Hedging with Forwards and Futures

Hedging in most cases is straightforward. You plan to buy 10,000 barrels of oil in six months and you wish to eliminate the price risk. If you take the buy-side of a forward/futures contract for 10,000 barrels of oil with a maturity of six months, you can eliminate the price risk. Alternatively, you are a US dollar based firm and you have a contract from which you will receive ¥500,000 yen in four months. You plan to sell yen and buy dollars. The exchange rate risk can be eliminate by taking the sell-side of futures contract with a maturity of four month to exchange ¥500,000 yen for US$ dollars at the futures/forward rate. In each of these examples, the price or exchange rate risk is eliminated with the use of a futures contract. The six-month oil futures contract will lock in the price of the oil and the four month yen/US$ futures contract will lock in the exchange rate.

At maturity the most likely scenario will be that in neither case will anyone actually take delivery of the underlying asset. For example, in the case of the oil, at maturity the oil hedger will buy the oil on the spot market at $-S_T$, and close out the futures position realizing a payoff of $+(S_T - F_T)$. The result of the hedge is a cost of $-S_T + (S_T - F_T) = -F_T$. If $S_T > F_T$, the positive inflow from the futures position will offset part of the cost. If $S_T < F_T$, then the hedger will have to pay the difference and again the net cost of purchasing the oil will be F_T.

In the above examples, the hedging was one for one and the maturity of the futures contract exactly matched the timing of the transaction. Often times the hedging approach is not as clear as it is in these examples. For example, the timing of the maturity of the available futures contracts may not be the same as the timing of the obligation. Suppose for the 10,000 barrels of oil the only futures contract available was for a maturity of eight months, T. If we use this to hedge our six months obligation, t, in six months we buy the oil at the spot, $-S_t$, and offset the original futures position by taking the sell side of the same contract which will yield $+(F_t - F_T)$. The futures price at time t, F_t, will be,

$$F_t = S_t \cdot e^{(c_t - y_t)\cdot(T-t)},$$

where c_t is cost of carry and y_t is convenience yield[1] at time t. As such, the net result of buying the oil at spot and hedge at time t is

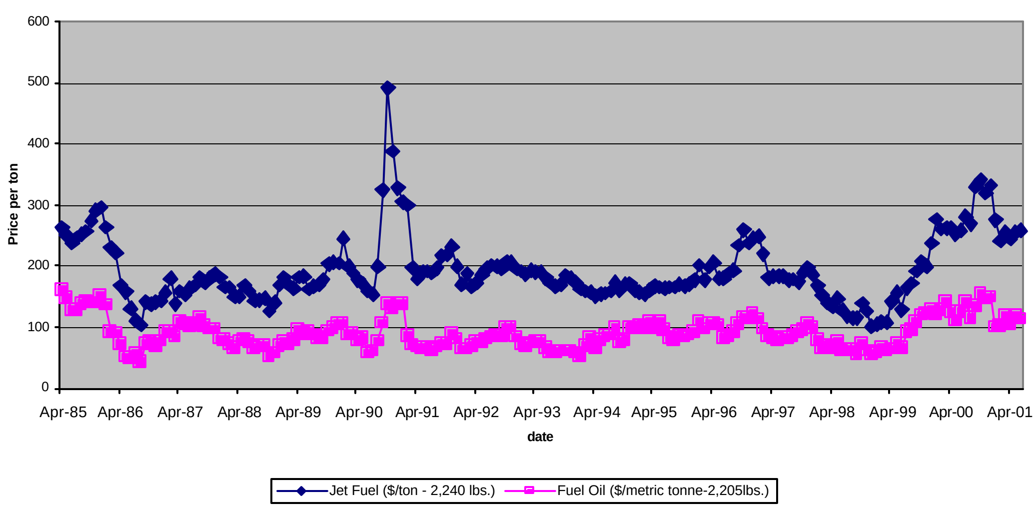
$$-S_t + (F_t - F_T) = -F_T + S_t \cdot (e^{(c_t - y_t)\cdot(T-t)} - 1) \neq -F_T \,.$$

The result is that the hedge is not perfect. It will depend on what is the cost of carry and convenience yield at time t and the result would be certainly different from the fixed cost of $-F_T$ that we had when the maturity of the futures contract exactly matched the obligation. Hence, this mismatch in maturities creates not quite the prefect hedge. The resulting difference from having an exact match of maturities is referred to a ***basis risk***.

