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Source: *The RAND Journal of Economics*, Vol. 29, No. 2 (Summer, 1998), pp. 306-323
Published by: Wiley on behalf of RAND Corporation
Stable URL: http://www.jstor.org/stable/2555890

RAND Journal of Economics
Vol. 29, No. 2, Summer 1998
pp. 306–323

Third-degree price discrimination in oligopoly: all-out competition and strategic commitment

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Price discrimination by imperfectly competitive firms may intensify competition, leading to lower prices for all consumers; the tradeoff of consumer groups' welfare that is characteristic of monopolistic discrimination need not arise. This escalation of competition may make firms worse off, and as a result firms may wish to avoid the discriminatory outcome. Under conditions similar to those in which unambiguous price and welfare effects may arise, unilateral commitments not to price discriminate—including the adoption of everyday low pricing or no-haggle policies—may raise firm profits by softening price competition.

1. Introduction

■ The effect of third-degree price discrimination on prices and welfare has long been of interest to economists; however, from the seminal work of Robinson (1933) to the more recent analyses of Schmalensee (1981) and Varian (1985), most research on this subject has addressed monopoly price discrimination. Despite more recent interest in competitive price discrimination (see, for example, Borenstein's (1985) simulation analysis of third-degree discrimination, Lederer and Hurter's (1986) work on spatial pricing, and Katz's (1984) and Stole's (1995) analyses of second-degree price discrimination), only Holmes (1989) extends the traditional literature's analytical approach to third-degree price discrimination in environments of imperfect competition.

Holmes demonstrates that in a symmetric oligopoly model the effects of price discrimination are quite similar to its effects in the monopoly case; in particular, the uniform price necessarily lies between the discriminatory prices, so that permitting price discrimination leads to higher prices for some consumers and lower prices for others. Katz (1984) demonstrates a similar result in a model that, while developed in terms of second-degree (quantity-dependent) price discrimination, amounts to a model of third-degree discrimination because he assumes there are two types of consumers that are perfectly separated. Thus, the literature suggests that competitive price discrimination, like monopolistic discrimination, has an ambiguous effect on consumer welfare, since some consumers are made worse off and some are made better off.

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I thank Bob Kennedy, Darwin Neher, Don Sull, Chuck Thomas, Mike Whinston, and seminar participants at Harvard, Boston University, and the 1997 Econometric Society meetings for helpful comments.

In contrast, I investigate the price and welfare effects of third-degree price discrimination in a general differentiated-goods oligopoly model, relaxing the assumptions of symmetric demand made by Holmes and Katz. I show that unambiguous price and welfare effects may arise when firms differ in their ranking of consumer groups by their demand elasticities at a given rival's price. Holding rivals' prices fixed, every firm has an incentive to raise price to its "strong market" and to lower price to its "weak market." In an asymmetric model, all firms need not rank the same group as the strong market. As a result, *some* firm may have an incentive to lower price to *every* group when permitted to price discriminate; if this elicits a strong enough competitive response, prices may fall for every group and consumer welfare may unambiguously increase for all consumers. While I focus on the case in which third-degree price discrimination by imperfectly competitive firms leads to all-out competition, characterized by lower prices for all consumers and unambiguous consumer welfare gains, it is also possible that all prices rise when firms are permitted to discriminate, leading to unambiguous consumer welfare losses.

When a monopolist discriminates, it necessarily earns higher profits than under uniform pricing, as it solves the same profit-maximization problem with fewer constraints. But outside of the monopolistic case, price discrimination may lead to lower profits for firms; as a result, imperfectly competitive firms may wish to avoid price discrimination and the all-out competition that may ensue, perhaps through commitments to uniform pricing. Examples of such commitments include most-favored-customer clauses, in which a firm promises a specific customer that it is receiving the lowest price offered to any customer by that firm, and no-haggle policies like the one adopted by the auto manufacturer Saturn, in which the firm promises customers that the posted sticker price is the final price. Perhaps even more prevalent among these kinds of commitments are "everyday low pricing" or EDLP policies. These policies may take the form of reduced or eliminated couponing activity—a tactic recently embraced by Procter and Gamble, Philip Morris' Kraft Foods, Kellogg, and General Mills, among others (Deveny and Gibson, 1994)—or of reduced frequency or elimination of sales, a strategy that has been adopted recently by many retailers, including Bloomingdale's and Dillard (Strom, 1992).

It is not immediately clear whether commitments not to discriminate are effective in avoiding the unprofitable consequences of competitive price discrimination. If, in fact, discrimination always raises a firm's profit, taking other prices and discrimination policies as fixed, but discrimination by both firms leads to all-out competition, firms find themselves in a prisoner's dilemma: price discrimination is a dominant strategy that results in lower equilibrium profits for the firms. One might therefore expect even credible commitments not to discriminate to be unprofitable, just as a commitment not to fink is not optimal in the prisoner's dilemma even if a technology exists for making this commitment credible.

A unilateral commitment might seem even less likely to be effective in avoiding the adverse effects of competitive price discrimination.[1] However, the asymmetry necessary for price discrimination to lead to all-out competition introduces a subtle constraint on the firms' available strategies, creating an opportunity for one firm to improve all firms' profits through a commitment not to price discriminate. When firms rank consumer groups differently by their elasticities, each firm wishes to target a different group for a discriminatory discount. If, however, the available technology for price

[1] In fact, the New York attorney general's office opened an investigation into Procter and Gamble's elimination of coupons, seeking to determine whether the firm was colluding with its rivals, since investigators apparently believed that the elimination of coupons "only works if everybody goes along with it." (Narisetti, 1996.)

discrimination permits only a single group (those with a low opportunity cost of time spent clipping coupons, say) to be targeted for such discounts, then a unilateral commitment to uniform pricing, together with this technological constraint, may implement the uniform-price equilibrium. Thus, even unilateral uniform-price commitments, including the policies mentioned above, may serve to soften price competition and raise firm profits by eliminating competitive price discrimination.

