\$100/(1 + 0.1) = \$90.91$ this year, because \$90.91 this year grows to \$100 next year at $r = 10\%$. On the other hand, \$100 received two years from now is worth $\$100/(1 + 0.1)^2 = \$82.64$ this year because \$82.64 this year grows to \$90.91 next year at $r = 10\%$,[8] and the \$90.91 next year grows to \$100 the year after (i.e., two years from now) at $r = 10\%$. That is \$82.64 today times $(1 + 0.1)^2$ or 1.21 equals \$100 two years from now.

If the project generated a net cash flow of \$100 in the third year also, this would be "worth" $\$100/(1 + 0.1)^3 = \$100/(1.331) = \$75.13$ this year because \$75.13 this year will grow to \$100 (except for rounding errors) in three years at $r = 10\%$. Similarly, \$100 in 10 years is worth $\$100/(1 + 0.1)^{10} = \$38.55$ this year because \$38.55 this year will grow to \$100 in 10 years at $r = 10\%$. Finally, \$100 in n years (where n is any number of years) is worth $\$100/(1 + 0.1)^n$ this year because this sum today will grow to \$100 in n years at $r = 10\%$.

The *NPV* for any net cash flow is inversely related to r. For example, if r had been 5% in the previous example, the net present value of the investment (NPV') would have been

$$NPV' = -\$150 + \frac{\$100}{(1 + 0.05)^1} + \frac{\$100}{(1 + 0.05)^2}$$
$$= -\$150 + \$95.24 + \$90.70 = \$35.94$$

(as compared with the *NPV* of \$23.55 with $r = 10\%$). On the other hand, if r had been 20%,

$$NPV'' = -\$150 + \frac{\$100}{(1 + 0.2)^1} + \frac{\$100}{(1 + 0.2)^2}$$
$$= -\$150 + \$83.33 + \$69.44 = \$2.77$$

The lower *NPV* when r is higher is due to the fact that with a higher r, the *net* cash flows from the project are "discounted" more heavily than the cost, because the net cash flows arise later in time than cost. Also note that the *net* cash flows should include the extra income generated by the machine minus the extra expense (such as maintenance and the higher cost of hiring more skilled workers) to operate the machine during each year. Similarly, the value today of the salvage value of the machine (if any) must also be included.

For example, suppose that the benefits and costs of an investment project (the purchase of a piece of machinery) are those given in Table 15.4. The table shows that the machine costs \$1,000 to purchase this year and also gives rise to \$200, \$300, \$300, and \$400 maintenance and other expenses in each of the subsequent four years. The revenues from the investment are \$600, \$800, \$800, and \$800, and the

[8] Actually, (\$82.64)(1.1) equals \$90.904 rather than \$90.91 (as indicated) because of rounding errors.

TABLE 15.4 BENEFIT-COST ANALYSIS OF AN INVESTMENT PROJECT

END OF YEAR	INVESTMENT (YEAR 0) AND COST	REVENUE	NET REVENUE	PRESENT VALUE COEFFICIENT $1/(1 + 0.1)^n$	PRESENT VALUE OF NET REVENUE
0	$1,000	...	−$1,000	...	−$1,000
1	200	$600	400	0.909	364
2	300	800	500	0.826	413
3	300	800	500	0.751	376
4	400	800	400	0.683	273
4	...	200*	200	0.683	137
					$563

*Salvage value.

salvage value of the machine is $200 at the end of the fourth year. The net revenue is the revenue from the investment minus the expenses in each year. The present value coefficient is $1/(1 + 0.1)^n$. For example, for the first year the present value coefficient is $1/(1 + 0.1)^1 = 0.909$. For the second year, it is $1/(1 + 0.1)^2 = 0.826$, and so on. The present value of the net revenue in each year is obtained by multiplying the net revenue (R) by the present value coefficient for that year. By adding together all present values of the net revenues, we get the net present value of the project (V_0) of $563. Since *NPV* is positive, the firm should purchase the machine.

Sometimes, complications may arise in applying the net present value rule for investment decisions. First, projects may be interdependent, so that the stream of net cash flows from a project depends on whether other projects are undertaken at the same time. In such a case, the net present value of a group of projects may have to be evaluated together and compared with the net present value of other groups of projects. Second, it may sometimes be difficult to accurately forecast the future stream of net cash flows from a project. Third, the firm may not have the resources and may not be willing or able to borrow to undertake all of the projects that have a positive net present value. The firm should then choose those projects with the highest net present value.

EXAMPLE 4
FIELDS OF EDUCATION AND HIGHER LIFETIME EARNINGS IN THE UNITED STATES

Table 15.5 gives the present value of the higher lifetime earnings with a college degree in various fields in the United States in 1995. Present values were calculated by capitalizing (i.e., finding the present discounted value of) the difference between the higher yearly salaries with a bachelor's degree in the various fields over the average salary of workers with only a high school diploma. The interest rate used to find the present values was 5%. Only the benefits of going to college were included; the earnings foregone or opportunity costs and other costs (tuition, books, and so on) of going to college were not included. Note that the higher earnings of the recipients of the bachelor's degree over the earnings of noncollege graduates cannot be attributed entirely to college education. At least in part, the higher incomes of college graduates may be due to their higher level of intelligence, longer working hours, and more inherited wealth than for noncollege graduates. Earnings differentials with and without a college degree declined during the 1970s and increased during the 1980s.

SOURCE: "Soaring Payoff from Higher Education," *The Margin*, January/February 1990, p. 6; J. Bound and G. Johnson, "Changes in the Structure of Wages in the 1980s: An Evaluation of Alternative Explanations," *American Economic Review*, June 1992, pp. 371–392; "Technological Change Boosts Wage Gap," The *Margin*, Spring 1993, pp. 42–43; and J. Mincer, "Investment in U.S. Education and Training," National Bureau of Economic Research, *Working Paper no. 4844*, October 1995.

TABLE 15.5 Present Values of Higher Lifetime Earnings with Bachelor's Degree in Various Fields, 1995

FIELD OF STUDY	HIGHER LIFETIME EARNINGS
Electrical engineering	$357,240
Computer science	310,500
Nursing	293,000
Mathematics	241,680
Chemistry	223,060
Accounting	194,780
Economics/finance	189,260
Marketing	144,260
Education	91,800
Communications	90,460

SOURCE: Calculated from data reported in Salary Survey, National Association of Colleges and Employers, September 1995.

15.4 DETERMINANTS OF THE MARKET RATES OF INTEREST

Until now we have discussed "the" interest rate; however, the rate of interest varies at different times and in different markets. Even at a given point in time and in a specific capital market, there is not a single rate of interest but many. That is, there is a different interest rate on different loans or investments depending on differences in (1) risk, (2) duration of the loan, (3) cost of administering the loan, and (4) tax treatment. We now briefly examine each of these in turn.

The major reason for differences in rates of interest at a given point in time and place is the risk of the loan. In general, the greater the risk, the higher the rate of interest. Two types of risk can be distinguished: default risk and variability risk. **Default risk** refers to the possibility that the loan will not be repaid. If the chance of default is 10%, the lender will usually charge a rate of interest 10% higher than on a loan with no risk of default, such as a government bond. Similarly, loans unsecured by collateral (such as installment credit) usually charge higher rates of interest than loans secured by collateral (such as home mortgages). **Variability risk** refers to the possibility that the yield or return on an investment, such as a stock, may vary considerably above or below the average. Given the usual aversion to risk, investors generally demand a premium or a higher yield for investments whose returns are more uncertain.

The second reason for differences in rates of interest is the duration of the loan. Loans for longer periods of time usually require higher rates of interest than loans for shorter durations. The reason is that the lender has less flexibility or liquidity with loans of longer duration, and so he or she will require a higher rate of interest. It is for this reason that savings deposits offer lower rates of interest than six-month certificates of deposit.[9]

The third reason for differences in rates of interest is the cost of administering the loan. Smaller loans and loans requiring frequent payments (such as installment loans) usually involve greater bookkeeping and service costs per dollar of the loan and, as a result, usually involve a higher interest charge. Finally, the tax treatment of interest and investment income can lead to differences in rates of interest among otherwise comparable loans and investments. For example, state and municipal bonds are exempted from federal income tax, and since investors look at the after-tax return, state and local governments can usually borrow at lower interest rates than corporations.

