Quiz 3

1. Prove or disprove the following statements.

(a) "*For all integers* n*, if* n *is odd then* $\frac{n-1}{2}$ *is odd.* "

(b) "*For all integers* n*, if* n *is odd then* $3n + 7$ *is even.*"

Solution:

(a) This statement is **false**. We can **disprove** this by showing that its negation is true. The negation of this statement is given by
" *there exists an integer* n *such that* n *is odd, but* $(n-1)/2$ *is even.* "

Since the negation is in the form of existential statement, we can prove this by finding n that is odd and $(n-1)/2$ is even.

Consider $n = 5$. We will see that

$$\frac{n-1}{2} = \frac{5-1}{2} = 2$$

is even. That is, we found a counterexample that makes the negation true and therefore the given statement is false.

Remark: Note that **not** all odd numbers can be used as counterexamples here.
We consider an **odd** number n of the form $n = 4a + 1$, for $a \in \mathbb{Z}$, so that

$$\frac{n-1}{2} = \frac{(4a+1)-1}{2} = 2a$$

is an even number (since it is in the form of $2a$ where $a \in \mathbb{Z}$). Hence the negation is true and the original statement is false. ■

(b) We can use the **direct proof** to show that this statement is **true**.

Suppose n is odd. That is, $n = 2b + 1$ for some integer b. We want to show that $3n + 7$ is even. In particular, by substituting $n = 2b + 1$,

$$3n + 7 = 7(2b + 1) + 3 = 14b + 7 + 3 = 14b + 10 = 2(7b + 5) = 2c$$

where $c = 7b + 5$ is an integer (because $b \in \mathbb{Z}$).

Therefore $3n + 7 = 2c$ is even. ■