The same asymmetry condition—that firms differ in their ranking of consumer groups by elasticities—permits both unambiguous price effects and the implementation of the uniform-price equilibrium by unilateral commitments not to discriminate. The plausibility of this condition is therefore central to the argument of this article. Many cases of interest, including discriminatory schemes like senior citizen discounts and airline ticket restrictions, may seem to be symmetric in some sense; all firms agree, for example, that senior citizens and leisure travelers are "low-price" groups. However, I show that the efficacy of such simple incentive-compatible discrimination schemes depends on a different condition: firms' rankings of consumer groups by equilibrium prices, not by elasticities, must coincide. I show that this condition does not imply the earlier condition and argue that in many markets, incentive-compatible price discrimination may plausibly lead to unambiguous price and wefare effects.

I begin Section 2 with an example of a real-world scenario in which price discrimination seems to have intensified competition. I then develop a simple model of a vertically differentiated duopoly to motivate the subsequent analysis and lay out the basic logic of the argument. Section 3 generalizes the results on the price and welfare effects of price discrimination to a more general model of imperfect competition. I then discuss incentive-compatible price discrimination and contrast the two symmetry conditions mentioned above. Section 4 demonstrates that commitments not to discriminate can effectively soften price competition and raise firm profits. Section 5 concludes.

2. A motivating example

■ The following excerpt from a 1992 *New York Times* article illustrates the type of competitive price discrimination I examine. In the retail markets discussed in this story, the relevant discriminatory tactic is the use of sales. By holding sales (especially if in a predictable pattern), retailers effectively set two prices: the everyday list price and the lower price paid by those who shop during sales. If consumers are segmented by their patience—their willingness to postpone a purchase to obtain a lower price—then sales effectively discriminate between more and less patient consumers (abstracting from the fact that some impatient consumers accidentally show up during sales). In discussing the implications of the rapid growth of discount retail stores in the 1980s, the article states:

> Conventional retailers found themselves selling the same brand-name goods their customers could buy in discount stores for 25 to 40 percent less, and thus the price wars began. . . . Merchants now treat their shoppers to a rich diet of one-day sales, pre- and post-holiday sales, seasonal sales and clearance sales, sacrificing their profit margins in the process.
>
> "Now they're competing with us, and the upshot is that everyone's bottom line has suffered," said Gene Kosack, president of NBO, the chain of off-price men's clothing stores. (Strom, 1992.)

Apparently, conventional retailers found themselves in competition with low-priced discounters for the business of some especially price-sensitive groups of consumers. They then attempted to win back those customers with sales, a form of price discrimination that effectively allowed the conventional retailer to target the most price-sensitive customers for low sales prices, while keeping everyday prices high and thereby avoiding the sacrifice of margins on purchases by other consumers that an across-the-board price cut would have entailed. So the sales, ignoring the potential costliness of spikes in production or distribution and ignoring the reaction of the discounters, should have been profit improving for the conventional retailer. But the equilibrium effect of this strategy was to intensify competition between conventional and discount retailers ("now they're competing with us. . . ."), with the net effect being, as the discount retailer bluntly put it, "that everyone's bottom line has suffered." The conventional retailers' ability to price discriminate made attacking discounters more attractive than if lowering list prices was the only competitive option; however, the discounters responded to this attack with more aggressive pricing of their own. As a result, the conventional retailers' price discrimination resulted in an escalation of competition that led to lower profits for all firms. While the following example formalizes this story in terms of high- and low-quality firms, one might think of them as conventional and discount retailers, or as a leading brand and a private-label firm in some consumer goods industry.

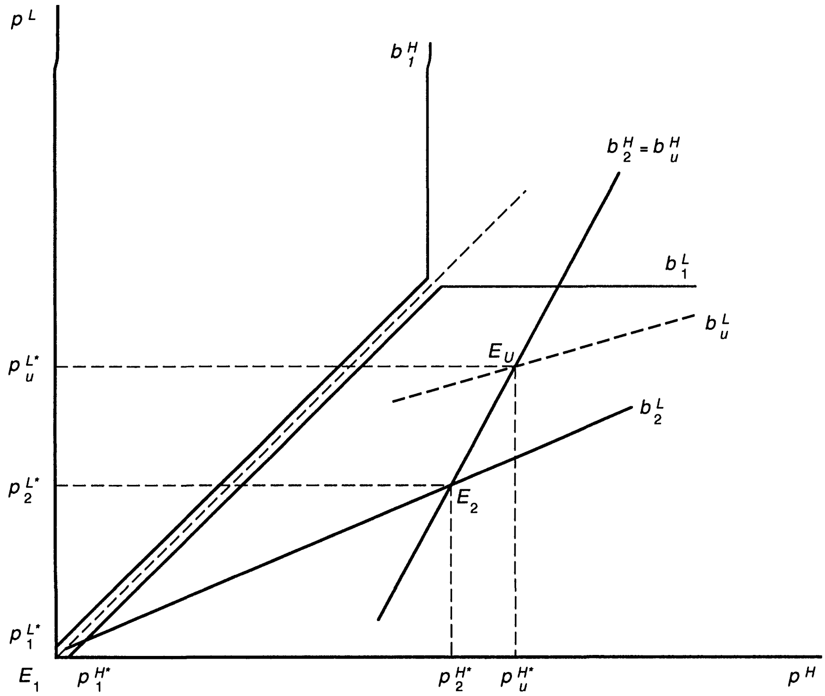
Suppose two vertically differentiated firms—one producing a high-quality product and the other a low-quality product—each sell to two groups of consumers: "choosy" consumers (market 2) that care about quality and "cheap" consumers (market 1) that do not. Both firms face constant marginal cost of zero. Cheap consumers are indifferent to quality and shop only on price. The firm with the lower price p^0 makes sales of $\alpha - \beta p^0$ to the cheap market, with sales split equally if the two firms set the same price in market 1. Choosy consumers vary in their preference for quality, with types θ distributed uniformly over $[0, \overline{\theta}]$ with a density of one. Type θ derives utility $v^L - p^L$ from the purchase of the low-quality good at price p^L and utility $v^L + \theta\Delta - p^H$ from the purchase of firm H's product at price p^H. The parameter Δ therefore represents the magnitude of the quality advantage of the high-quality firm over its rival.

