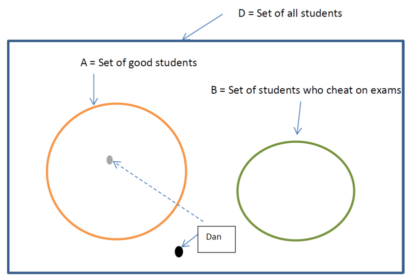
Solution: Quiz 2

1. Let the domain D be the set of students. Suppose Dan $\in D$.
   Use the **diagram** to show that the following argument is valid or invalid.
   "None of the good students cheats on the exam."
   "Dan does not cheat on the exam."
   $\therefore$ "Dan is a good student."

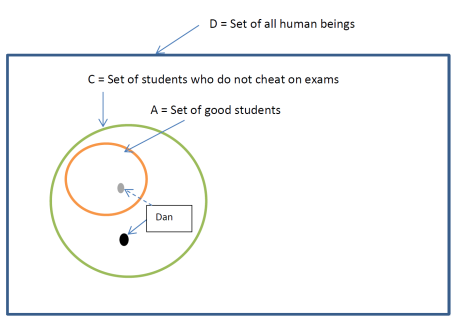
   Solution: To use the diagram, let A be the set of good students, and let B be the set of students who cheat on exams.
   Then
   - the first premise "None of the good students cheats on the exam." implies "**All good students do not cheat on the exams**" (i.e. being good students implies not cheating on the exam), or "$A \subseteq B^c$." or "$A \cap B = \emptyset$" where B^c is the compliment of set B.
   - the second premise "Dan does not cheat on the exams." implies that Dan $\notin B$,
   - the conclusion implies that "Dan is in the set A."

   From the diagram, since we only know that "Dan" is not an element in B , it is possible that "Dan" is either an element in A or not an element in A. That is, the conclusion could be false. Therefore the argument is **invalid**.



   Remarks: The following is an alternative solution. let A be the set of good students, and let C be the set of students who **do not** cheat on exams.
   Then, (i) the first premise implies "$A \subseteq C$." (ii) the second premise implies Dan $\in C$, (iii) the conclusion implies "Dan is in the set C."

   From the diagram, since we only know that "Dan" is in C , it is possible that "Dan" is either an element in A or not an element in A. That is, the conclusion could be false and the argument is **invalid**.



2. Let $A = \{0, 1\}$ and $B = \{-1, -2, -3\}$. Let $D = \{(1, -1), (0, -1)\}$ be the domain of variable (x, y) for the predicate $P(x, y)$ which is defined as

$$P(x, y) : \sim [\exists a \in A,\ \forall b \in B, \quad ab > x + y]\,.$$

Determine the truth set of the predicate $P(x, y)$.

Solution: First, we can simplify $P(x, y)$ by finding the negation of $[\exists a \in A,\ \forall b \in B, \quad ab > x + y]$:

$$P(x, y) : \sim [\exists a \in A,\ \forall b \in B, \quad ab > x + y] \;=\; [\forall a \in A,\ \exists b \in B, \quad ab \leq x + y]\,.$$

To find the truth set T_p of $P(x, y)$, we have to consider all elements in D that make $P(x, y)$ true.

(i) For $(x, y) = (1, -1) \in D$, we have $x + y = 0$ and $P(1, -1) : [\forall a \in A,\ \exists b \in B, \quad ab \leq 0]$ is **true** because:
- for $a = 0 \in A$, there exists $b \in B$, e.g. $b = -1$, (or $b = -2, -3$) such that $ab = 0 \leq 0$ is true;
- for $a = 1 \in A$, there exists $b \in B$,e.g. $b = -1$, such that $ab = -1 \leq 0$ is true
(we can also use $b = -2, -3 \in B$).
$\therefore (1, -1) \in T_p$

(ii) For $(x, y) = (0, -1) \in D$, we have $x + y = -1$ and $P(0, -1) : [\forall a \in A,\ \exists b \in B, \quad ab \leq -1]$ is **false** because there exists $\boxed{a = 0 \in A}$ that makes $ab \leq -1$ false for all values of $b \in B = \{-1, -2, -3\}$:
- for $b = -1$, $ab = 0 \leq -1$ is false;
- for $b = -2$, $ab = 0 \leq -1$ is false;
- for $b = -3$, $ab = 0 \leq -1$ is false.
$\therefore (0, -1) \notin T_p$

Hence $T_p = \{(x, y) \in D :\ P(x, y) \text{ is true.}\} = \{(1, -1)\}$. ■