A potentially more significant basis risk comes from a situation where an investor must use futures contracts on a different asset to hedge another asset. For example, airlines often wish to hedge their jet fuel costs. They sell tickets well in advance but the actual cost of delivering the flight will depend largely on the cost of jet fuel on the date of the travel. Airlines can eliminate this risk by using futures. However, they face a problem in hedging jet fuel. There are no futures contracts traded on jet fuel. The nearest substitute is heating fuel oil. Thus, an airline could attempt to hedge their fuel cost exposure using Heating Oil futures contracts. However, they do face some risk that the changes in the Heating Oil futures contracts will not exactly match the changes in the price of Jet Fuel. The difference between the price of Jet Fuel and the price of heating Oil futures at the date that the jet fuel is purchased is also referred to as ***basis risk***.

As an example, Exhibit 1 shows the spot prices for jet fuel and for heating (fuel) oil from 1985 to 2001. The price movements are similar but not quite the same. Heating oil prices are lower and appear to be less volatile.

Exhibit 1.
Jet Fuel vs. Heating (Fuel) Oil



[1] Please see Forward and Futures note page 7. Also Hull (5th edition), Chapter 3 page 60.

Hence using heating oil futures contracts on a one-to-one basis may not provide a good hedge for jet fuel.
For an ideal hedge, over our time horizon we would like the change in the futures price to exactly match the change in the value of the asset we wish to hedge, i.e.,

$$\Delta Spot = \Delta Futures$$

Exhibit 2 shows the spot prices for jet fuel and for heat oil 90-day futures and 60-day futures. Let's assume that an airline wishes to hedge jet fuel 30 days forward in time and the only contracts available are 90-day futures contracts for heating oil. The change in the spot price for jet fuel over a month is just the price at the end of the month less the price at the beginning of the month. The change in the value of a futures contract is slightly different. A 90-day contract at the beginning of the month is a 60-day contract at the end of the month. Hence if we use a 90-day contract to hedge for 30 days the change in the price is the difference between the futures price for a 60 day contract at the end of the month less the futures price for 90 day contract at the beginning of the month.

From exhibit 2, it is clear that the price changes of the spot jet fuel prices and heating oil futures are not the same. This raises the question of whether we can use a hedge ratio, h, different from 1.0 to hedge the jet fuel prices or

$$\Delta Spot = h \cdot \Delta Futures .$$

But how do we choose the best h? The usual solution is to choose h such that it minimizes the following:

$$\underset{h}{Min} \quad E\left[(\Delta S - h \cdot \Delta F)^2\right].$$

This results in a value of h that minimizes the squared differences between the price changes. Another way of stating the same thing is to choose h such that it minimizes the variance of the hedge[2]. In choosing h, it places a big penalty on big differences between

[2] The minimization can be rewritten as

$$E\left[(\Delta S - h \cdot \Delta F)^2\right] = E\left[\Delta S^2\right] + h^2 \cdot E\left[\Delta F^2\right] - 2 \cdot h \cdot E\left[\Delta S \cdot \Delta F\right]$$

Assuming E[ΔS]=0, and E[ΔF]=0, then

$$\sigma_S^2 = E\left[\Delta S^2\right],$$
$$\sigma_F^2 = E\left[\Delta F^2\right],$$
$$Cov(\Delta S, \Delta F) = E\left[\Delta S \cdot \Delta F\right] = \sigma_S \sigma_F \rho_{S,F}$$

Substituting back in the original problem results yields

$$E\left[(\Delta S - h \cdot \Delta F)^2\right] = \sigma_S^2 + h^2 \cdot \sigma_F^2 - 2 \cdot h \cdot \sigma_S \sigma_F \rho_{S,F}$$

ΔS and ΔF. Note that we could have chosen a very different objective function. However, this particular objective function happens to be very convenient in a number of ways. The actual solution[3], $\hat{h}$, to this formulation is fairly straightforward.

$$\hat{h} = \frac{\sigma_s}{\sigma_F} \cdot \rho_{S,F} ,$$

where σ_S is the standard deviation of the spot price changes, σ_F is the standard deviation of the futures price changes and $\rho_{S,F}$ is the correlation between the spot price changes and the futures price changes.