Consumer type $\hat{\theta} = (p^H - p^L)/\Delta$ is indifferent between the two firms' products. As a result, firms H and L face demand functions $x_H = \overline{\theta} - \hat{\theta} = \overline{\theta} - (p_H - p_L)/\Delta$ and $x_L = \hat{\theta} - 0 = (p_H - p_L)/\Delta$, respectively, as long as all consumers make a purchase in equilibrium and $\hat{\theta} \in [0, \overline{\theta}]$. I maintain these assumptions throughout; given Δ and $\overline{\theta}$, the former necessarily holds for large enough v^L, since all equilibrium prices are independent of v^L. Subsequent analysis demonstrates that the latter assumption holds in the neighborhood of equilibrium prices, which give $\hat{\theta} = \overline{\theta}/3$.

These assumptions permit the derivation of best-response functions for the two firms, which allows the graphical representation of this problem in Figure 1. Best-response functions with respect to the choosy market alone are denoted $b_2^i(p^{-i})$. It is easy to show that $b_2^H = (p^L + \Delta\overline{\theta})/2$ and $b_2^L = p^H/2$. Equilibrium in the choosy market is given by the intersection of these best-response functions. In response to a rival's cheap market price at or below marginal cost, cheap market best responses permit any price weakly greater than the rival's, and in response to prices above the cheap market monopoly price ($p_1^m = \alpha/(2\beta)$), they require the monopoly price to be set. However, cheap market best responses are not well defined for intermediate rivals' prices due to the familiar discontinuity associated with homogeneous-good price competition. Graphically, these are depicted as lines just below the 45-degree line, capturing the fact that each firm wants to undercut its rival by the narrowest possible margin. Analytically, what is important is that the unique equilibrium prices in this market are equal to marginal cost for both firms. The equilibrium price vectors of a game in which prices are set independently for the two markets are denoted $E_1 = (p_1^{L*}, p_1^{H*})$ and $E_2 = (p_2^{L*}, p_2^{H*})$. Note the implicit assumption in Figure 1 that $p_1^m > p_2^{L*}$.

Now consider the uniform-pricing game, in which firms may set only a single price. Given prices p_2^* that would prevail if the firms considered only the choosy market, the low-quality firm finds itself naturally in a monopoly position with respect

FIGURE 1



to the cheap consumers, who do not value quality and shop only on price. If firm L remains a monopolist with respect to that market, it would maximize its profits against the sum of its demand from the choosy market and the entire demand of the cheap market. This yields a best-response function $b_u^L = (\alpha\Delta + p^H)/(2(1 + \beta\Delta))$. The intersection of this best-response function with b_2^H then yields a possible uniform-price equilibrium. This point, denoted $E_u = (p_u^{L*}, p_u^{H*})$ in Figure 1, is in fact the (unique) uniform-price equilibrium if the relative profitability of the two markets is such that it does not pay for the high-quality firm to undercut the low-quality firm to steal the cheap market, i.e., if $\pi_2^H(p_u^{L*}, p_u^{H*}) - \pi_2^H(p_u^{L*}, p_u^{L*}) < \pi_1\ (p_u^{L*})$, where π_j^i is firm i's profit in market j under the given prices. This condition holds for a wide range of parameter values, including those in the numerical example to follow, and I assume that it holds throughout this example.[2] Analysis of this simple model yields several observations that motivate subsequent theoretical analysis.

Observation 1. In this example, price discrimination leads to all-out competition, with prices for all consumers lower than in the uniform-price equilibrium. As a result, consumer welfare is unambiguously higher.

Since E_1 and E_2 both lie strictly to the southwest of E_u, prices fall in this model when the firms are permitted to price discriminate; I term this situation "all-out competition." When the firms can set only a single price, the high-quality firm gives up on the cheap market because stealing sales from the low-quality firm in that market is too costly in terms of lost margins in the choosy market. Firm L then takes the cheap

[2] In general, it fails to hold if (1) market 1 is large relative to market 2, so that stealing market 1 yields a large benefit, or (2) Δ is small, so that the price differential between firms is small and undercutting in the choosy market is therefore cheap.

market into account in determining its optimal price and, finding itself a monopolist with respect to this market and disadvantaged in the choosy market, raises its price above its choosy market best response, yielding uniform prices higher than the single-market equilibrium prices. When the firms are able to price discriminate, they compete fiercely for sales to the cheap market, lowering the equilibrium price in market 1 to marginal cost. The low-quality firm no longer has any reason to shade its price above its choosy market best response, and market 2 equilibrium prices fall as well.

Observation 2. In this example, price discrimination leads to lower profits for all firms, compared to the uniform-price equilibrium.

Both firms are made worse off by competitive price discrimination and the all-out competition that ensues in this model. The high-quality firm sells only to market 2 and now prices off its best-response function at a lower rival's price; it therefore earns lower profits. The low-quality firm must be worse off because it has lost all profits from market 1 and is now optimizing in market 2 against a lower rival's price ($\pi_2^L(p_u^*) + \pi_1^L(p_u^*) > \pi_2^L(b_2^L(p_u^H), p_u^H) > \pi_2^L(p_2^*)$).

Why does this example generate all-out competition when imperfectly competitive firms price discriminate? As suggested in the introduction, it is the asymmetry of this model—specifically, the asymmetry in the firms' rankings of the best-response functions of the two markets—that generates the unambiguous price and consumer welfare effects. In this example, the firms rank consumer groups differently, or in the language of the traditional literature, the two firms have different "strong markets." While the high-quality firm sees the choosy market as its strong market, preferring to set a higher price in the choosy market than in the cheap market at any rival's price, the low-quality firm sees the cheap market as its strong market. As a result, the low-quality firm, finding itself a monopolist with respect to the cheap market because of its quality disadvantage in the choosy market, shades its uniform price upward from its choosy market optimum. Discrimination that breaks up this uniform-price equilibrium therefore leads to lower prices.

