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BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.1)

77.625 3.2125 $\hat{\beta}_1+\hat{\beta}_2\bar{X}$ 3.2125 $Y_i-\hat{Y}_i$

Student	Y_i	X_i	Y_1X_i	X_i^2	$x_i = X_i - \bar{X}$	$y_i = Y_i - \bar{Y}$	x_i^2	$x_i y_i$	$\hat{y}_i$	$\hat{u}_i$
1	2.8	63	176.4	3,969	-14.625	-0.4125	213.89	6.03		-0.4125
2	3.4	72	244.8	5,784	-5.625	0.1875	31.64	-1.05		0.1875
3	3.0	78	234	6,084	0.375	-0.2125	0.14	-0.08		-0.2125
4	3.5	81	283.5	6,561	3.375	0.2875	11.39	0.97		0.2875
5	3.6	87	313.2	7,569	9.375	0.3875	87.89	3.63		0.3875
6	3.0	75	225	5,625	-2.625	-0.2125	6.89	0.56		-0.2125
7	2.7	75	202.5	5,625	-2.625	-0.5125	6.89	1.35		-0.5125
8	3.7	90	333	8,100	12.375	0.4875	153.14	6.03		0.4875
sum	25.7	621	2,012.4	48,717	0	0	511.87	17.44		0
mean	3.2125	77.625	251.55	6,089.625	0	0	63.98	2.18		0

to find $\hat{\beta}_2$ using short-cut $= \dfrac{\Sigma x_i y_i}{\Sigma x_i^2} = \dfrac{17.44}{511.87} = \dfrac{17.44}{511.87} = 0.0341$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{X}$$
$$= 3.2125 - (0.0341)(77.625)$$
$$= 3.2125 - 2.647$$
$$= 0.5681$$

$\therefore\ \hat{y}_1 = 3.2073 + 0.00006656\,X_i$ can be interpreted that

If exam point increased by 1 point, GPA on average will increase by 0.0341 and when total microeconomics exam point is equal to zero, student's GPA is 0.5681

1.2) • $\hat{y}_1 = \hat{\beta}_1 + \hat{\beta}_2\bar{X}$

$$= 3.2073 + 0.00006656(77.625)$$
$$= 3.2125$$

• $\hat{u}_i = Y_i - \hat{Y}_i$ in the model

to find $\hat{u}_i$, we know that $\hat{Y}_i$ is approximately 3.2125

For example student 1 : $y_i = 2.8$

$y_i - \hat{y}_1 = 2.8 - 3.2125 = -0.4125$

student 2: $y_i = 3.4$

$y_i - \hat{y}_i = 3.4 - 3.2125 = 0.1875$

• to prove that $\Sigma_{i=1}^{n} \hat{u}_i = 0$, we sum $\hat{u}_i$ of each student of total of 8 students such as [-0.4125 + 0.1875 - 0.2125 + 0.2875 + ... 0.4875] and get -0.003 which approximately equal to 0

1.3)

$$Var(\hat{u}_i) = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = \frac{0.4347253}{8-2} = 0.0725$$

$$Var(\hat{\beta}_1) = \frac{\sum_{i=1}^{n} X^2 \hat{\sigma}^2}{n \sum_{i=1}^{n} x_i^2} = \frac{48717(0.0725)}{8(511.875)} = 0.862$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n} x_i^2} = \frac{0.0725}{511.875} = 0.00014163$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

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2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

X_i	Y_i	$y_i - \bar{y}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})(y_i - \bar{y})$	$y_i - \hat{y}_i$	$(y_i - \hat{y}_i)^2$	x_i^2
10	0	-9.1	-10	100	91	-0.1364	0.0186	100
12	2	-7.1	-8	64	56.8	0.0745	0.0055	144
14	5	-4.1	-6	36	24.6	1.2854	1.6523	196
16	6	-3.1	-4	16	12.4	0.4964	0.2464	256
18	7	-2.1	-2	4	4.2	-0.2927	0.0857	324
22	10	0.9	2	4	1.8	-0.8709	0.7585	484
24	10	0.9	4	16	3.6	-2.66	7.0756	576
26	15	5.9	6	36	35.4	0.551	0.3036	676
28	16	6.9	8	64	55.2	-0.2381	0.0567	784
30	20	10.9	10	100	109	1.9728	3.8919	900

2.1.) $\hat{\beta}_2 = \dfrac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \dfrac{394}{440} = 0.8955$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20) = -8.8091$

$y_i = -8.8097 + 0.8955 x_i + \hat{u}_i$ $\rightarrow$ when x_i increase by 1 y_i will increase by 0.8955 and y-intercept is -8.8091

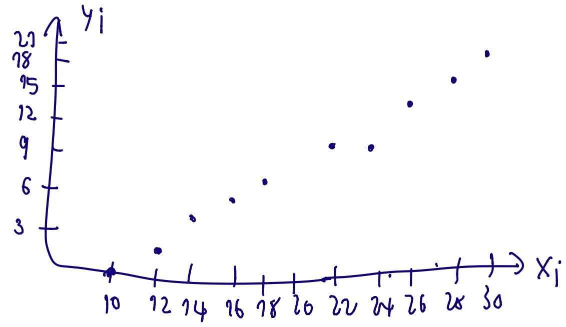
2.2) $\hat{y}_i = -8.8091 + 0.8955 X_i$

$\hat{u}_i = u_i - \hat{u}_i$

$\sum_{i=1}^{10} \hat{u}_i = -0.1364 + 0.0745 + 1.2854 + .. + 1.9728$

≈ 0

2.3)



The line passes $(\bar{X}, \bar{Y})$ where $\bar{X} = 20$, $\bar{Y} = 9.1$

2.4) $X_i = 16$

$y_i = -8.8091 + 0.8955 X_i$

$= -8.8091 + 0.8955(16)$

$= 5.5189$

2.5) $Var(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{1.7679}{440} = 0.004$

$Var(\hat{\beta}_1) = \dfrac{\Sigma X^2}{n \Sigma x_i^2} \sigma^2 = \dfrac{4440}{10(440)} (1.7679) = 1.7619$

$var(\hat{u}_i) = \sigma^2 = 1.7679$

3. proof: The unbiased estimator of B_1 is $\hat{B}_1$ is to find $E(\hat{B}_1) = B_1$

$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}$, $\hat{\beta}_2 = \Sigma k_i y_i$

$\hat{\beta}_1 = \bar{Y} - (\Sigma k_i y_i) \bar{X}$

$\hat{\beta}_1 = \dfrac{\Sigma y_i}{n} - \Sigma \bar{X} k_i y_i$

$\hat{\beta}_1 = \Sigma \left(\dfrac{y_i}{n} - \bar{X} k_i y_i \right)$

$\hat{\beta}_1 = \Sigma \left(\dfrac{1}{n} - \bar{X} k_i \right) y_i$

$\therefore$ the assumption is that $\hat{\beta}_1$ is linear