Thus, at a given point in time and in a given capital market there is a large number of interest rates depending on relative risk, term structure, administration costs, and tax treatment. Yet all of these rates of interest are related. If individuals and firms collectively decide to save less (a leftward shift in the aggregate supply curve of savings), interest rates will rise. Interest rates will also rise if the time preference of consumers shifts in favor of the present or if the net productivity or yield of capital increases. In addition, a rise in short-term rates will lead to higher long-term rates, and vice versa. Furthermore, higher interest rates for comparable instruments in one market than in another market will lead to an outflow of funds from the latter to the former. These flows of funds will reduce (and may eventually eliminate) interest rate differences between the two markets. Specifically, the supply curve of funds will shift to the left (i.e., the supply of funds decreases) and interest rates will rise in the market with lower

[9] Regulation may also account for part of the difference.

rates of interest. The opposite occurs in the market with the higher rates of interest, causing interest rates to fall there.

Finally, a distinction must be made between real and nominal or money interest rates. Until this point, we have been discussing the **real rate of interest (r).** This refers to the premium on a unit of a commodity or real consumption income today compared to a unit of the commodity or real consumption income in the future. However, in the everyday usage of the term, the interest rate refers to the nominal or money rate of interest. The **nominal rate of interest (r')** refers to the premium on a unit of a monetary claim today compared to a unit of monetary claim in the future. The nominal rate of interest (r') is affected by the anticipated rate of price inflation (i), while the real rate of interest is not. Thus, the nominal rate of interest equals the real rate of interest plus the anticipated rate of price inflation. That is,

$$r' = r + i \qquad [15.7]$$

The reason for this is that during the period of the loan, the general price level may rise (i.e., inflation may occur) so that the loan is repaid with dollars of lower purchasing power than the dollars borrowed. Therefore, the nominal rate of interest must be sufficiently high to cover any increase in the price level (or in the price of real claims) during the loan period. It is primarily to avoid this complication (and to deal with the real rate of interest) that we chose to borrow and lend a *commodity* in sections 15.1 and 15.2.

Anyone who borrows money now and repays in money in the future must expect to pay an additional monetary amount to cover any anticipated increase in the monetary price of real claims by the time of repayment. Only if anticipated inflation is zero will $r' = r$. Since some price inflation is always occurring, r' usually exceeds r. For example, if $r' = 11\%$ and $i = 6\%$, then $r = 5\%$. We have concentrated on the real rate of interest throughout most of the chapter because it is the real, and not the nominal, rate of interest that primarily affects incentives to borrow and lend, and to save and invest.

EXAMPLE 5
NOMINAL AND REAL INTEREST RATES IN THE UNITED STATES: 1981–1994

Table 15.6 shows the nominal annual interest rates on three-month U.S. Treasury Bills, the change in the Consumer Price Index, and the real interest rate (the difference between the nominal interest rate and the change in the Consumer Price Index) in the United States from 1981 to 1994. The implicit assumption made here is that the anticipated rate of inflation is equal to the actual rate of inflation. Since the nominal interest rate was greater than the change in the Consumer Price Index (except for 1993, when it was equal), the real interest rate was positive (except for

1993, when it was zero). Note that the nominal interest rate moved in the same direction as the change in the Consumer Price Index, except in 1986–1987, 1989–1990, and 1992–1994. As a result, the real interest rate reflected changes in the nominal interest rate and in the Consumer Price Index, as expected. When interest payments are tax deductible, it is the real after-tax interest rate that affects incentives to lend, borrow, and invest.

TABLE 15.6 Nominal and Real Interest Rates on Three-Month U.S. Treasury Bills: 1981–1994

YEAR	NOMINAL INTEREST RATE	CHANGE IN CONSUMER PRICE INDEX	REAL INTEREST RATE
1981	14.0	10.3	3.7
1982	10.7	6.2	4.5
1983	8.6	3.2	5.4
1984	9.6	4.3	5.3
1985	7.5	3.6	3.9
1986	6.0	1.9	4.1
1987	5.8	3.6	2.2
1988	6.7	4.1	2.6
1989	8.1	4.8	3.3
1990	7.5	5.4	2.1
1991	5.4	4.2	1.2
1992	3.5	3.0	0.5
1993	3.0	3.0	0.0
1994	4.3	2.6	1.7

Council of Economic Advisers, *Economic Report of the President* (Washington, D.C.: U.S. Government Printing Office, 1995), pp. 345, 358.

EXAMPLE 6
INVESTMENT RISKS AND RETURNS IN THE UNITED STATES

Table 15.7 shows that riskier assets, such as common stocks or long-term corporate bonds, have provided higher average real rates of return than more liquid and less risky assets, such as U.S. (three-month) Treasury Bills, in the United States from 1926 to 1994. While corporate bonds can provide higher returns than stocks during some years over many years the reverse is usually true. Risk is measured here as the standard deviation of the real return as a percentage of the mean or average real return. Common stocks involve a great deal of variability risk while long-term corporate bonds involve a greater default risk and are less liquid (and therefore provide higher average real returns) than U.S. Treasury Bills. Clearly, risk-averse investors must balance expected return against risk in their investment decisions.

TABLE 15.7 INVESTMENT RISKS AND RETURNS IN THE UNITED STATES, 1926–1994

ASSET	REAL RATE OF RETURN (%)	RISK (STANDARD DEVIATION, %)
Common Stocks	8.9	20.5
Long-term Corporate Bonds	2.7	9.8
U.S. Treasury Bills	0.6	4.2

SOURCE: Ibbotson & Associates, *Stocks, Bonds, Bills, and Inflation, 1995 Yearbook* (Chicago, 1995), p. 119.

15.5 THE COST OF CAPITAL

We now examine how a firm estimates the cost of raising capital to invest. This is an essential element of the capital budgeting process. The firm can raise investment funds internally (i.e., from undistributed profits) or externally (i.e., by borrowing and/or from selling stocks). The cost of using internal funds is the opportunity cost or foregone return on these funds outside the firm. The cost of external funds is the lowest rate of return that lenders and stockholders require to lend to or invest their funds in the firm. In this section, we examine how the cost of debt (i.e., the cost of raising capital by borrowing) and the cost of equity capital (i.e., the cost of raising capital by selling stocks) are determined. The estimation of the cost of debt is fairly straightforward. On the other hand, there are at least three methods of estimating the cost of equity capital: the risk-free rate plus premium, the dividend valuation model, and the capital asset pricing model (CAPM). These methods will be examined in turn.

Cost of Debt

The **cost of debt** is the return that lenders require to lend their funds to the firm. Since the interest payments made by the firm on borrowed funds are deductible from the firm's taxable income, the *after-tax* cost of borrowed funds to the firm (k_d) is given by the interest paid (r) multiplied by 1 minus the firm's marginal tax rate, t. That is,

$$k_d = r\,(1 - t) \qquad [15.8]$$

For example, if the firm borrows at a 12.5% interest rate and faces a 40% marginal tax rate on its taxable income, the after-tax cost of debt capital to the firm is

$$k_d = 12.5\%(1 - 0.40) = 7.5\%$$

...irm. ...
...aising equi...
...ne stock). For si...
...d treat both types of equ...
...s opposed to the interest paid...
...ense (i.e., dividends are paid out a...
...e is no tax adjustment in determining the c...
...e method employed to estimate the cost of equ...
...sk-free rate (r_f) plus a risk premium (r_p). That is,

$$k_e = r_f + r_p$$

The risk-free rate (r_f) is usually taken to be the six-month U.S. Tre...
...is is because the obligation to make payments of the interest an...
...nment securities is assumed to occur with certainty. The risk p...
...st be paid in raising equity capital has two components. The firs...
...ts because of the greater risk that is involved in investing in a...
...uch as bonds) as opposed to investing in federal government se...
...nd component is the additional risk resulting from purchasing t...
...ther than the bonds of the firm. Stocks involve a greater risk than
...dends on stocks are paid only after the firm has met its contrac...
...ake interest and principal payments to bondholders. Because
...firm's profits, stocks are more risky than bonds, so that
...an additional risk premium. If the premiums associated
...are labeled p_1 and p_2, we can restate the formula for

$$r_f + p_1 + p_2 \quad [15.10]$$

...ally measured by the excess of the rate of
...ate of return on government bonds (r_f).
..., the firm's stocks rather than bonds
...entage points. This is the historical
... plus capital gains) on stocks as
...ivate companies. For example,
...s is 8% and the firm's bonds

