MULTI-REGIONAL INPUT-OUTPUT (MRIO) MODELS

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Nesting a city input–output table in a multiregional framework: a case example with the city of Bogota

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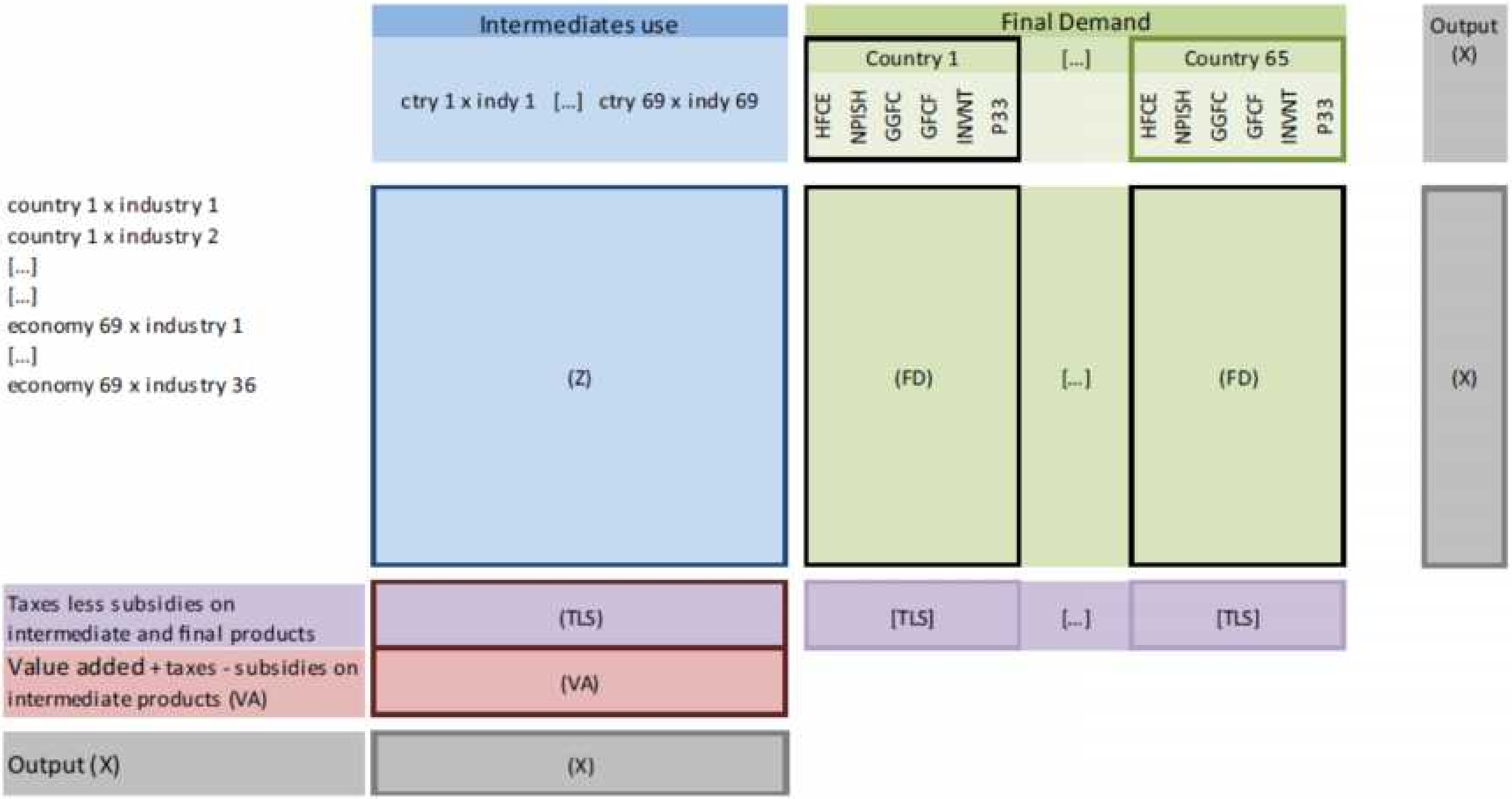

Fig. 1 Structure of ICIO Tables, 2018 edition. The ICIO table includes a 2484 square intermediate transaction matrix (*Z*); a 65 × 2848 taxes less subsidies on intermediate and final products matrix (TLS); a 1 × 2484 vector of value added—including net taxes—(VA), a 1 × 2484 vector of total output (*X*); a 2484*414 final demand matrix—6 final demand categories for each country (source: OCDE (2018))

Source: Monsalve et al. (2020)

