Solution key HW#4

CHAPTER 15: THE TERM STRUCTURE OF INTEREST RATES

PROBLEM SETS.

1. In general, the forward rate can be viewed as the sum of the market's expectation of the future short rate plus a potential risk (or 'liquidity') premium. According to the expectations theory of the term structure of interest rates, the liquidity premium is zero so that the forward rate is equal to the market's expectation of the future short rate. Therefore, the market's expectation of future short rates (i.e., forward rates) can be derived from the yield curve, and there is no risk premium for longer maturities.

   The liquidity preference theory, on the other hand, specifies that the liquidity premium is positive so that the forward rate is greater than the market's expectation of the future short rate. This could result in an upward sloping term structure even if the market does not anticipate an increase in interest rates. The liquidity preference theory is based on the assumption that the financial markets are dominated by short-term investors who demand a premium in order to be induced to invest in long maturity securities.

7.

Maturity	Price	YTM	Forward Rate
1	$943.40	6.00%	
2	$898.47	5.50%	$(1.055^2/1.06) - 1 = 5.0\%$
3	$847.62	5.67%	$(1.0567^3/1.055^2) - 1 = 6.0\%$
4	$792.16	6.00%	$(1.06^4/1.0567^3) - 1 = 7.0\%$

8. The expected price path of the 4-year zero coupon bond is shown below. (Note that we discount the face value by the appropriate sequence of forward rates implied by this year's yield curve.)

Beginning of Year	Expected Price	Expected Rate of Return
1	$792.16	($839.69/$792.16) – 1 = 6.00%
2	$\frac{\$1{,}000}{1.05 \times 1.06 \times 1.07} = \$839.69$	($881.68/$839.69) – 1 = 5.00%
3	$\frac{\$1{,}000}{1.06 \times 1.07} = \$881.68$	($934.58/$881.68) – 1 = 6.00%
4	$\frac{\$1{,}000}{1.07} = \$934.58$	($1,000.00/$934.58) – 1 = 7.00%

9. If expectations theory holds, then the forward rate equals the short rate, and the one year interest rate three years from now would be

$$\frac{(1.07)^4}{(1.065)^3} - 1 = .0851 = 8.51\%$$

11. a. $P = \dfrac{\$9}{1.07} + \dfrac{\$109}{1.08^2} = \$101.86$

b. The yield to maturity is the solution for y in the following equation:

$$\frac{\$9}{1+y} + \frac{\$109}{(1+y)^2} = \$101.86$$

[Using a financial calculator, enter n = 2; FV = 100; PMT = 9; PV = –101.86; Compute i] YTM = 7.958%

c. The forward rate for next year, derived from the zero-coupon yield curve, is the solution for f_2 in the following equation:

$$1 + f_2 = \frac{(1.08)^2}{1.07} = 1.0901 \Rightarrow f_2 = 0.0901 = 9.01\%.$$

Therefore, using an expected rate for next year of r_2 = 9.01%, we find that the forecast bond price is:

$$P = \frac{\$109}{1.0901} = \$99.99$$

d. If the liquidity premium is 1% then the forecast interest rate is:

$E(r_2) = f_2$ – liquidity premium = 9.01% – 1.00% = 8.01%

The forecast of the bond price is:

$$\frac{\$109}{1.0801} = \$100.92$$

13.

Year	Forward Rate	PV of $1 received at period end
1	5%	$1/1.05 = $0.9524
2	7%	$1/(1.05×1.07) = $0.8901
3	8%	$1/(1.05×1.07×1.08) = $0.8241

a. Price = ($60 × 0.9524) + ($60 × 0.8901) + ($1,060 × 0.8241) = $984.14

b. To find the yield to maturity, solve for y in the following equation:

$984.10 = [$60 × Annuity factor (y, 3)] + [$1,000 × PV factor (y, 3)]

This can be solved using a financial calculator to show that y = 6.60%

c.

Period	Payment received at end of period:	Will grow by a factor of:	To a future value of:
1	$60.00	1.07 × 1.08	$69.34
2	$60.00	1.08	$64.80
3	$1,060.00	1.00	$1,060.00
			$1,194.14

$$\$984.10 \times (1 + y_{\text{realized}})^3 = \$1{,}194.14$$

$$1 + y_{\text{realized}} = \left(\frac{\$1{,}194.14}{\$984.10}\right)^{1/3} = 1.0666 \Rightarrow y_{\text{realized}} = 6.66\%$$

d. Next year, the price of the bond will be:

[$60 × Annuity factor (7%, 2)] + [$1,000 × PV factor (7%, 2)] = $981.92

Therefore, there will be a capital loss equal to: $984.10 – $981.92 = $2.18

The holding period return is: $\dfrac{\$60 + (-\$2.18)}{\$984.10} = 0.0588 = 5.88\%$

18. a.

Maturity (years)	Price	YTM	Forward rate
1	$925.93	8.00%	
2	$853.39	8.25%	8.50%
3	$782.92	8.50%	9.00%
4	$715.00	8.75%	9.50%
5	$650.00	9.00%	10.00%

b. For each 3-year zero issued today, use the proceeds to buy:

$782.92/$715.00 = 1.095 four-year zeros

Your cash flows are thus as follows:

Time	Cash Flow	
0	$ 0	
3	-$1,000	The 3-year zero issued at time 0 matures; the issuer pays out $1,000 face value
4	+$1,095	The 4-year zeros purchased at time 0 mature; receive face value

This is a synthetic one-year loan originating at time 3. The rate on the synthetic loan is 0.095 = 9.5%, precisely the forward rate for year 4.

c. For each 4-year zero issued today, use the proceeds to buy:

$715.00/$650.00 = 1.100 five-year zeros

Your cash flows are thus as follows:

Time	Cash Flow	
0	$ 0	
4	-$1,000	The 4-year zero issued at time 0 matures; the issuer pays out $1,000 face value
5	+$1,100	The 5-year zeros purchased at time 0 mature; receive face value

This is a synthetic one-year loan originating at time 4. The rate on the synthetic loan is 0.100 = 10.0%, precisely the forward rate for year 5.

19. a. For each three-year zero you buy today, issue:

$782.92/$650.00 = 1.2045 five-year zeros

The time-0 cash flow equals zero.

b. Your cash flows are thus as follows:

Time	Cash Flow	
0	$ 0	
3	+$1,000.00	The 3-year zero purchased at time 0 matures; receive $1,000 face value
5	-$1,204.50	The 5-year zeros issued at time 0 mature; issuer pays face value

This is a synthetic two-year loan originating at time 3.

c. The effective two-year interest rate on the forward loan is:

$1,204.50/$1,000 − 1 = 0.2045 = 20.45%

d. The one-year forward rates for years 4 and 5 are 9.5% and 10%, respectively. Notice that:

$1.095 \times 1.10 = 1.2045 =$

1 + (two-year forward rate on the 3-year ahead forward loan)

The 5-year YTM is 9.0%. The 3-year YTM is 8.5%. Therefore, another way to derive the 2-year forward rate for a loan starting at time 3 is:

$$f_3(2) = \frac{(1 + y_5)^5}{(1 + y_3)^3} - 1 = \frac{1.09^5}{1.085^3} - 1 = 0.2046 = 20.46\%$$

[Note: slight discrepancies here from rounding errors in YTM calculations]