...ne for r_f...

yield 11%, the total risk premium (r_p) involved in purchasing the firm's stocks rather than government bonds is

$$r_p = p_1 + p_2 = (11\% - 8\%) + 4\% = 3\% + 4\% = 7\%$$

so that the firm's cost of equity capital is

$$k_e = r_f + p_1 + p_2 = 8\% + 3\% + 4\% = 15\%$$

Cost of Equity Capital: The Dividend Valuation Model

The equity cost of capital to a firm can also be estimated by the **dividend valuation model.** To derive this model, we begin by pointing out that, with perfect information, the value of a share of the common stock of a firm should be equal to the present value of all future dividends expected to be paid on the stock, discounted at the investor's required rate of return (k_e). If the dividend per share (D) paid to stockholders is expected to remain constant over time, the present value of a share of the common stock of the firm (P) is then

$$P = \sum_{t=1}^{\infty} \frac{D}{(1 + k_e)^t} \tag{15.11}$$

If dividends are assumed to remain constant over time and to be paid indefinitely, equation [15.11] can be rewritten as

$$P = \frac{D}{k_e} \tag{15.12}$$

If dividends are instead expected to increase over time at the annual rate of g, the price of a share of the common stock of the firm will be greater and is given by

$$P = \frac{D}{k_e - g} \tag{15.13}$$

Solving equation [15.13] for k_e, we get the following equation to measure the equity cost of capital to the firm:

$$k_e = \frac{D}{P} + g \tag{15.14}$$

That is, the investor's required rate of return on equity is equal to the ratio of the dividend paid on a share of the common stock of the firm to the price of a share of the stock (the so-called dividend yield) plus the expected growth rate of dividend payments by the firm (g). The value of g is the firm's historic growth rate or the earnings growth forecasts of securities analysts (based on the expected sales, profit margins, and competitive position of the firm) published in *Business Week*, *Forbes*, and other business publications.

For example, if the firm pays a dividend of \$20 per share on common stock that sells for \$200 per share and the growth rate of dividend payments is expected to be 5% per year, the cost of equity capital for this firm is

$$k_e = \frac{\$20}{\$200} + 0.05 = 0.10 + 0.05 = 0.15 \text{ or } 15\%$$

Cost of Equity Capital: The Capital Asset Pricing Model (CAPM)

Another method commonly used to estimate the equity cost of capital is the **capital asset pricing model (CAPM).** This model considers not only the risk differential between common stocks and government securities but also the risk differential between the common stock of the firm and the average common stock of all firms, or broad-based market portfolio. The risk differential between common stocks and government securities is measured by $(k_m - r_f)$, where k_m is the average return on all common stocks and r_f is the return on government securities.

The risk differential between the common stock of a particular firm and the common stock of all firms is called the **beta coefficient,** β. This is the ratio of the variability in the return of the common stock of the firm to the variability in the average return on the common stocks of all firms. Beta coefficients for individual stocks can be obtained from the *Value Line Investment Survey,* Merrill Lynch, or other brokerage firms.

A beta coefficient of 1 means that the variability in the returns on the common stock of the firm is the same as the variability in the returns on all stocks. Thus, investors holding the stock of the firm face the same risk as holding a broad-based market portfolio of all stocks. A beta coefficient of 2 means that the variability in the returns on (i.e., risk of holding) the stock of the firm is twice that of the average stock. On the other hand, holding a stock with a beta coefficient of 0.5 is half as risky as holding the average stock.

The cost of equity capital to the firm estimated by the capital asset pricing model (CAPM) is then measured by

$$k_e = r_f + \beta(k_m - r_f) \qquad [15.15]$$

where k_e is the cost of equity capital to the firm, r_f is the risk-free rate, β is the beta coefficient, and k_m is the average return on the stock of all firms. Thus, CAPM postulates that the cost of equity capital to the firm is equal to the sum of the risk-free rate plus the beta coefficient (β) times the risk premium on the average stock $(k_m - r_f)$. Note that multiplying β by $(k_m - r_f)$ gives the risk premium on holding the common stock of the particular firm.

For example, suppose that the risk-free rate (r_f) is 8%, the average return on common stocks (k_m) is 15%, and the beta coefficient (β) for the firm is 1. The cost of equity capital to the firm (k_e) is then

$$k_e = 8\% + 1(15\% - 8\%) = 15\%$$

That is, since a beta coefficient of 1 indicates that the stock of this firm is as risky as the average stock of all firms, the equity cost of capital to the firm is 15% (the same as the average return on all stocks). If $\beta = 1.5$ for the firm (so that the risk involved in holding the stock of the firm is 1.5 times larger than the risk on the average stock), the equity cost of capital to the firm would be

$$k_e = 8\% + 1.5(15\% - 8\%) = 18.5\%$$

On the other hand, if $\beta = 0.5$

$$k_e = 8\% + 0.5(15\% - 8\%) = 11.5\%$$

In this example, and in the examples using the risk-free rate plus premium and the dividend valuation model, the equity cost of capital was found to be the same (15%). This is seldom the case. That is, the different methods of estimating the equity cost of capital to a firm are likely to give somewhat different results. Firms are thus likely to use all three methods and then attempt to reconcile the differences to arrive at an equity cost of capital for the firm.

Weighted Cost of Capital

In general, a firm is likely to raise capital from undistributed profits, by borrowing, and by the sale of stocks, and so the marginal cost of capital to the firm is a weighted average of the cost of raising the various types of capital. Since the interest paid on borrowed funds is tax deductible while the dividends paid on stocks are not, the cost of debt is generally less than the cost of equity capital. The risk involved in raising funds by borrowing, however, is greater than the risk on equity capital because the firm must regularly make payments of the interest and principal on borrowed funds before paying dividends on stocks. Thus, firms do not generally raise funds only by borrowing but also by selling stock (as well as from undistributed profits).

Firms often try to maintain or achieve a particular long-term capital structure of debt to equity. For example, public utility companies may prefer a capital structure involving 60% debt and 40% equity, whereas auto manufacturers may prefer 30% debt and 70% equity. The particular debt-equity ratio that a firm prefers reflects the risk preference of its managers and stockholders and the nature of the firm's business. Public utilities accept the higher risk involved in a higher debt-to-equity ratio because of their more stable flow of earnings than automobile manufacturers. When a firm needs to raise investment capital, it borrows and sells stocks so as to maintain or achieve a desired debt-to-equity ratio.

The **composite cost of capital** to the firm (k_c) is then a weighted average of the cost of debt capital (k_d) and equity capital (k_e) as given by

$$k_c = w_d k_d + w_e k_e \qquad [15.16]$$

where w_d and w_e are, respectively, the proportion of debt and equity capital in the firm's capital structure. For example, if the (after-tax) cost of debt is 7.5%, the cost of equity capital is 15%, and the firm wants to have a debt-to-equity ratio of 40:60, the composite or weighted marginal cost of capital to the firm is

$$k_c = (0.40)(7.5\%) + (0.60)(15\%) = 3\% + 9\% = 12\%$$

That is, the proportion of debt to equity that the firm seeks to achieve or maintain in the long run is not usually defined for individual projects but for all the investment projects that the firm is considering. Note that the marginal cost of capital eventually rises as the firm raises additional amounts of capital by borrowing and selling stocks because of the higher risk that lenders and investors face as the firm's debt-to-equity ratio increases.

AT THE FRONTIER

DERIVATIVES: USEFUL BUT DANGEROUS

During the past few years, news about derivatives has been all over the financial and front pages of our newspapers. **Derivatives** are financial instruments or contracts whose value is derived from the price of such underlying financial assets as stocks, bonds, commodities, and currencies. Derivatives are offered by banks and brokerage firms to corporations and investors who wish to protect themselves against such risks as a decrease in the international value of the dollar, an increase in the price of a commodity, or an increase in interest rates. Used properly, derivatives can be a very useful risk-management tool; used without a clear understanding of all their implications, they can be very dangerous and can lead to huge losses. Despite the dangers and risks involved in their use, derivatives have been growing at the torrid rate of 40% per year during the past few years and are clearly here to stay. The value of the underlying assets on which derivatives are based (called notional value) now exceeds $35 trillion (as compared with the $6.4 trillion of the U.S. gross domestic product or GDP in 1995).