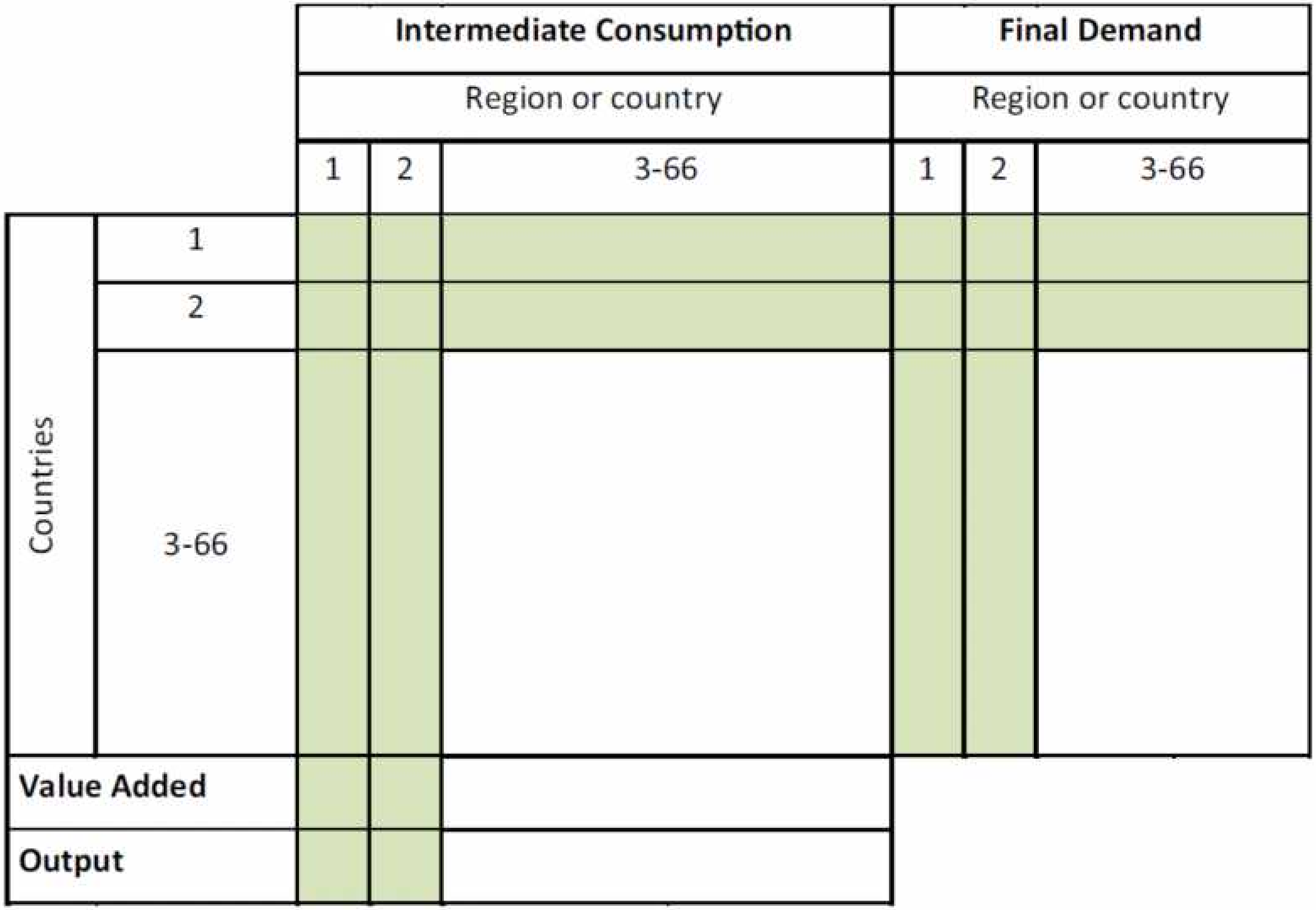

Fig. 2 The augmented MRIO table source: own elaboration. Bogotá is region 1, RoC is region 2

Source: Monsalve et al. (2020)

		1 [...] 32	33 [...] 64	65 [...] 2112	2113	2114	2115 [...] 2178
		Intermediate Consumption			**Final Demand**		
		Bogota	Rest of Colombia	Rest of the World	Bog	RoC	RoW
1 [...] 32	Bogota	[1]	[2]	[3]	[9]	[10]	[11]
33 [...] 64	Rest of Colombia	[4]	[5]	[6]	[12]	[13]	[14]
65 [...] 2112	Rest of the World	[7]	[8]		[15]	[16]	
2113	**Value Added**						
2114	**Output**						