Exhibit 3 shows the calculation of the optimal hedge using the historical data in Exhibit 2. The basic statistics[4] are estimated as follows:

Means: $\overline{\Delta S} = \sum_{t=1}^{48} \frac{\Delta S_t}{48}$ and $\overline{\Delta F} = \sum_{t=1}^{48} \frac{\Delta F_t}{48}$

Standard Deviations: $\sigma_S = \sqrt{\frac{1}{48} \sum_{t=1}^{48} \left(\Delta S_t - \overline{\Delta S}\right)^2}$ and $\sigma_F = \sqrt{\frac{1}{48} \sum_{t=1}^{48} \left(\Delta F_t - \overline{\Delta F}\right)^2}$

Covariance: $Cov(\Delta S, \Delta F) = \frac{1}{48} \sum_{t=1}^{48} \left(\Delta S_t - \overline{\Delta S}\right) \cdot \left(\Delta F_t - \overline{\Delta F}\right)$

Correlation: $\rho_{S,F} = \frac{Cov(\Delta S, \Delta F)}{\sigma_S \cdot \sigma_F}$

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[3] The solution to the minimization problem is to take the first derivative of the hedge variance with respect to h, set it equal to zero, and solve for h.

$$\partial E(\Delta S - h \cdot \Delta F)^2 / \partial h = 0$$

$$\partial(\sigma_S^2 + h^2 \sigma_F^2 - 2h\sigma_S \sigma_F \rho_{S,F}) / \partial h = 0$$

$$2h\sigma_F^2 - 2\sigma_S \sigma_F \rho_{S,F} = 0$$

$$\hat{h} = \frac{\sigma_S}{\sigma_F} \rho_{S,F}$$

[4] The statistics shown below are based on the population. If everything was recalculated on a sample basis the estimated hedge ratio would be the same. Be careful using statistical functions in excel. You need to make sure that the estimates of standard deviations and correlations have the same basis, population or sample.

Hedge: $\hat{h} = \frac{\sigma_S}{\sigma_F} \cdot \rho_{S,F}$

It is also possible to estimate the optimal hedge using regression analysis. The basic equation is

$$\Delta S = \alpha + h \cdot \Delta F$$

Since the basic OLS regression for this equation estimates the value of $\hat{h}$ as

$$\hat{h} = \frac{\sigma_S}{\sigma_F} \cdot \rho_{S,F},$$

we can use OLS regression. This is the solution to the minimizing the original objective function. Hence, this is one of the reasons that the objective function of minimizing the squared differences is so appealing. Exhibit 4 shows the output of an Excel regression using the data in Exhibit 3. Note that the results are the same. The optimal hedge ratio[5] is 1.0264. This is very close to a value of 1.00, which is what we would expect for two very similar commodities where the prices would tend to move together.

It is useful to note that the regression analysis also provides us with some information as to how good a hedge we are creating. The r-square[6] of the regression tells how much of the variance in the change in spot price is explained by the variance in the change of the futures price. In this case the r-squared statistic is .443 or 44.3%. A good hedge might result in an r-square value of .80. Hence, in this case, while the optimal hedge ratio is close to 1.00, the hedge itself might not be that effective. There is the potential here for a lot of basis risk. Nonetheless, the appropriate hedge is 1.0264 heating oil futures contracts for each ton[7] of jet fuel.

I have one comment on the analysis presented in this section. Here we used the price changes in the futures contract for Heating Oil. Actually, for most practical purposes we could have used simply the changes in the spot prices of Heating Oil to calculate the optimal hedge. It is often very difficult to get a good consistent historic series of futures prices.

Equity Portfolio Hedging

Hedging portfolios is the same as hedging commodities. Consider a portfolio with a value today of $25,345,456. We wish to hedge this portfolio using S&P 500 futures

[5] I used the excel regression function with ΔS as the y variable and ΔF as the x variable.
[6] The r-square of the regression is estimated as the square of the correlation coefficient between ΔS and ΔF. From exhibit 3, the correlation coefficient is .666. Squaring this yields .443.
[7] Note that since we calculated the optimal hedge ratio based on price changes, the difference in the tonnage between the long ton (2,240 lbs.) for jet fuel and the metric tonne for heating oil was accounted for in the analysis.

contracts. While our portfolio is similar to the S&P 500, it is not the same. If we follow what we did above, the optimal hedge is

$$\hat{h} = \frac{s_{P}}{s_{S\&P}} \cdot r_{p,S\&P}.$$

For equity portfolios the optimal hedge is in terms of returns. For example, assume we have a portfolio with a current value of $10, 968,000. You wish to use S&P 500 futures contracts to hedge the risk over the next month. Exhibit 5 shows the monthly values for the portfolio and the index for the last four years. In this case, instead of using price changes we will calculate the optimal hedge ratio using monthly returns[8]. From exhibit 5, the optimal hedge ratio is 1.0258.