If, instead, the low-quality firm's best-response function for the cheap market were lower (if $p_1^m < p_2^{L*}$), the firms would both rank market 2 as their strong market (in the vicinity of p_2^*), and this model would lack this source of asymmetry. In that case, the low-quality firm's uniform-price best response would involve shading down from the choosy market equilibrium price. Price discrimination that broke up that uniform-price equilibrium would lead to higher prices in the choosy market and lower prices in the cheap market; price discrimination would in that case have ambiguous price and consumer welfare effects, as cheap consumers were made better off and choosy consumers worse off. Clearly, the upward pressure on uniform prices (in the vicinity of E_2) results from the low-quality firm's incentive to raise price to exploit its monopoly position in market 1. This incentive arises only if the low-quality firm sees the cheap market as its strong market, which suggests a third observation about this model.

Observation 3. Asymmetry of the firms' ranking of the market best-response functions is necessary for price discrimination to have unambiguous price effects.

While this model is asymmetric in one important sense, I argue that it is also symmetric in important ways. In particular, both firms set lower prices in the same market *in the discriminatory equilibrium.* As a result, the discriminatory equilibrium could be implemented with any of several common price-discrimination schemes. For example, if the choosy consumers faced a prohibitively high cost of clipping and saving coupons and the cheap consumer faced no cost of doing so, coupons together with list prices p_2^* could effectively implement the discriminatory equilibrium prices, since only market 1 would avail themselves of these discounted prices. That is, symmetry of equilibrium prices permits incentive-compatible price discrimination when the technology of discrimination requires each firm to target the same market for its lower price. I return to this matter in more detail below, but note now only that this symmetry of equilibrium prices is not inconsistent with the asymmetry that leads to all-out competition in this model.

Observation 4. Asymmetry of the firms' ranking of the market best-response functions is not inconsistent with symmetry of their ranking of the market prices in the discriminatory equilibrium.

3. The price and welfare effects of competitive price discrimination

■ In this section I investigate the effects of price discrimination in a more general differentiated-goods oligopoly model. The quality-differentiation interpretation of the firms is no longer straightforward, and I consequently refer to the firms as simply A and B. These firms compete in price to sell a differentiated good to a consumer population composed of two types, 1 and 2. The restriction of the model to two firms and two consumer groups is in keeping with the existing literature's assumptions and simplifies the exposition, but it is not essential for the results presented in this section. In this model, as in the example of Section 2, differentiation of the goods between the firms provides the market power necessary for price discrimination to arise. However, each firm sells the same good to both sets of consumers. This contrasts with models of multiproduct firms with quality-differentiated product lines, where differences in consumers' willingness to pay for quality allow a firm to price its products in a way that ensures consumer self-selection (see, for example, McAfee and Deneckere (1996) and Corts (1995)). Issues of self-selection and incentive compatibility are ignored in this section, and firms are assumed to be able to both identify types and prevent resale.

Assume marginal costs are constant so that profits are separable across markets, and denote by $\pi_k^i(p^i, p^{-i})$ the profit earned by firm i from sales to consumer type k when those consumers face prices p^i, p^{-i} from the two firms. The following assumptions, standard for price-setting games in differentiated-goods industries, are made directly on the (twice continuously differentiable) profit functions:

Assumption 1. $$\frac{d^2\pi_j^i(p_j^i, p_j^{-i})}{dp_j^{i2}} < 0 \qquad \forall p_j^{-i}$$

Assumption 2. $$\frac{d^2\pi_j^i(b_j^i(p_j^{-i}), p_j^{-i})}{dp_j^i dp_j^{-i}} > 0 \qquad \forall p_j^{-i}$$

Assumption 3. $$\frac{d^2\pi_j^i(b_j^i(p_j^{-i}), p_j^{-i})}{dp_j^i dp_j^{-i}} < -\frac{d^2\pi_j^i}{dp_j^{i2}} \quad \forall p_j^{-i},$$

where b_j^i denotes firm i's single-market best-response function defined by

$$b_j^i(p_j^{-i}) = \arg\max_{p^i} \pi_j^i(p^i, p_j^{-i}).$$

Assumption 1 ensures that the profit-maximization problem in each market is well defined and that the best-response mappings are functions. Assumption 2 is an assumption that this game exhibits strategic complementarity. Assumption 3 is a stability condition that ensures uniqueness of the equilibrium price vector in each market. Together, these assumptions ensure that the best-response functions b_j^i, which give the firms' optimal prices with respect to a single consumer type, have slope $b_j^{i\prime}$ satisfying $0 < b_j^{i\prime} < 1$ for all p^{-i}. Thus, there exist unique equilibrium prices $p_j^* = (p_j^{A*}, p_j^{B*})$ in the game in which the firms are permitted to price discriminate between the two groups. Figure 2 depicts such best-response functions and equilibrium price vectors for a symmetric oligopoly game.

To establish a baseline for evaluation of the price and welfare effects of price discrimination, consider the uniform-price game, in which each firm sets a single price p_u^i to maximize $\pi_u^i(p_u^i, p_u^{-i}) = \pi_1^i(p_u^i, p_u^{-i}) + \pi_2^i(p_u^i, p_u^{-i})$. Note that Assumption 1 implies π_u^i is concave, but that conditions analogous to Assumption 2 and Assumption 3 need not hold for π_u^i. As a result, uniform-price equilibrium price vectors need not be unique. Define the uniform-price best-response function

$$b_u^i(p_u^{-i}) = \arg\max_{p^i} \pi_1^i(p^i, p_u^{-i}) + \pi_2^i(p^i, p_u^{-i}).$$

The following propositions characterize the set of uniform-price equilibrium price vectors, denoted by