Fig. 3 Zones to fill in the augmented MRIO table (source: own elaboration)

Source: Monsalve et al. (2020)

Appendix: Multiregional input–output framework

Current development of the multiregional input–output (MRIO) databases has favored the generalization of its implementation in multiple economic and environmental applications (López et al. 2017). The disaggregation level, in terms of both intermediate and final goods and services that the different available MRIO models feature, allows for better identification of global production chains (Tukker and Dietzenbacher 2013). The success of the MRIO models is based on the traceability of current international fragmentation of production processes among industries between different countries. In a MRIO table, the exports of a given sector of a country are the imports of the same or another sector in another country. The detailed direct and indirect linkages derived from that inter industrial relationships have provided institutions, organizations, governments, and countries with a significant amount of useful information to develop different environmental applications (López et al. 2017).

Source: Monsalve et al. (2020)

The MRIO table scheme is shown in Fig. 6, which represent a table with two countries and two sectors per country. By columns, in the intermediate goods block, we can differentiate between domestic and imported intermediate inputs. Note that intermediate inputs of region 1 from region 2 are equal to intermediate exports from region 2 to region 1. In the first column of the final demand block, in turn, we can find the domestically supplied final demand of region 1 and the imported final demand of region 1 from region 2 (which is equal to the exports of final goods from region 2 to region 1). The value-added block provides information about the income generation by region and by industry in the production process. The sum-up by rows has to be equal to the sum-up by columns (Peters et al. 2011; López et al. 2017).

Source: Monsalve et al. (2020)

		REGION 1		REGION 2		FINAL DEMAND		TOTAL
		INDUSTRY 1	INDUSTRY 2	INDUSTRY 1	INDUSTRY 2	REGION 1	REGION 2	OUTPUT
REGION 1	INDUSTRY 1	DOMESTIC INTERMEDIATE INPUTS OF REGION 1		INTERMEDIATE IMPORTS OF REGION 2 = INTERMEDIATE EXPORTS OF REGION REGION 1		DOMESTIC FINAL DEMAND OF REGION 1	EXPORTED FINAL DEMAND OF REGION 1 = IMPORTED FINAL DEMAND OF REGION 2	
	INDUSTRY 2							
REGION 2	INDUSTRY 1	INTERMEDIATE IMPORTS OF REGION 1 = INTERMEDIATE EXPORTS OF REGION REGION 2		DOMESTIC INTERMEDIATE INPUTS OF REGION 2		EXPORTED FINAL DEMAND OF REGION 2 = IMPORTED FINAL DEMAND OF REGION 1	DOMESTIC FINAL DEMAND OF REGION 2	
	INDUSTRY 2							
VALUE ADDED		VALUE ADDED OF REGION 1		VALUE ADDED OF REGION 2				
TOTAL INPUTS								

Fig. 6 Multiregional input–output scheme (source: own elaboration)

Source: Monsalve et al. (2020)

The MRIO model can be easily explained by starting from the fundamental equation of Leontief's quantity model, an n linear equations system with n variables, shown in Eq. 14:

$$x = (I - A)^{-1} y, \tag{14}$$

where x is the total output, A is the technical coefficients matrix, $(I - A)^{-1}$ is the inverse matrix of Leontief and y is the final demand vector.

Taking into account the production of several regions in a multiregional context (López et al. 2017), the MRIO can be expressed as:

$$\begin{pmatrix} x^1 \\ \cdots \\ x^2 \end{pmatrix} = \begin{pmatrix} A^{11} & \vdots & A^{12} \\ \cdots & \vdots & \cdots \\ A^{21} & \vdots & A^{22} \end{pmatrix} \cdot \begin{pmatrix} x^1 \\ \cdots \\ x^2 \end{pmatrix} + \begin{pmatrix} y^{11} + y^{12} \\ \cdots \\ y^{21} + y^{22} \end{pmatrix}, \tag{15}$$

where x^r is the output vector of each of the regions. A^{rr} is the domestic technical coefficients matrix of region r, and A^{rs} is region r technical coefficients matrix of imported intermediate goods from region s. Finally, y^{rr} stands for the domestic final demand vector of region r, while y^{rs} are the vectors of final demand of region r from region s (i.e., region s exports of final goods and services to region r.)