Exhibit 6 shows the estimate of the hedge ratio using regression analysis. Note that the regression model is

$$R_{P} = a + b \cdot R_{S\&P}.$$

This regression model is also a way to estimate Beta for a portfolio using the S&P 500 portfolio as a proxy for the market portfolio. Hence, in this context one interpretation of the optimal hedge ratio is ***Beta***.

Since the $ value of each S&P 500 futures contract is the index value times $250, the actual number of S&P 500 futures contracts to be written is determined by taking the hedge ratio times the ratio of the portfolio $ value divided by the current value of the index underlying the futures contract, the S&P 500 in this case. For the example,

Number of Contracts[9] = $\hat{h} \cdot \frac{\$10{,}968{,}000}{(934.53 \cdot \$250)} = 1.0258 \cdot 46.95 = 48.16$ contracts or approximately 48 contracts.

[8] We use monthly returns because the scale differences in the value of the portfolio and the value of the index. This much easier to scale each of the series and use returns.

[9] Since we calculated the hedge ratio using percentage returns, the hedge ratio does not account for the size differential between the portfolio and the index. Hence we need to take this into account when we estimate the number of contracts required.

Exhibit 2

Jet Fuel and Heating Oil Futures Prices 1997-2001

	Jet Fuel	Fuel Oil 90 day futures	Fuel Oil 60 day futures		Jet Fuel	Fuel Oil Futures
	$/ton - 2,240 lbs.	$/metric tonne-2,205lbs.	$/metric tonne-2,205lbs.		$/ton - 2,240 lbs.	$/metric tonne-2,205lbs.
	Price	Price	Price		Price Change	Price Change**
Jun-97	184.50	81.50	80.93			
Jul-97	178.00	82.00	81.43		-6.50	-0.07
Aug-97	179.00	85.00	84.41		1.00	2.41
Sep-97	174.00	91.50	90.86		-5.00	5.86
Oct-97	190.00	96.50	95.82		16.00	4.32
Nov-97	197.50	105.50	104.76		7.50	8.26
Dec-97	186.00	98.00	97.31		-11.50	-8.19
Jan-98	167.50	79.50	78.94		-18.50	-19.06
Feb-98	151.00	64.00	63.55		-16.50	-15.95
Mar-98	140.00	68.50	68.02		-11.00	4.02
Apr-98	134.00	66.00	65.54		-6.00	-2.96
May-98	147.50	75.00	74.48		13.50	8.47
Jun-98	127.50	61.00	60.57		-20.00	-14.43
Jul-98	119.00	64.00	63.55		-8.50	2.55
Aug-98	116.00	63.00	62.56		-3.00	-1.44
Sep-98	116.50	57.00	56.60		0.50	-6.40
Oct-98	139.00	73.50	72.99		22.50	15.99
Nov-98	126.00	61.50	61.07		-13.00	-12.43
Dec-98	101.50	57.00	56.60		-24.50	-4.90
Jan-99	105.50	59.50	59.08		4.00	2.08
Feb-99	109.50	67.50	67.03		4.00	7.53
Mar-99	108.50	61.00	60.57		-1.00	-6.93
Apr-99	141.50	64.00	63.55		33.00	2.55
May-99	157.50	73.00	72.49		16.00	8.49
Jun-99	129.50	65.00	64.55		-28.00	-8.46
Jul-99	164.00	91.50	90.86		34.50	25.86
Aug-99	172.50	96.00	95.33		8.50	3.83
Sep-99	192.00	108.00	107.24		19.50	11.24
Oct-99	208.50	120.50	119.66		16.50	11.66
Nov-99	199.50	123.50	122.64		-9.00	2.14
Dec-99	237.50	128.50	127.60		38.00	4.10
Jan-00	277.00	120.00	119.16		39.50	-9.34
Feb-00	260.50	128.00	127.10		-16.50	7.10
Mar-00	262.00	141.00	140.01		1.50	12.01
Apr-00	264.00	124.00	123.13		2.00	-17.87
May-00	253.00	110.00	109.23		-11.00	-14.77
Jun-00	258.00	125.00	124.13		5.00	14.13
Jul-00	280.50	140.00	139.02		22.50	14.02
Aug-00	269.50	116.00	115.19		-11.00	-24.81
Sep-00	330.50	135.50	134.55		61.00	18.55
Oct-00	340.50	153.00	151.93		10.00	16.43
Nov-00	319.50	148.50	147.46		-21.00	-5.54
Dec-00	332.50	149.50	148.45		13.00	-0.05
Jan-01	276.00	100.00	99.30		-56.50	-50.20
Feb-01	242.00	101.00	100.29		-34.00	0.29
Mar-01	254.50	119.50	118.66		12.50	17.66
Apr-01	244.50	104.50	103.77		-10.00	-15.73
May-01	255.00	119.00	118.17		10.50	13.67
Jun-01	258.50	113.00	112.21		3.50	-6.79