Derivatives come in two basic categories: option-type contracts and forward-type contracts. Both types may be listed on the exchanges or negotiated privately between institutions (i.e., over the counter). *Options* give buyers the right, but not the obligation, to buy (a call) or sell (a put) an asset at a specific price over a given time period. The option price or premium is usually a small percentage of the price of the value of the underlying asset. Although the buyer can never lose more than the premium he or she paid for the option (by not exercising it), the seller's potential

Continued . . .

Derivatives: Useful but Dangerous Continued . . .

losses are unlimited. For example, you can buy a call on GE stock for a specified time, at a specified price, and for a specified premium. If within the specified time the price of the GE stock falls below the specified price, you do not exercise the call option to buy the stock and your losses are limited to the premium that you paid for the call. On the other hand, if the price of GE stock rises above the specified price, you exercise the call and buy the stock and gain the difference between the higher market price of the stock and the agreed call price.

Forward-type contracts, on the other hand, include forwards, futures, and swaps. *Forwards* commit both buyer and seller to trade a fixed amount of a given asset for a specific price at a specific future date. For example, a U.S. corporation that has to make a payment in German marks in three months faces the risk that the dollar price of the German mark might be much higher in three months. The corporation can hedge its foreign exchange risk by entering today into a contract to purchase the quantity of German marks that it will need at a price agreed upon today for delivery and payment in three months. In three months, the U.S. corporation will get the specified quantity of German marks by paying the agreed price, whatever the dollar price of the German mark is on that day.

Futures are standardized forward agreements to buy or sell a fixed amount of an asset (currency, bond, stock, or commodity) traded on an exchange. *Swaps* are agreements involving an exchange of streams of payments over time according to terms agreed today. The most common type of swap is an interest rate swap, in which one party holding a fixed-interest rate mortgage and believing that interest rates will fall, exchanges it for a flexible exchange rate mortgage owned by another party who believes that interest rates will rise. The first party will then pay the other's flexible rate mortgage in exchange for the second party paying the fixed rate mortgage of the first.

Many derivatives are the simple options, forwards, futures, and swaps discussed above (called "plain vanilla" derivatives). They form the bread and butter of the derivatives market and are used mostly to hedge risks; they provide dealers only thin profit margins. Some of these derivatives, such as currency options and futures, have been used by U.S. and foreign corporations for decades and have been spared criticism. The derivatives that have caught most of the attention and criticism in recent years are the customized, over-the-counter, exotic derivatives whose dazzling growth to over $20 trillion since the late 1980s has provided huge profits for dealers and tremendous risks for users. Examples of exotic derivatives are options that depend on the amount by which one asset will outperform another, or that gamble the price of oil will not fall below a specified price or that interest rates do not rise above a given level. The list of derivative products is very long and new ones are being created every day.

By allowing users to make big bets with little or no money down (i.e., by providing huge leverage), derivatives became very popular during the 1992–1993 bull market in bonds, when heavy bets were made that interest rates would continue to fall. As long as interest rates did fall, these derivatives provided huge profits for

Continued . . .

Derivatives: Useful but Dangerous Continued . . .

users and fat fees for providers. But when interest rates abruptly reversed course in early 1994, a number of companies that had invested heavily in these exotic derivatives faced huge losses. A fund managed by Orange County, California, lost more than $1.5 billion and was forced to declare bankruptcy. Procter & Gamble lost nearly $200 million on leveraged interest derivatives and sued Bankers Trust (the bank that sold it the derivatives) for failing to properly explain the risks involved. Air Products lost more than $100 million on derivatives, Sandoz lost almost $80 million, Federal Paper Board $47 million, and Gibson Greetings $23 million (which successfully sued Bankers Trust). As part of the Gibson Greetings' legal settlement, Bankers Trust was required to detail the risks of exotic derivatives before the derivatives are bought and to report the value of those derivatives to buyers every day, even if not asked. Indeed, the requirements of the settlement could become the de facto standard for the entire industry.

Despite the Gibson Greetings settlement there are still little or no disclosure requirements in the United States and abroad on derivatives users. Most derivatives are treated as "off-balance-sheet" items and mentioned in footnotes in companies' financial statements. The systemic danger to which this gives rise is that a party to a derivative transaction may be unable to meet the terms of the transaction and lead to the insolvency of its counterparty (the other party of the derivative transaction), and that this may trigger a domino effect of defaults that could threaten the stability of the entire financial system. The Federal Reserve System and other regulators now face the delicate task of regulating the derivatives market without leading this very lucrative business for many U.S. banks (which are world leaders in this market) to relocate abroad. One proposal is for firms to report their derivatives profits or losses on their earnings statements, rather than carrying them on their books at cost, as they do now.

SOURCES: "The Risk That Won't Go Away," *Fortune*, March 7, 1994, pp. 40–60; "Untangling Derivatives Mess," *Fortune*, March 20, 1995, pp. 51–68; and "The Bankers Trust Tapes," *Business Week*, October 16, 1995, pp. 106–111.

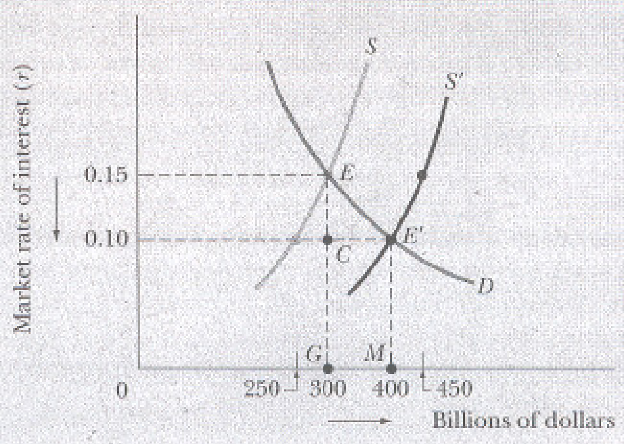
15.6 EFFECTS OF FOREIGN INVESTMENTS ON THE RECEIVING NATION

Foreign investments reduce the supply of investment funds in the investing nation and increase them in the receiving nation. Foreign investments flow from the nation where the rate of return on investment is lower to the nation where the rate of return is higher. During the 1980s, the United States was the recipient of a large net inflow of foreign investments because of the higher rate of return on investments in the United States than abroad. We can examine the effect of foreign investments on the receiving nation with Figure 15.7. For simplicity, we assume that the rate of interest on borrowed capital is the same as the rate of return on both borrowed and equity capital in the nation.

In Figure 15.7, *D* is the nation's demand curve for investment funds, while *S* is the domestic supply curve of investment funds. *D* and *S* intersect at point *E*

FIGURE 15.7 EFFECTS OF FOREIGN INVESTMENTS ON THE RECEIVING NATION

D is the demand curve for and S is the supply curve of investment funds in the nation in the absence of foreign investments. D and S intersect at point E, defining the equilibrium rate of return (r) of 0.15 or 15% and level of investments of \$300 billion. With \$150 billion of foreign investments, S shifts to S', defining the new equilibrium point of E', at which total investments in the nation rise to \$400 billion (\$250 billion domestic and \$150 billion foreign) and r falls to 0.10 or 10%. The \$150 billion foreign investment increases the total output of the nation by $GEE'M$ (about \$12.5 billion), of which $GCE'M$ = \$10 billion is the return on foreign investments and $EE'C$ (about \$2.5 billion) is the net gain of the nation.



indicating that, in the absence of foreign investments, \$300 billion is invested in the nation at the rate of return (r) of 0.15 or 15%. With \$150 billion of foreign investment, the nation's supply curve shifts to the right to S'. The intersection of D and S' defines the new equilibrium point of E', which indicates that total investments in the nation are \$400 billion and the rate of return on domestic and foreign investments is 0.10 or 10%. Thus, foreign investments reduced the rate of return on domestic investments from 15% to 10% (and this led to a reduction in the quantity supplied of domestic investments to \$250 billion).

The \$150 billion foreign investment increases the total output of the receiving nation by $GEE'M$ (about \$12.5 billion), of which $GCE'M$ = \$10 billion is paid out to foreign investors as the return on their investment, and $EE'C$ (about \$2.5 billion) represents the net benefit or gain to the nation receiving the foreign investments. Some of this net benefit goes to domestic labor because the inflow of foreign investments increases the capital-labor ratio and thus the productivity and wages of domestic labor. Another benefit (not shown in the figure) accruing to the nation receiving the foreign investments is the taxes paid by foreigners on the income (the return on investments) earned in the nation.