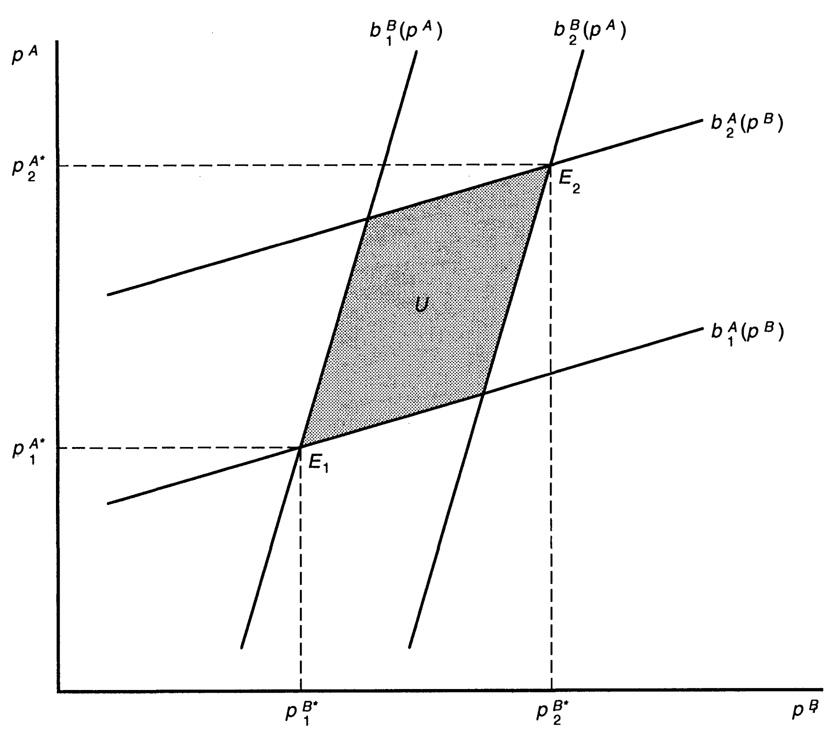
$$P_u^* = \{p_u^* \,|\, (p_u^{A*}, p_u^{B*}) = (b_u^A(p_u^{B*}), b_u^B(p_u^{A*}))\}.$$

Define the region $U \subset \mathcal{R}^2$ by $U = U^A \cap U^B$, where U^i is the region between firm i's two single-market best-response functions in $p^A \times p^B$ space:

$$U^i = \{(p^A, p^B) \,|\, p^i = \alpha b_1^i(p^{-i}) + (1 - \alpha) b_2^i(p^{-i}), \quad \text{for some } \alpha \in (0, 1)\}.$$

Proposition 1. Assume the profit functions π_j^i, for $i = A, B$ and $j = 1, 2$, satisfy

FIGURE 2



Assumptions 1–3. Then a uniform-price equilibrium exists, and all uniform-price equilibrium price vectors lie in the intersection of the regions bounded by each firm's single-market best-response functions. Formally, P_u^* is nonempty and $P_u^* \subset U$.

Analytical proofs of this and most subsequent propositions are found in the Appendix; graphical arguments appear in the text. Proposition 1 states that all uniform-price equilibrium price vectors must lie in region U, defined by the intersection of the regions between the single-market best-response functions and depicted as the shaded region in Figure 2. Intuitively, each firm's optimal uniform price involves a tradeoff of profits earned in each of the two markets, and its uniform-price best response lies between its two single-market best-response functions. In an imperfectly competitive setting, this is the partial equilibrium analog of the observation made in the literature on discriminating monopolists that the optimal uniform price involves raising the price to some consumers and lowering the price to others. In imperfect competition, the equilibrium uniform price is determined by the intersection of these uniform-price best-response functions, and such an equilibrium must then lie in the intersection of the regions bounded by the firms' single-market best-response functions. In fact, every element of this region U is a uniform-price equilibrium for some profit functions generating that region U.

Proposition 2. Assume the profit functions π_j^i, for $i = A, B$ and $j = 1, 2$, satisfy Assumptions 1–3 and define region U as above. Then for any $\hat{p} \in U$ there exist weights α_j^i such that $\hat{p}$ is a uniform-price equilibrium price vector in the game with $\hat{\pi}_j^i = \alpha_j^i \pi_j^i$.

Proof. The vector $\hat{p}$ is a uniform-price equilibrium price vector in the game defined by $\hat{\pi}_j^i$ if and only if it solves

$$\frac{d\hat{\pi}_1^i(\hat{p}^i, \hat{p}^{-i})}{dp^i} + \frac{d\hat{\pi}_2^i(\hat{p}^i, \hat{p}^{-i})}{dp^i} = \alpha_1^i \frac{d\pi_1^i(\hat{p}^i, \hat{p}^{-i})}{dp^i} + \alpha_2^i \frac{d\pi_2^i(\hat{p}^i, \hat{p}^{-i})}{dp^i} = 0,$$

for $i = A, B$. For any $\hat{p} \in U$, $d\pi_1^i(\hat{p}^i, \hat{p}^{-i})/dp^i > 0$ and $d\pi_2^i(\hat{p}^i, \hat{p}^{-i})/dp^i < 0$, or vice versa. It follows that the above expression is satisfied at every $\hat{p} \in U$ for some α_j^i. *Q.E.D.*

Together, Propositions 1 and 2 state that a vector $\hat{p}_u$ is a uniform-price equilibrium price vector for some profit functions generating region U if and only if $\hat{p}_u \in U$. The proof that any vector in U can be a uniform-price equilibrium price vector rests on the observation that it is the relative slopes of a firm's two single-market profit functions that determine each firm's uniform-price best-response. Since the slopes of the profit functions can vary without violating Assumptions 1–3 or altering the best-response functions, profit functions exist that generate equilibrium price vectors p_u^* at all points in U.