** Price change for futures compares the 60 day price at time t to the 90 day price in time t-1.

Exhibit 3				
Optimal Hedge Ratio				
Jet Fuel and Heating Oil Futures Prices 1997-2001				
		Jet Fuel	Fuel Oil Futures	
		$/ton - 2,240 lbs.	$/metric tonne-2,205lbs.	
		Price Change	Price Change**	(PJ-MJ)(PH-MH)
Jun-97				
Jul-97		-6.50	-0.07	0.41
Aug-97		1.00	2.41	(1.31)
Sep-97		-5.00	5.86	(38.48)
Oct-97		16.00	4.32	62.85
Nov-97		7.50	8.26	49.36
Dec-97		-11.50	-8.19	106.47
Jan-98		-18.50	-19.06	381.47
Feb-98		-16.50	-15.95	287.32
Mar-98		-11.00	4.02	(50.71)
Apr-98		-6.00	-2.96	22.17
May-98		13.50	8.47	101.62
Jun-98		-20.00	-14.43	310.30
Jul-98		-8.50	2.55	(25.85)
Aug-98		-3.00	-1.44	6.44
Sep-98		0.50	-6.40	6.64
Oct-98		22.50	15.99	335.50
Nov-98		-13.00	-12.43	180.43
Dec-98		-24.50	-4.90	126.99
Jan-99		4.00	2.08	5.18
Feb-99		4.00	7.53	18.56
Mar-99		-1.00	-6.93	17.55
Apr-99		33.00	2.55	80.99
May-99		16.00	8.49	123.06
Jun-99		-28.00	-8.46	249.11
Jul-99		34.50	25.86	853.03
Aug-99		8.50	3.83	26.79
Sep-99		19.50	11.24	202.33
Oct-99		16.50	11.66	174.70
Nov-99		-9.00	2.14	(22.75)
Dec-99		38.00	4.10	150.32
Jan-00		39.50	-9.34	(353.68)
Feb-00		-16.50	7.10	(128.57)
Mar-00		1.50	12.01	(0.50)
Apr-00		2.00	-17.87	(8.18)
May-00		-11.00	-14.77	184.96
Jun-00		5.00	14.13	48.93
Jul-00		22.50	14.02	294.31
Aug-00		-11.00	-24.81	310.90
Sep-00		61.00	18.55	1,104.38
Oct-00		10.00	16.43	139.15
Nov-00		-21.00	-5.54	124.36
Dec-00		13.00	-0.05	(0.28)
Jan-01		-56.50	-50.20	2,912.39
Feb-01		-34.00	0.29	(11.21)
Mar-01		12.50	17.66	193.81
Apr-01		-10.00	-15.73	181.31
May-01		10.50	13.67	122.63
Jun-01		3.50	-6.79	(13.26)
Mean		1.54	-0.02	
Standard Deviation		20.66	13.40	
			Covariance	184.21
			Correlation	0.666
			Hedge	**1.0264**

Exhibit 4
Optimal Hedge Ratio Using Regression Analysis

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.665658193
R Square	0.44310083
Adjusted R Square	0.430994326
Standard Error	15.74669319
Observations	48

ANOVA

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	1	9075.333	9075.333	36.60023	2.44E-07
Residual	46	11406.08	247.9583		
Total	47	20481.42			

	Coefficients	***Standard Error***	***t Stat***	***P-value***	***Lower 95%***	***Upper 95%***	***Lower 95.0%***	***Upper 95.0%***
Intercept	1.564717811	2.272843	0.688441	0.494633	-3.01027	6.139708	-3.01027	6.139708
X Variable 1	**1.0264**	0.169657	6.049812	2.44E-07	0.684894	1.367899	0.684894	1.367899