In 1985, the United States changed from being a net creditor to being a net debtor nation (i.e., the amount that foreigners lent and invested in the United States began to exceed the amount that the United States lent and invested abroad) for the first time since 1914. Indeed, with a net foreign debt of almost $600 billion in 1994, the United States was the most indebted nation in the world. This large debt sparked a lively debate among economists, politicians, and government officials in the United States regarding the benefits, and risks of this recent development.

In terms of benefits, foreign investments allowed the United States to finance about half of its budget deficit without the need for still higher interest rates and more "crowding out" of private investments. To the extent that foreign investments went into directly productive activities and the return on this investment was greater than the interest and dividend payments flowing to foreign investors, this investment was beneficial to the United States.[11] To the extent, however, that foreign investments simply financed larger consumption expenditures in the United States, the interest and dividend payments flowing to foreign investors represent a real burden or drain on future consumption and growth in the United States. Some experts are concerned that a growing share of capital inflows to the United States since 1983 cannot be clearly identified as productive investments, and to that extent they may represent a real burden on the U.S. economy in the future.

There is also the danger that foreigners, for whatever reason, may suddenly withdraw their funds. This would lead to a financial crisis and much higher interest rates in the United States. Some economists and government officials also fear that foreign companies operating in the United States will transfer advanced American technology abroad. They further fear some loss of domestic control over political and economic matters. The irony is that these fears were precisely the complaints usually heard from Canada, smaller European nations, and developing countries with regard to the large American investments in their countries during the 1950s and 1960s. With the great concern often voiced in the United States today about the danger of foreign investments, the tables now seem to have turned.[12]

EXAMPLE 7
THE RISE AND FALL OF FOREIGN DIRECT INVESTMENTS IN THE UNITED STATES

Figure 15.8 shows that the level of foreign direct investments (FDI) in the United States rose rapidly from $8.1 billion 1983 to the all-time high of $72.7 billion in 1988, declined to $15.3 billion in 1992, and then rose again to $47.2 billion in 1994. Foreign direct investments include

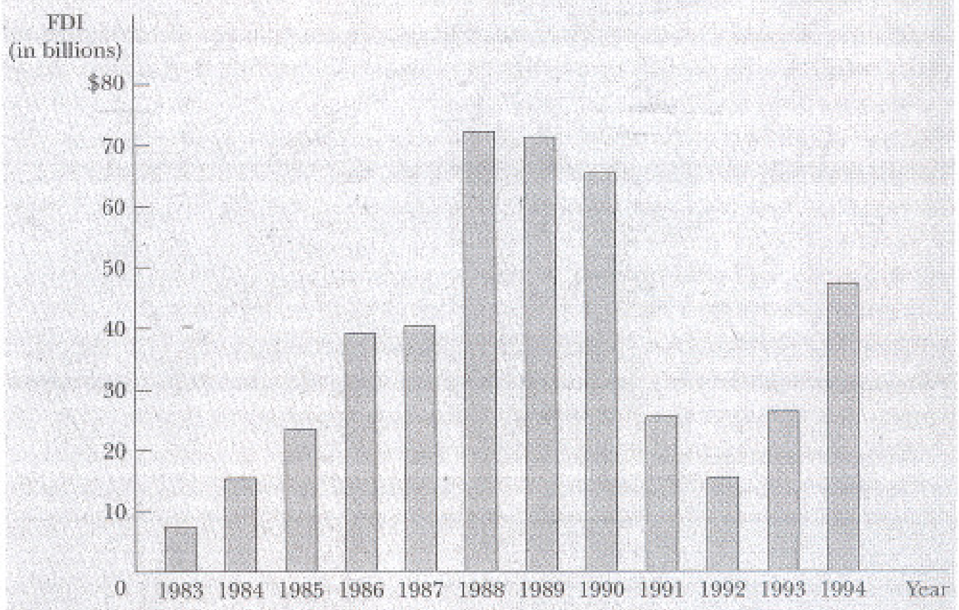
[11] Of course, the United States would have benefitted even more if it had financed its domestic investments entirely through domestic savings. But with inadequate domestic savings, the second best situation was to receive foreign investments and pay the return on investments to foreign investors.

[12] See "A Note on the United States as a Debtor Nation," *Survey of Current Business* (Washington, D.C.: U.S. Government Printing Office, 1985), p. 28.

acquisitions, the formation of new businesses, and the construction of new plants. They do not include the purchase of stocks and bonds. Slow growth and recession made FDI in the United States less attractive to foreigners in 1991 and 1992. But with the resumption of rapid growth in the United States in 1993 and 1994, FDI increased again (even though they did not reach the high achieved in 1988, when many Americans became concerned that foreigners, particularly the Japanese, were "buying up" America). At the same time, U.S. FDI abroad remained high and increased from $27 billion in 1990 to $56 billion in 1994 in order to take advantage of very high growth in the emerging markets of Asia, especially China.

FIGURE 15.8 Foreign Direct Investments in the United States, 1983–1994

The flow of foreign direct investments (FDI) to the United States rose from $8.1 billion in 1983 to an all-time high of $72.7 billion in 1988 before declining to $15.3 billion in 1992 and then rising again to $47.2 billion in 1994.



Source: U.S. Department of Commerce, *Survey of Current Business* (Washington, D.C.: May 1995, p. 57).

15.7 SOME APPLICATIONS OF FINANCIAL MICROECONOMICS

In this section, we discuss some important applications of the theory presented in the chapter. These applications include investment in human capital, the effect of investment in human capital on hours of work, the pricing of exhaustible resources, and the management of nonexhaustible resources. These applications clearly indicate the usefulness and applicability of the theory.

Investment in Human Capital

Investment in human capital is any activity on the part of a worker or potential worker that increases his or her productivity. It refers to expenditures on education, job training, health, migration to areas of better job opportunities, and so on. Like any other investment, investments in human capital involve costs and entail returns. For example, going to college involves explicit and implicit, or opportunity, costs. The explicit costs are tuition, books, fees, and all other out-of-pocket expenses of attending college. The implicit costs are the earnings or opportunities foregone while attending college (the individual could have worked or could have worked more by not attending college). As we have seen in the example in Chapter 7.1, the implicit costs of attending college are nearly as high as the explicit costs. The returns of attending college take the form of higher lifetime earnings with a college education than without a college education (see Example 3 in this chapter).

As with any other investment, we can find the present value of the stream of net cash flows from a college degree. Net cash flows are negative during the college years (because of the explicit and implicit or opportunity costs of attending college) and positive during the working life of the college graduate until retirement. The same is generally true for other investments in human capital. That is, they also lead to a stream of net cash flows and should be undertaken only if their net present value is positive or higher than the present value of other investments (such as the purchasing of a stock). Using this method, it was estimated that the return to a college education was about 10% to 15% per year during the 1950s and 1960s. This was substantially higher than the return on similarly risky investments (such as the purchasing of a stock). During the 1970s, and as a result of the sharp increases in tuition and relatively lower starting salaries, the returns to a college education declined to about 7% per year, but they increased during the 1980s and early 1990s.[13]

These studies, however, face a number of statistical problems. For example, not all expenditures for education represent an investment (as, for example, when a physics student takes a course in Shakespeare). In addition, at least part of the higher earnings of college graduates may be due to their being more intelligent or from working harder than noncollege graduates (see the next section). Nevertheless, there are benefits from a college education that cannot be easily measured. For example,

[13] R. B. Freeman, "The Decline in the Economic Rewards to College Education," *The Review of Economics and Statistics*, February 1977, pp. 18–29; "The Soaring Payoff from Higher Education," *The Margin*, January/February, 1990, p. 6; "Managing Your Career," Special Edition of the National Employment Weekly, *Wall Street Journal*, Fall 1992; "Technological Change Boosts Wage Gap," *The Margin*, Spring 1993, pp. 42–43; and J. Mincer, "Investment in U.S. Education and Training," National Bureau of Economic Research, *Working Paper no. 4844*, October 1995.

college graduates seem to enjoy their jobs more than noncollege graduates, have happier marriages, and generally suffer less mental illness. In spite of these measurement difficulties, however, the concept of investment in human capital is very important and commonly used. Most differences in labor incomes can be explained by differences in human capital. Juries routinely determine the amount of damages to award injury victims (or their survivors, in cases of fatal accidents) on the basis of the human capital or income lost by the injured party. Developing countries complain about the brain drain or the emigration to rich nations of their young and skilled people (who embody a great deal of human capital), and so on.