For simplicity of exposition, I now restrict attention to situations in which both firms have a clear ranking of the consumer groups. Specifically, I assume that neither firm's single-market best-response functions cross:

Assumption 4. $$b_j^i(p^{-i}) > b_{-j}^i(p^{-i}) \quad \forall\, p^{-i},$$

for some j and for $i = A, B$. In the language of the traditional literature on discriminating monopolists, market j is termed firm i's strong market when this condition holds, and the other market is termed firm i's weak market. Models satisfying Assumption 4 can be divided into two classes based on whether both firms rank the same consumer group as their strong market.

Definition. A model satisfying Assumptions 1–4 exhibits best-response symmetry if $b_j^i(p) > b_{-j}^i(p)$ implies $b_j^{-i}(p) > b_{-j}^{-i}(p)$ for any p. All other models are said to exhibit best-response asymmetry.

Best-response symmetry simply requires that both firms rank the same group of consumers as the strong market, and this property clearly holds under the much stronger symmetry assumptions made by Holmes (1989).[3] The following proposition shows that best-response symmetry is a sufficient condition for price discrimination's equilibrium effects in imperfect competition to be qualitatively similar to the effects demonstrated in the existing literature. Specifically, in models exhibiting best-response symmetry, price discrimination in imperfect competition leads to an increase in prices to one group and decreases in prices to the other group. Without loss of generality, assume market 2 is the strong market, i.e., $b_2^i(p) > b_1^i(p)$ for $i = A, B$ and for all p. Denote prices in the discriminatory equilibrium by p_j^{i*}, for $i = A, B$ and $j = 1, 2$.

Proposition 3. Assume best-response symmetry holds for profit functions π_j^i satisfying Assumptions 1–4. Then, for both firms, price discrimination leads to higher equilibrium prices in the strong market and lower equilibrium prices in the weak market. Formally, all uniform-price equilibrium price vectors $p_u^* \in P_u^*$ satisfy $p_1^{i*} < p_u^{i*} < p_2^{i*}$, for $i = A, B$.

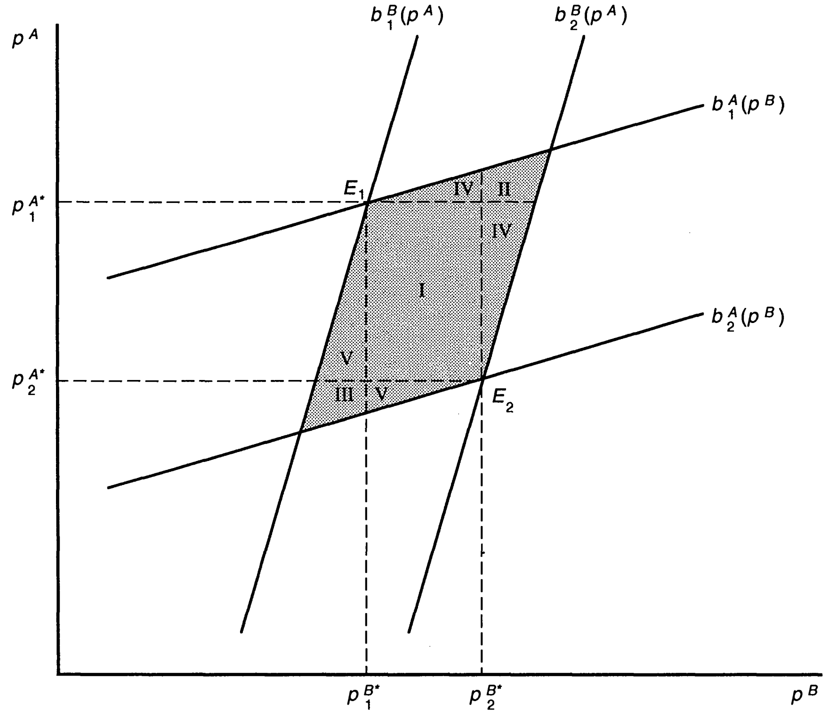
A graphical argument makes this point clear. The best-response functions depicted in Figure 2 satisfy best-response symmetry and yield single-market equilibrium prices denoted by p_j^{i*}. Since the best-response functions are upward sloping, region U (which contains all uniform-price equilibria by Proposition 1) must lie entirely to the northeast of p_1^* and entirely to the southwest of p_2^*. Starting from any uniform-price equilibrium in region U, allowing price discrimination leads to prices p_1^* and p_2^*, which are necessarily higher in market 2 and lower in market 1 for both firms. Thus, in models satisfying best-response symmetry, the usual sort of welfare ambiguity surrounds the effects of third-degree price discrimination. Even when the net effect on (a cardinal measure of) total social surplus can be determined (as it can be under some circumstances in the models studied by Holmes (1989) and Katz (1984)), it is always true that some consumers are made better off and some consumers are made worse off. The following proposition, which formalizes Observations 1 and 3 from Section 2, demonstrates that this does not hold in models exhibiting best-response asymmetry.

Proposition 4. Assume best-response asymmetry holds for profit functions π_j^i satisfying Assumptions 1–4. Then for some profit functions generating the same best-response functions, price discrimination leads to lower (or higher) prices for all consumers. Formally, there exists some $\hat{p}$ such that $\hat{p}^i < p_j^{*i}$ for $i = A, B, j = 1, 2$ and some weights α_j^i such that $\hat{p}$ is a uniform-price equilibrium price vector in the game with $\hat{\pi}_j^i = \alpha_j^i \pi_j^i$. The same holds for the existence of some $\hat{p}$ such that $\hat{p}^i > p_j^{*i}$ for $i = A, B, j = 1, 2$.

Again, a graphical argument makes the reasoning behind this proposition clear. Figure 3 depicts a model exhibiting best-response asymmetry, and as before the set of possible uniform-price equilibria is the shaded region U. When best-response asymmetry holds, the single-market equilibrium prices are at the northwest and southeast corners of region U in Figure 3, and there necessarily exist price vectors in U that give

[3] Holmes's demand structure is fully symmetric: not only is the ranking of types by best-response functions common across firms, but own-price and cross-price elasticities are also identical. As a result, his model also exhibits symmetry in equilibrium prices, which is not a necessary feature of models with best-response symmetry.