Exhibit 5
Equity Portfolio Hedging

	Portfolio	S&P 500 Return index	Portfolio Monthly return	S&P 500 Monthly return	(Rp-MP)*(Rs&p-Ms&p)
Jan-99	$ 9,278,400	1670.01			
Feb-99	9,234,100	1730.81	-0.0048	0.0364	-0.0004
Mar-99	8,789,900	1682.86	-0.0481	-0.0277	0.0013
Apr-99	8,957,200	1763.31	0.0190	0.0478	0.0007
May-99	10,000,100	1847.63	0.1164	0.0478	0.0057
Jun-99	10,044,200	1767.8	0.0044	-0.0432	0.0000
Jul-99	10,530,300	1888.15	0.0484	0.0681	0.0031
Aug-99	10,239,500	1817.27	-0.0276	-0.0375	0.0011
Sep-99	9,962,500	1824.2	-0.0271	0.0038	-0.0002
Oct-99	9,574,200	1759.76	-0.0390	-0.0353	0.0014
Nov-99	9,649,400	1858.86	0.0079	0.0563	0.0001
Dec-99	9,859,500	1921.49	0.0218	0.0337	0.0006
Jan-00	10,105,300	2002.11	0.0249	0.0420	0.0009
Feb-00	9,960,500	1940.24	-0.0143	-0.0309	0.0005
Mar-00	9,902,000	1901.51	-0.0059	-0.0200	0.0002
Apr-00	10,615,600	2077.97	0.0721	0.0928	0.0064
May-00	10,793,400	2027.39	0.0167	-0.0243	-0.0002
Jun-00	10,700,000	2003.45	-0.0087	-0.0118	0.0001
Jul-00	10,828,400	2033.58	0.0120	0.0150	0.0001
Aug-00	10,835,900	1991.43	0.0007	-0.0207	0.0001
Sep-00	11,678,800	2108.76	0.0778	0.0589	0.0045
Oct-00	11,198,400	1992.94	-0.0411	-0.0549	0.0024
Nov-00	11,280,800	1973.72	0.0074	-0.0096	0.0000
Dec-00	10,706,400	1828.81	-0.0509	-0.0734	0.0039
Jan-01	11,247,700	1837.36	0.0506	0.0047	0.0004
Feb-01	12,363,100	1913.11	0.0992	0.0412	0.0042
Mar-01	11,709,700	1730.91	-0.0529	-0.0952	0.0053
Apr-01	10,872,700	1599.35	-0.0715	-0.0760	0.0056
May-01	12,248,900	1769.12	0.1266	0.1061	0.0133
Jun-01	12,638,100	1763.87	0.0318	-0.0030	0.0000
Jul-01	12,488,900	1731.53	-0.0118	-0.0183	0.0003
Aug-01	12,343,600	1704.24	-0.0116	-0.0158	0.0002
Sep-01	11,792,600	1591.18	-0.0446	-0.0663	0.0031
Oct-01	9,949,600	1459.33	-0.1563	-0.0829	0.0128
Nov-01	10,952,400	1524.96	0.1008	0.0450	0.0046
Dec-01	11,739,300	1591.48	0.0718	0.0436	0.0031
Jan-02	12,471,300	1618.98	0.0624	0.0173	0.0012
Feb-02	12,299,800	1584.06	-0.0138	-0.0216	0.0003
Mar-02	12,356,900	1600.02	0.0046	0.0101	0.0000
Apr-02	13,085,500	1622.23	0.0590	0.0139	0.0009
May-02	12,896,000	1538.65	-0.0145	-0.0515	0.0010
Jun-02	12,168,500	1476.26	-0.0564	-0.0405	0.0023
Jul-02	11,280,200	1375.87	-0.0730	-0.0680	0.0051
Aug-02	10,007,000	1258.22	-0.1129	-0.0855	0.0097
Sep-02	10,272,000	1304.85	0.0265	0.0371	0.0009
Oct-02	9,407,900	1209.59	-0.0841	-0.0730	0.0062
Nov-02	10,080,600	1287.13	0.0715	0.0641	0.0045
Dec-02	10,968,000	1337.34	0.0880	0.0390	0.0035
		Mean	0.0053	-0.0035	
		Stand. Deviation	0.0596	0.0501	
				Covariance	0.0026
				Correlation	0.8621
			Hedge	**(Beta)**	**1.0253**

Exhibit 6
Regression Results for Equity Portfolio Hedging

Regression Statistics	
Multiple R	0.8623
R Square	0.7435
Adjusted R Square	0.7378
Standard Error	0.0308
Observations	47

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.1240	0.1240	130.4440	6.89E-15
Residual	45	0.0428	0.0010		
Total	46	0.1668			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0100	0.0045	2.2194	0.0315	0.0009	0.0191	0.0009	0.0191
Beta	**1.02581**	0.0898	11.4212	0.0000	0.8449	1.2067	0.8449	1.2067