Investment in Human Capital and Hours of Work

People may work more hours as a result of investment in human capital. This can be shown with the aid of Figure 15.9. In Figure 15.9, the movement from left to right on the horizontal axis measures hours of leisure per day. The movement from right to left measures the hours of work. The sum of the hours of leisure and the hours of work always adds up to the 24 hours of the day. The vertical axis measures the daily income of the individual.

We begin by assuming that the individual portrayed in Figure 15.9 has a daily property income of *FC* ($30). If the hourly wage is $2.50, the individual's budget line is *FB* (so that the negative of the slope of the budget line gives the wage rate). Before investing in education, the individual maximizes utility or satisfaction at point *E*, where indifference curve *U* is tangent to budget line *FB*. The individual works 8 hours per day and has a daily income of $50 (*FC* or $30 from property income plus *HE* or $20 from working 8 hours at the wage rate of $2.50 per hour).

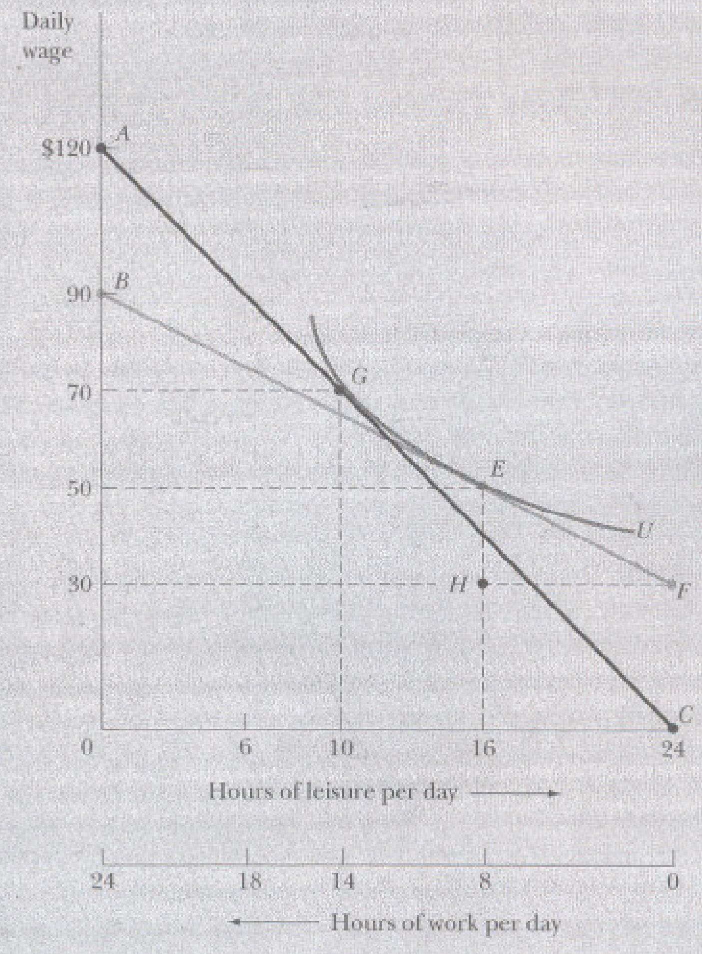
Suppose that now the individual decides to invest all of his or her endowed property income in education (i.e., sacrifice all of his or her nonhuman capital) and that as a result he or she can earn a wage rate of $5 per hour. With education, the budget line of the individual is now *CA* (see the figure), reflecting zero property income available for consumption and the wage of $5 per hour (the negative of the slope of budget line *CA*). Assuming that the individual's tastes remain unchanged as a result of the education, the individual will now maximize utility at point *G*, where indifference curve *U* is tangent to budget line *CA*. The individual now works 14 hours per day for a daily income of $70 (all of which is labor income).

Thus, education seems to induce individuals to work more hours (i.e., have fewer leisure hours) and earn higher incomes. Having made the investment in education, the individual will work more hours and earn a higher income to maximize utility. This seems to be confirmed in empirical studies. For example, Lindsay found that physicians work on average 62 hours per week, far more than the average worker.[14] The same seems to be true for other professionals as opposed to nonprofessionals.

[14] C. M. Lindsay, "Real Returns to Medical Education," *Journal of Human Resources*, Summer 1982, p. 338.

FIGURE 15.9 EDUCATION AND HOURS OF WORK

The individual has a daily property income of FC = \$30 and faces budget line FB (with the negative of the slope giving the wage of \$2.50 per hour). The individual maximizes utility at point E, where indifference curve U is tangent to FB. The individual works 8 hours and has a daily income of \$50 (of which HE = \$20 is labor income). Suppose the individual invests all property income in education and as a result earns \$5 per hour. The budget line is now CA. The individual maximizes utility at point G, where U is tangent to CA, and works 14 hours for a daily income of \$70.



Pricing of Exhaustible Resources

One of the great concerns of modern societies is that the world's resources will become depleted. Resources can generally be classified as exhaustible or nonexhaustible. **Exhaustible resources** are those, such as petroleum and other minerals, that are available in fixed quantities and are nonreplenishable. **Nonexhaustible**

resources are those such as fertile land, forests, rivers, and fish, which can last forever if they are properly managed. We first examine the pricing of exhaustible or nonrenewable resources and then look at the pricing of nonexhaustible or renewable resources under proper management.

During the 1970s (and to a large extent as a result of the petroleum crisis), there was great concern that exhaustible resources would soon be depleted. Doomsday models were built that predicted when various exhaustible resources would run out, thereby threatening the living standard and the very future of humanity.[15] Economists, while not entirely shrugging off the danger, were skeptical for the most part. They pointed out that as the prices of exhaustible resources tend to rise over time, this would lead to conservation and to the discovery of substitutes. Thus, doomsday models were not to be taken too seriously. Let us see why economists proved to be generally right.

We begin by pointing out that the owner of an exhaustible resource will keep it in the ground and available for future use if the present value of the resource in future use is greater than its current price. For example, suppose that the price of the resource is \$100 per unit today, it is expected to be \$120 next year, and the market rate of interest is 10% per year. The owner will sell the resource next year, since the present value of a unit of the resource sold next year is $\$120/(1 + 0.1) = \$109.09$ and this exceeds its price of \$100 today.

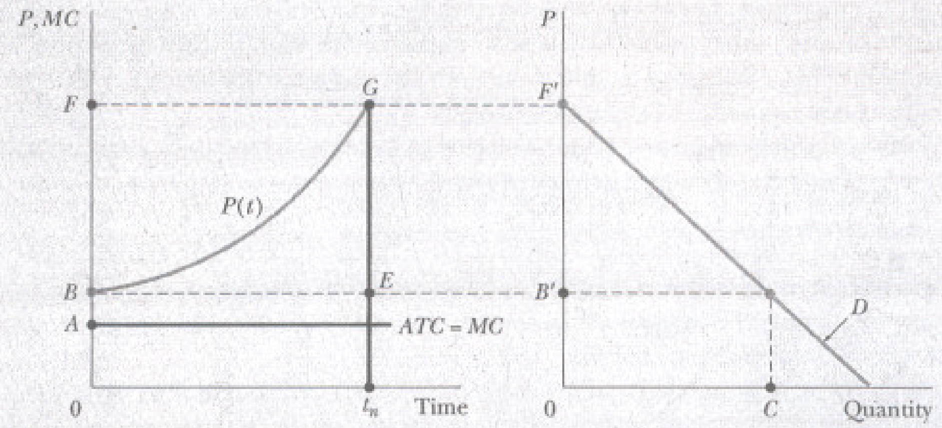
In a perfectly competitive market, the net price of the resource (i.e., the price minus the cost of extraction) will rise at a rate equal to the market rate of interest, and this will spread available supplies over time. If the net price of a resource is expected to rise faster than the market rate of interest, more of the resource will be held off the market for future sale. This increases the current price and reduces the future price until the present value of the future net price is equal to the present net price. On the other hand, if the net price of the resource is expected to rise at a slower rate than the market rate of interest, more of the resource will be sold in the present. This will reduce the present price and increase the future price until the present value of the expected future net price equals the present net price. This is shown in Figure 15.10.