FIGURE 3



equilibrium uniform prices greater than all single-market prices and less than all single-market prices. In region II (III), equilibrium uniform prices are higher (lower) than all single-market equilibrium prices, and both firms lower (raise) prices to all consumers when allowed to price discriminate. Since consumers face a choice among the same set of products, higher prices from both firms lead to unambiguous consumer welfare losses, and lower prices from both firms likewise lead to unambiguous gains.[4]

Competitive price discrimination clearly generates very different price and welfare effects under best-response asymmetry, but when is this condition likely to hold? It necessarily requires that firms be asymmetrically differentiated, but this may hold in many plausible scenarios. For example, when products are differentiated in the eyes of some consumers but not in the eyes of others, as in the example of Section 2, best-response asymmetry necessarily holds for a high enough monopoly price in the undifferentiated "cheap" market (specifically, $p_1^m > p_2^{L*}$) when there is any asymmetry in equilibrium prices in the differentiated "choosy" market. While assuming a high p_1^m may seem unnatural in the sense that this requires the cheap market to have a high monopoly price, recall that this is a comparison of the monopoly price in the cheap market with the competitive price in the choosy market. In addition, best-response asymmetry is not inconsistent with the following definition of price symmetry, which I argue more accurately characterizes many common forms of apparently symmetric price discrimination.

Definition. A model satisfying Assumptions 1–4 exhibits price symmetry if $p_j^{i*} < p_{-j}^{i*} \Rightarrow p_j^{-i*} < p_{-j}^{-i*}$. All other models are said to exhibit price asymmetry.

[4] If marginal costs are identical across firms, these price effects translate directly into unambiguous total social welfare gains or losses. More generally, however, a cost distortion arises as sales are shifted between firms with different costs in moving to the discriminatory equilibrium (see Katz, 1984).

Price symmetry holds when both firms charge the same group a lower price in the discriminatory equilibrium. This is consistent with the senior citizen discount and airline ticket restriction examples, since casual empiricism suggests that all firms employing such discriminatory tactics charge lower prices to senior citizens than to other customers and charge lower prices to leisure travelers than to business travelers. In fact, price symmetry is a necessary condition for a broad class of common incentive-compatible price-discrimination schemes, including carding (requiring verification of each customer's membership in a common group that qualifies for a discount, as in student and senior citizen discounts), couponing (where groups of consumers differ in their utility cost of utilizing a coupon), and bad-bundling (a scheme in which the good in question is bundled with a common "bad" and offered at a lower price, as in airline ticket restrictions). All three of these schemes are subsumed by a single model in which a consumer of type k faces cost $c_k \geq 0$ of obtaining the lower price. This can be interpreted as the utility cost of clipping, saving, and redeeming a coupon, or as the disutility associated with consuming or disposing of the bundled "bad." In a carding scheme, the cost to the group identified for the discount is the (possibly zero) cost of carrying and presenting one's identification, while the cost to the other group is the (possibly infinite) cost of obtaining credible fake identification.

Without loss of generality, consider a model in which $p_2^{A*} > p_1^{A*}$. From firm A, type k consumers effectively face price $p_k^{Ae} = \min\{p_2^{A*}, p_1^{A*} + c_k\}$. Firm A then achieves equilibrium incentive-compatible price discrimination only if $c_1 < p_2^{A*} - p_1^{A*} < c_2$, i.e., if the discount is large enough to induce *only* consumers from market 1 to incur the cost associated with obtaining the lower price. For firm B to achieve equilibrium incentive-compatible price discrimination, it must be either that $c_1 < p_2^{B*} - p_1^{B*} < c_2$ (if price symmetry holds) or that $c_2 < p_1^{B*} - p_2^{B*} < c_1$ (if price asymmetry holds). Since $c_1 < c_2$ if firm A achieves equilibrium incentive-compatible price discrimination, the latter inequality cannot be satisfied simultaneously, and price symmetry is a necessary condition for equilibrium incentive-compatible price discrimination. This proves the following proposition.

Proposition 5. Price symmetry is a necessary condition for carding, couponing, or bad-bundling to achieve equilibrium incentive-compatible price discrimination by both firms.

The intuition behind this proposition is straightforward. If both firms use the same scheme to price discriminate, then both firms can achieve incentive-compatible discrimination only by offering the same market the lower price. For example, if all consumers in each market face the same cost of redeeming a coupon, there is no way for firm A to offer a coupon that only market 1 redeems while firm B offers a coupon that only market 2 redeems.[5] The logic of this proposition fails if firms can use different criteria to screen their customers. For example, one firm might offer a discount to all men, while the other offered a discount to all women. This is a form of carding that allows firms to target different groups for discounts, thereby achieving incentive-compatible price discrimination in the case of price asymmetry. Similarly, each firm might bundle a different bad with its product to target different groups for the lower price. If each firm can adopt some incentive-compatible scheme that implements a lower effective price for its weak market, price symmetry is not required for discrimination and this reduces to a model with identifiable types.

[5] Note that not every consumer in market 1 redeems the coupon in equilibrium in a differentiated-goods model; however, every consumer in market 1 would redeem the coupon if he or she chose to purchase from firm A.

The objective here is not to lay out a general theory of incentive-compatible price discrimination, but to point out that price symmetry is a different kind of symmetry from best-response symmetry, and that price symmetry (not best-response symmetry) is a necessary condition for the efficacy of commonly observed price-discrimination schemes like senior citizen discounts (carding with respect to a common consumer group) and airline ticket restrictions (bad-bundling with common "bads"). The following proposition, which formalizes Observation 4 in Section 2, demonstrates that price symmetry does not imply best-response symmetry.

Proposition 6. A model satisfying Assumptions 1–4 may exhibit both price symmetry and best-response asymmetry.

