1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

$$\rightarrow C = a + bY_d$$
$$C^* = a + bY^*_d$$

$$\rightarrow Y_d = Y - T$$
$$Y_d^* = Y^* - T_0$$

$$\rightarrow Y = C + I + G$$
$$Y^* = a + bY_d^* + I_0 + G_0$$

- Numerically, if a = 1, T_0 = \$0 I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

$$\rightarrow Y^*_d = Y^* - 0$$
$$= 6$$

$$\rightarrow C^* = 1 + 0.5Y^*_d$$
$$C^* = 1 + 0.5Y^*$$
$$= 1 + 0.5(6)$$
$$= 4$$

$$\rightarrow Y^* = 1 + 0.5Y_d^* + 1 + 1$$
$$Y^* = 0.5Y_d^* + 3$$
$$Y^* = 0.5Y^* + 3$$
$$0.5Y^* = 3$$
$$Y^* = 6$$

Question: What if G is now changed to \$2, how big is the change in Y^* and C^*?

Answer:

New Y^* = 8 → C^* is remain the same

A \$1 increase in G causes an increase in Y^* by 2

→ $Y^* = 0.5Y_d^* + 4$

$Y^* = 0.5Y^* + 4$

$0.5Y^* = 4$

$Y^* = 8$

In the later part, we will try to figure out the value of *multiplier* when we don't assign any numerical values of exogenous variables.

- The idea is simple. *We just apply the derivative method to the equilibrium solution function.*
- That is, we calculate the value of $\frac{\partial Y^*}{\partial I_0}, \frac{\partial Y^*}{\partial G_0}, \frac{\partial Y^*}{\partial a}, \frac{\partial Y^*}{\partial b}$.

Exercise 2.A:

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

(i) elasticity of supply is always greater than 1 if $c > 0$,

(ii) elasticity of supply is always eqaul to 1 if $c = 0$,

(iii) elasticity of supply is always less to 1 if $c < 0$.

2.A.1) Black line : $p = a - bQ$

$$Q = \frac{a-p}{b}$$

$$\text{Point-PED(B)} = -\frac{1}{b} \times \frac{P}{Q} = -\frac{1}{b} \times \frac{P}{\frac{a-p}{b}}$$

Red line : $p = a - cQ$

$$Q = \frac{a-p}{c}$$

$$\text{Point-PED(R)} = 1 \times \frac{P}{Q}$$

$$= -\frac{1}{c} \times \frac{P}{\frac{a-p}{c}}$$

$$\text{Point-PED(B)} = \text{Point-PED(R)}$$

$$\left(-\frac{1}{\cancel{b}}\right)(\cancel{p})\left(\frac{\cancel{b}}{\cancel{a-p}}\right) = \left(-\frac{1}{\cancel{c}}\right)(\cancel{p})\left(\frac{\cancel{c}}{\cancel{a-p}}\right)$$

$$-1 = -1$$

2.A.2)

i) $Q = \frac{P-C}{d}$ if $c > 0$,

$$PES = \frac{1}{d} \times \frac{P_0}{\frac{P_0 - C}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(P_0)\left(\frac{\cancel{d}}{P_0 - C}\right)$$

$$= \frac{P_0}{P_0 - C} \quad \therefore P_0 > P_0 - C$$

ii) if $c = 0$;

$$Q = \frac{P}{d}$$

$$PES = \frac{1}{d} \times \frac{P_0}{\frac{P_0}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(\cancel{P_0})\left(\frac{\cancel{d}}{\cancel{P_0}}\right) = 1$$

iii) if $c < 0$; $p = -c + dQ$

$$Q = \frac{P+C}{d}$$

$$PES = \frac{1}{d} \times \frac{P_0}{\frac{P_0 + C}{d}}$$

$$= \left(\frac{1}{\cancel{d}}\right)(P_0)\left(\frac{\cancel{d}}{P_0 + C}\right)$$

$$= \frac{P_0}{P_0 + C}$$

$$\therefore P_0 < P_0 + C$$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

$$TR(Q) = P(Q) \times Q$$
$$= (10 - Q) \times Q$$
$$= 10Q - Q^2$$

At $Q = 5$, TR is max

$$TR = 25$$

$$\text{slope} = \frac{dTR}{dQ} = 10 - 2Q$$

maximum occurs when $\frac{dTR}{dQ} = 0$

$$10 - 2Q = 0$$
$$10 = 2Q$$
$$Q = 5$$

- What is the break-even output?

$$TC = 4Q \times Q$$
$$TR = TC$$
$$10Q - Q^2 = 4Q \times Q$$
$$10Q - Q^2 = 4Q^2$$
$$10Q = 5Q^2$$
$$10 = 5Q$$
$$Q = 2$$

- What is the profit-maximizing level of output?

1. $MR = MC$

$$MC = \frac{\Delta TC}{\Delta Q} = 4$$

2. $\pi = TR - TC$

$$\frac{d\pi}{dQ} = 0 \rightarrow Q^*$$

$$\pi = 10Q - Q^2 - 4Q^2$$
$$= 10Q - 5Q^2$$
$$\frac{d\pi}{dQ} = 10 - 10Q$$
$$0 = 10 - 10Q$$
$$10Q = 10$$
$$Q = 1$$

Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

(c) What tax rate is consistent with the maximum tax revenue?

a) $60 = 350t - 500t^2$

$50t^2 - 35t + 6 = 0$

$(5t - 2)(10t - 3) = 0$

$5t = 2 \quad | \quad 10t = 3$

$t = \frac{2}{5} \quad | \quad t = \frac{3}{10}$

∴ When tax rate equal to 0.4 and 0.3, it is consistent with raising tax renevenues equal to \$60 billion.

b) $61.25 = 350t - 500t^2$

$500t^2 - 350t + 61.25 = 0$

$100t^2 - 70t + 12.25 = 0$

$20t^2 - 14t + 2.45 = 0$

$2000t^2 - 1400t + 245 = 0$

$400t^2 - 280t + 49 = 0$

$(20t - 7)(20t - 7) = 0$

$20t = 7$

$t = \frac{7}{20}$

∴ When tax rate equal to 0.35 it is consistent with raising tax renevenues equal to \$60 billion.

c) $TR(t) = 350t - 500t^2$

$\text{Slope} = \frac{\partial TR}{\partial t} = 350 - 1000t$

maximum occurs when $\frac{\partial TR}{\partial t} = 0$

$350 - 1000t = 0$

$1000t = 350$

$t = \frac{35}{100}$

At $t = \frac{35}{100}$, TR is max

$TR(t) = 350\left(\frac{35}{100}\right) - 500\left(\frac{1225}{10000}\right)$

$= \frac{490 - 245}{4} = \frac{245}{4} = 61.25$