In the left panel of the figure, time is measured along the horizontal axis, and the price of the exhaustible resource and its average total cost (assumed to be constant, and thus equal to marginal cost) are measured along the vertical axis. The right panel shows the market demand curve for the resource (input). The net price or benefit to the owners of the resource is given by the difference (*AB* at time zero) between the (gross) price of the resource and the assumed constant cost of extracting it. The owner can obtain these net benefits now or in the future

[15] See J. W. Forrester, *World Dynamics* (Cambridge, Mass.: Wright-Allen Press, 1971); D. H. Meadows et al., *The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind* (New York: Universe Books, 1972); and M. Mesarovic and E. Pestel, *Mankind at the Turning Point: The Second Report to the Club of Rome* (New York: The American Library, 1974).

FIGURE 15.10 THE PRICE OF EXHAUSTIBLE RESOURCES

In the left panel, time is measured along the horizontal axis, and the price of the exhaustible resource and its average cost (assumed constant and equal to MC) is measured along the vertical axis. The right panel shows the demand curve for the resource. At $P = OB$, the net benefit is AB per unit and the quantity demanded is OC. Over time, the net benefit or net price rises at the same rate as the market rate of interest until at $P = OF$, the supply of the resource is exhausted (point G in the left panel), and the quantity demanded is zero (point F' in the right panel).



(by leaving the resource in the ground). For the owner of the resource to be indifferent between extracting the resource now or in the future, the net benefit or net price of the resource must appreciate over time at a rate equal to the market rate of interest.

The right panel of Figure 15.10 shows that at the resource (gross) price $P = 0B'$, the quantity demanded of the resource is $0C$. Over time, the net price rises at the same rate as the market rate of interest (from AB to EG in the left panel) until at $P = 0F'$ the supply of the resource is exhausted (point G in the left panel) and the quantity demanded of the resource is zero (point F' in the right panel). Thus, in perfectly competitive markets, exhaustion of the resource coincides with zero quantity demanded. If exhaustion occurs before time t_n at $P = 0F$, owners of the resource could have sold the resource at a higher price (and net benefit) over time than indicated by line BG. On the other hand, if the resource is not exhausted by t_n at $P = 0F$, owners would have gained by selling the resource at a lower price over time.[16] In the real world, the net price of most resources

[16] See H. Hotelling, "The Economics of Exhaustible Resources," *Journal of Political Economy*, April 1931, pp. 137–175.

increases at a smaller rate than the market rate of interest (and the net price of many resources actually falls) over time because of new discoveries, technological improvements in extraction, and conservation.

Management of Nonexhaustible Resources

Nonexhaustible or renewable resources such as forests and fish grow naturally over time. Unless the rate of utilization of the resource exceeds its rate of natural growth, the resource will never be depleted.[17] If the renewable resource is trees, the question is when should the trees be cut? The answer (as you might suspect by now) is that the trees should be allowed to grow as long as the rate of growth in the net value of the trees exceeds the market rate of interest. Cutting the trees when the rate of growth in their net value exceeds the market rate of interest would be equivalent to taking money out of a bank paying a higher rate of interest and depositing the money in another bank that pays a lower rate of interest. We can analyze this situation with the aid of Figure 15.11.

The top panel of Figure 15.11 shows the net value of the trees if harvested at time t. This is given by the $V(t)$ curve. The net value is the total market value of the trees minus the cost of harvesting them. We assume zero maintenance or management costs. The top panel shows that $V(t)$ grows at an increasing rate at first. At time $t = 3$ (point A), diminishing returns begin. $V(t)$ reaches the maximum value of \$14 million at $t = 9$ (point B), after which disease, age, and decay set in.

When should the trees be cut? The answer is not at $t = 9$ when $V(t)$ is maximum. This would be the case only if the market rate of interest were zero. With a positive market rate of interest, the correct answer is to cut the trees when the growth in the net value of the standing trees (ΔV) is equal to the growth of the net receipts from cutting the trees and investing the proceeds at the market rate of interest (rV). That is, the trees should be cut when

$$\Delta V = rV \qquad [15.17]$$

or

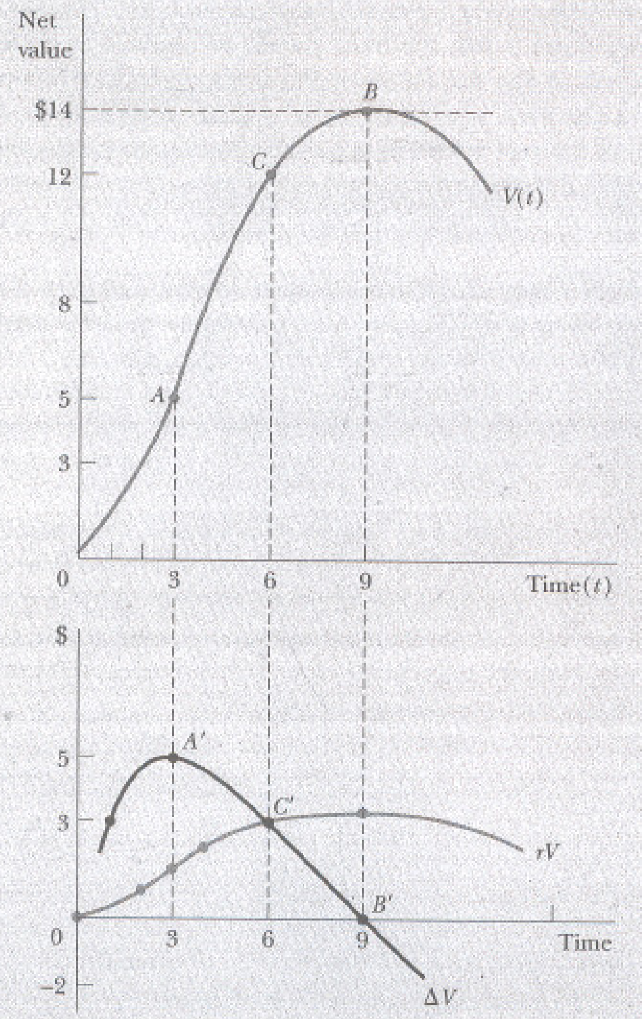
$$\Delta V/V = r \qquad [15.17A]$$

This says that trees should be cut when the *rate* of growth in the value of the standing trees ($\Delta V/V$) equals the market *rate* of interest (r).

[17] The term "renewable" is, perhaps, more appropriate than "nonexhaustible" because if the rate of utilization of the resource exceeds its natural growth rate, the resource can be exhausted. For example, if you cut all the trees or catch all the fish now, there will be no trees or fish in the future.

FIGURE 15.11 OPTIMAL MANAGEMENT OF A STANDING FOREST

The top panel shows the net value of the trees if harvested at time t, $V(t)$. The trees should be cut when the growth in the net value of the standing trees (ΔV) is equal to the growth in the net receipts from cutting the trees and investing the proceeds at the market rate of interest (rV). This equilibrium occurs at $t = 6$ when the ΔV or MV curve crosses the rV curve (point C' in the bottom panel) and $\Delta V/V = r$.



In terms of Figure 15.11, the trees should be cut at $t = 6$ when the ΔV curve crosses the rV curve (point C' in the bottom panel). The ΔV curve (in the bottom panel) is the marginal value curve or slope of the $V(t)$ curve in the top panel (i.e., $\Delta V = MV$). The rV curve in the bottom panel is 0.25 or 25% of the $V(t)$ curve, at $r = 25\%$. To the left of point C', MV exceeds rV (i.e., $\Delta V/V > r$) and it pays for the firm to let the trees continue to grow. To the right of point C', MV is smaller than rV (i.e., $\Delta V/V < r$) and it pays for the firm to cut the trees. The optimal choice is to cut the trees at point $t = 6$ (point C', where $MV = rV$). This is the usual marginal rule applicable in all optimization decisions.