Source: Monsalve et al. (2020)

Following Peters (2008), expression 15 can be rewritten for two different industries in two different regions as follows:

$$\begin{pmatrix} x_1^1 \\ x_2^1 \\ \cdots \\ x_1^2 \\ x_2^2 \end{pmatrix} = \begin{pmatrix} a_{11}^{11} & a_{12}^{11} & \vdots & a_{11}^{12} & a_{12}^{12} \\ a_{21}^{11} & a_{22}^{11} & \vdots & a_{21}^{12} & a_{22}^{12} \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ a_{11}^{21} & a_{12}^{21} & \vdots & a_{11}^{22} & a_{12}^{22} \\ a_{21}^{21} & a_{22}^{21} & \vdots & a_{21}^{22} & a_{22}^{22} \end{pmatrix} \cdot \begin{pmatrix} x_1^1 \\ x_2^1 \\ \cdots \\ x_1^2 \\ x_2^2 \end{pmatrix} + \begin{pmatrix} y_1^{11} + y_1^{12} \\ y_2^{11} + y_2^{12} \\ \cdots \\ y_1^{21} + y_1^{22} \\ y_2^{21} + y_2^{22} \end{pmatrix}, \tag{16}$$

where x_j^r denotes the output of industry j in each of the regions r; a_{ij}^{rr} is an element of the domestic technical coefficients matrix for each region r and the i and j industries; and a_{ij}^{sr} is the analog measure but involving two different regions s and r. Each of the technical coefficient represents a share of the output of the industry in a region r or s, bought as an intermediate input by the industry j in a r or s region. The final demand components y_i^{rr} and y_i^{rs} are referred to the domestic final demand of a region r of i goods, and the imported final demand of region r from region s of i goods, respectively.

Source: Monsalve et al. (2020)

The previous expression can be solved in a multiplicative way using the concept of the inverse matrix of Leontief, where $L = (I - A)^{-1}$ (López et al. 2017). The information provided by L is the same as in a single region model, that is total (direct and indirect) requirements to supply the production of an additional unit of final demand, but now including domestic and imported requirements. The resulting expression would be:

$$\begin{pmatrix} x_1^1 \\ x_2^1 \\ \cdots \\ x_1^2 \\ x_2^2 \end{pmatrix} = \begin{pmatrix} l_{11}^{11} & l_{12}^{11} & \vdots & l_{11}^{12} & l_{12}^{12} \\ l_{21}^{11} & l_{22}^{11} & \vdots & l_{21}^{12} & l_{22}^{12} \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ l_{11}^{21} & l_{12}^{21} & \vdots & l_{11}^{22} & l_{12}^{22} \\ l_{21}^{21} & l_{22}^{21} & \vdots & l_{21}^{22} & l_{22}^{22} \end{pmatrix} \cdot \begin{pmatrix} y_1^{11} + y_1^{12} \\ y_2^{11} + y_2^{12} \\ \cdots \\ y_1^{21} + y_1^{22} \\ y_2^{21} + y_2^{22} \end{pmatrix}, \tag{17}$$

where l elements represent the components of the domestic (l^{rr}) or imported (l^{rs}) output multipliers of the Leontief inverse matrix. The output multiplier shows the difference between the initial and the total effect of an exogenous impact in an industry. We can define the total effect as the direct and indirect (those generated in other industries) impact generated in the output of an industry j to supply a unit increase of the final demand in the industry i (Miller and Blair 2009).

Source: Monsalve et al. (2020)

The participation of Bogota and Colombia in the global value chains, developed in the case study of this paper, is going to be showed on next expression:

$$\begin{pmatrix} va_1^1 \\ va_2^1 \\ \cdots \\ va_1^2 \\ va_2^2 \end{pmatrix} = \begin{pmatrix} v_1^1 & 0 & \vdots & 0 & 0 \\ 0 & v_2^1 & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & v_1^2 & 0 \\ 0 & 0 & \vdots & 0 & v_2^2 \end{pmatrix} \cdot \begin{pmatrix} l_{11}^{11} & l_{12}^{11} & \vdots & l_{11}^{12} & l_{12}^{12} \\ l_{21}^{11} & l_{22}^{11} & \vdots & l_{21}^{12} & l_{22}^{12} \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ l_{11}^{21} & l_{12}^{21} & \vdots & l_{11}^{22} & l_{12}^{22} \\ l_{21}^{21} & l_{22}^{21} & \vdots & l_{21}^{22} & l_{22}^{22} \end{pmatrix} \cdot \begin{pmatrix} y_1^{11} + y_1^{12} \\ y_2^{11} + y_2^{12} \\ \cdots \\ y_1^{21} + y_1^{22} \\ y_2^{21} + y_2^{22} \end{pmatrix}, \qquad (18)$$

where v_j^r is value added coefficients vector for each region r and j industries, and va_1^1, denotes the value added of industry j in each of the regions r.

Source: Monsalve et al. (2020)