SUMMARY

1. Given the consumer's income or endowment for this year and the next, and the rate of interest, we can define the consumer's budget line. The rate of interest is the premium received next year for lending or borrowing one dollar this year. The optimal consumer's choice involves lending or borrowing so as to reach the highest possible indifference curve showing the consumer's time preference between present and future consumption. The wealth of an individual is given by the sum of the present income and the present value of future income. If the rate of interest rises, the borrower will borrow less and the lender will usually lend more. The equilibrium rate of interest is determined at the intersection of the market demand curve for borrowing and the market supply curve for lending.
2. For an isolated individual, optimal saving and investment is given by the point where the production-possibilities curve is tangent to an indifference curve. With saving and investment, and borrowing and lending, the optimal choice of the individual is first to maximize wealth (by reaching the market line that is tangent to the production-possibilities curve) and then to borrow or lend along the market line until the individual reaches the highest indifference curve possible. The equilibrium rate of interest is given by the intersection of (1) the aggregate demand curve for borrowing and the aggregate supply curve of lending, or (2) the aggregate demand curve for investment and the aggregate supply curve of savings.
3. A firm should undertake an investment only if the net present value of the investment is positive. The net present value (NPV) of the investment is the value today from the stream of the net cash flows (positive and negative) from the investment. In choosing between any two projects, the firm will maximize attained wealth by undertaking the project with the highest net present value. The separation theorem refers to the independence of the optimum production decision from the individual's preferences in perfect capital markets.
4. The rate of interest usually varies at different times and in different markets. Even at a given point in time and in a specific capital market, there is not a single rate of interest, but many. That is, there is a different interest rate on loans or investments depending on differences in (1) default and variability risks, (2) duration of the loan, (3) cost of administering the loan, and (4) tax treatment. Interest rates rise if society decides to save less or to borrow and invest more. The nominal rate of interest equals the real rate of interest plus the anticipated rate of price inflation.
5. The cost that a firm incurs for using internal funds is the foregone return on these funds invested outside the firm. The cost of external funds is the rate of return that lenders and stockholders require to lend or invest funds in the firm. The after-tax cost of borrowed funds is given by the interest paid times $(1 - t)$, where t is the firm's marginal tax rate. The cost of equity capital can be measured by (1) the risk-free rate plus a risk premium, (2) the dividend

valuation model, and (3) the capital asset pricing model (CAPM). The composite cost of capital is the weighted average of the cost of debt and equity capital.

6. Foreign investments result in a reduction in the rate of return on domestic investments but a net gain for the recipient nation. Labor in a nation receiving foreign investments shares in the gains through higher wages from the higher productivity resulting from an increased capital-labor ratio. The nation also collects taxes on foreign earnings. The United States is now the largest debtor nation in the world. Concern has been voiced on the dangers arising from a sudden withdrawal of foreign investments, technology transfer, and foreign domination. These dangers may be exaggerated.
7. Investment in human capital refers to expenditures on education, job training, health, or migration to areas of better job opportunities that increase the productivity of an individual. Like any other investment, investments in human capital involve costs and entail returns. Education seems to induce people to work more hours. The net price of exhaustible resources tends to rise at the same rate as the market rate of interest, and this spreads the available supply of the resource over time and stimulates the discovery of substitutes. A nonexhaustible resource should be harvested when the growth in its net value equals the market rate of interest.

KEY TERMS

Endowment position
Rate of interest (r)
Wealth
Production-possibilities curve
Saving
Investment
Market line
Capital budgeting
Net present value (NPV)
Separation theorem
Default risk
Variability risk
Real rate of interest (r)
Nominal rate of interest (r')
Cost of debt
Dividend valuation model
Capital asset pricing model (CAPM)
Beta coefficient (β)
Composite cost of capital
Derivatives
Investment in human capital
Exhaustible resources
Nonexhaustible resources

REVIEW QUESTIONS

1. How much interest must an individual be paid to save part of his or her income this year if his or her time preference is zero? What happens to the individual's satisfaction this year and next if the individual saves part of this year's income and spends it next year? What happens to the individual's combined satisfaction for this year and next if the individual saves part of this year's income and spends it next year?
2. In what way is intertemporal optimum consumer choice analogous or similar to optimum consumer choice at one point in time?
3. Is microeconomic theory concerned primarily with the real or with the nominal interest rate? Why?
4. What is the present discounted value of an inheritance of \$10,000 to be paid in two years, if the market rate of interest is 10%?
5. Should you prefer \$100 one year from today or \$110 two years from today if the market rate of interest is 5% per year? Why?

6. What is the rate of interest or discount if an individual is indifferent between receiving \$11,111.11 today or \$1,000 at the end of each year in perpetuity?
7. Why is the cost of debt capital usually lower than the cost of equity capital for a firm?
8. What additional risks do the stockholders of a firm face in comparison to holders of government securities?
9. A corporation can sell bonds at an interest rate of 9%, and the interest rate on government securities is 7%. What is the cost of equity capital for this firm?
10. If labor and capital are the only inputs, what are the total gains of labor in Figure 15.7 when the nation receives \$100 billion of foreign investments?
11. What are some of the benefits and costs of foreign investments not captured by Figure 15.7? When all benefits and costs are considered, can we still say that foreign investments are beneficial for the receiving nation?
12. Does a nation gain or lose from foreign investments when all benefits and costs from foreign investments are considered?

PROBLEMS

1. Suppose that an individual is endowed with $Y_0 = 7.5$ units of a commodity this year and $Y_1 = 2.75$ units next year. Draw a figure showing that the individual lends 2.5 units of this year's endowment for 2.75 units next year. What is the rate of interest? On the same figure show that the individual lends 3 units for 4.2 units. What would the rate of interest be then?
2. Suppose that an individual is endowed with $Y_0 = 2.5$ units of a commodity this year and $Y_1 = 8.25$ units next year. Draw a figure showing that the individual borrows 2.5 units this year and repays 2.75 units next year. What is the rate of interest? On the same figure, show that the individual borrows 2 units this year and repays 2.80 units next year. What would the rate of interest be then?
3. Assume that (1) the consumer of Problem 2 (call him or her individual B or the borrower) is a different individual from the consumer of Problem 1 (call him or her individual A or the lender), and that (2) both individual A and B have the same tastes or time preference. Draw a figure showing how the equilibrium rate of interest is determined if A and B are the only individuals in the market. What would happen at $r = 40\%$? At $r = 5\%$?

*4. Assume that (1) individuals A and B have identical endowments of a commodity of $Y_0 = 5$ this year and $Y_1 = 6$ next year, and that (2) the optimal choice for individual B is to borrow 2.5 units this year and repay 3 units next year, while the optimal choice for individual A is to lend 2.5 units this year and receive 3 units next year. Draw a figure similar to Figures 15.1 and 15.2 for the above. What is the equilibrium rate of interest if A and B are the only individuals in the market? On the same figure show that at $r = 50\%$, individual B wants to borrow 2 units instead of 2.5 this year and repay 3 units next year, while individual A wants to lend 3 units this year and receive 4.5 units next year. Why is $r = 50\%$ not the equilibrium rate of interest?

*5. a. Why does a lender's supply curve of loans (lending) bend backward at sufficiently high rates of interest?
   b. Why is a borrower's demand curve for loans (borrowing) negatively sloped throughout?

* = Answer provided at the end of the book.

6. Draw a figure similar to Figure 15.5 showing that a rise in the rate of interest will reduce the individual's level of investment and borrowing.
7. Starting from Figure 15.5, draw a figure showing that if indifference curve U_5 had been tangent to market line HW'_0 to the right of point A, the individual would have been dissaving.
8. Starting from Figure 15.5, draw a figure showing that if indifference curve U_5 had been tangent to market line HW'_0 to the left of point H, the individual would have been saving more than he or she invested.

*9. Reestimate the net present value of the project given in Table 15.4 for $r = 5\%$.

*10. A firm expects to earn $200 million after taxes for the current year. The company has a policy of paying out half of its net after-tax income to the holders of the company's 100 million shares of common stock. A share of the common stock of the company currently sells for eight times current earnings. Management and outside analysts expect the growth rate of earnings and dividends for the company to be 7.5% per year. Calculate the cost of equity capital to this firm.

11. A company pays the interest rate of 11% on its bonds, the marginal income tax rate that the firm faces is 40%, the rate on government bonds is 7.5%, the return on the average stock of all firms in the market is 11.55%, the estimated beta coefficient for the common stock of the firm is 2, and the firm wishes to raise 40% of its capital by borrowing. Determine:
    a. The cost of debt.
    b. The cost of equity capital.
    c. The composite cost of capital for this firm.
12. Draw a figure showing the effect of the following on the price of an exhaustible resource.
    a. A decrease in the market rate of interest.
    b. An increase in the demand for the resource.

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