This proof is most easily understood graphically. In Figure 3, the discriminatory equilibrium prices are in the northwest and southeast corners of the shaded region, since best-response asymmetry holds. If b_1^A is moved down until it is near b_2^A, the northwest corner of the shaded region, which defines p_1^*, will clearly lie strictly to the southwest of p_2^*. In this case, price symmetry holds, even though there is asymmetry in the firms' ranking of the consumers by their best-response functions. Again, this demonstrates that best-response asymmetry may be a plausible and interesting condition, in the sense that it is consistent with an important and intuitively appealing definition of symmetry in equilibrium prices. The excessively strong symmetry assumptions used in the existing literature blur the distinction between asymmetry in equilibrium prices and asymmetry in best-response functions. With the best-response asymmetry condition identified precisely, it is clear that the previously unstudied asymmetric case, in which price discrimination may lead to lower (or higher) prices for all consumers and unambiguous consumer welfare gains (or losses), may be of significant importance and interest.

4. Strategic commitments not to price discriminate

■ While a monopolist is always better off discriminating, imperfectly competitive firms may find the uniform-price equilibrium more profitable than the discriminatory equilibrium. However, in a game in which decisions about whether to discriminate and what prices to charge are made simultaneously, price discrimination is a dominant strategy: given the other firm's strategy, each firm is better off unconstrained in its price choice. While this discrimination is unilaterally profit improving, in equilibrium it leads to lower profits for both firms; in this sense, the firms find themselves in a prisoner's dilemma.

One might argue that it is more natural to model such a game in two stages, with firms deciding whether to put in place some scheme for discrimination in advance of the actual price-setting decisions. This would allow meaningful commitments not to discriminate, as a firm could announce in the first stage, for example, that it would not hold sales or would not issue coupons. To focus on the issue at hand, I will assume that such commitments can be made credibly, and I examine the possibility that such commitments not to discriminate may improve profits for the firms.

Simply permitting credible precommitments does not render this a trivial problem, because breaking the game into two stages may simply shift the prisoner's dilemma into the first stage. For example, consider the classical prisoner's dilemma story but add a stage prior to the prisoners' independent interrogations in which each prisoner has the opportunity to commit credibly not to fink, a commitment that will then be observed by the other prisoner before their interrogations. This does not resolve their dilemma, since in this modified game it is a dominant strategy in the first stage not to commit not to fink. Regardless of the other prisoner's committing or not committing not to fink, one still does best in the second stage by finking and thus does best in the first stage by making no commitments. The crux of the prisoner's dilemma is not the prisoners' inability to commit but rather their inability to *coordinate* their actions so that neither pursues the dominant strategy. In the context of the price-discrimination game, even when one firm commits not to discriminate, the other firm may find it profitable to take advantage of the other firm's weakness by discriminating anyway. Thus, it is not immediately clear whether first-stage commitments not to discriminate are helpful in implementing the more profitable uniform-price equilibrium.

In describing the use of the nondiscrimination commitments discussed in the introduction, the popular press and business literature (e.g., Ortmeyer, Quelch, and Salmon, 1991) have focused largely on their effects on the efficiency of the firm's manufacturing and distribution activities. For example, EDLP policies lead to smoother sales patterns, thereby eliminating costly spikes in production, reducing costly stock-outs, and permitting more efficient distribution of merchandise. Other such policies might be valuable for their direct appeal to consumers. The very name of Saturn's "no-haggle" (as opposed to, say, "no-bargain" or "no-discount") policy seems intended to address the unpleasant image some consumers have of car dealers and the bargaining process that is typical of car sales in the United States. In addition, both EDLP policies and explicitly intertemporal most-favored-customer clauses (which guarantee future price reductions will be made retroactive to past sales) are sometimes argued to be valuable in overcoming the durable-good monopolist's Coase-conjecture problem by permitting the firm to rule out price reductions in the future (or at least make them expensive), thereby inducing earlier purchases at higher prices. While the above considerations are surely important to firms that adopt these policies, I demonstrate, in contrast, a "strategic" rationale for these policies, whereby commitments not to discriminate soften price competition and raise firm profits.

It is possible to show that the prisoner's-dilemma nature of the price-discrimination game may persist when commitments to uniform pricing are allowed (see Corts (1996) for a numerical example based on Section 2); however, this is not always the case. It is possible that the equilibria in the subgames in which a single firm discriminates are preferred by the nondiscriminating firm to the equilibrium in which both firms discriminate. In this case, the commitment game is *not* a prisoner's dilemma, and equilibria with unilateral commitments to uniform pricing may arise. This is effectively the result demonstrated by Cooper (1986). While he does not frame his argument in terms of the broader price-discrimination literature, he shows that a unilateral commitment to an explicitly intertemporal most-favored-customer policy (which amounts to a commitment to uniform pricing) may soften price competition and raise firm profits. When the two-stage game is not a prisoner's dilemma but has equilibria with unilateral commitments to uniform pricing in equilibrium, the uniform-price equilibrium is not implemented. Instead, one firm intentionally ties its own hands to reap the benefits of the rival's less aggressive (though still discriminatory) response to this constrained behavior.

I demonstrate in this section that even when the two-stage commitment game between discriminating firms is a prisoner's dilemma, unilateral commitments to uniform pricing may arise in equilibrium if firms are constrained to employ only incentive-compatible price discrimination schemes. I show that when best-response symmetry holds, this constraint is irrelevant, as both firms may employ the same scheme to discriminate in favor of a common group of consumers. The prisoner's-dilemma nature of the payoffs therefore dictates a unique equilibrium in which both firms discriminate. However, the incentive-compatibility constraint fundamentally alters the structure of the game when best-response asymmetry holds. In that case, requiring incentive-compatibile price discrimination prevents one firm from discriminating unilaterally when faced with the rival's uniform price, since discrimination in favor of its weak market is technologically infeasible. This effectively permits the rival to choose between the discriminatory and uniform-price subgames. Given the prisoner's-dilemma structure, that firm will always choose to implement the uniform-price equilibrium through a unilateral commitment to uniform pricing.

Consider a model in which price symmetry holds (without loss of generality, let $p_1^{i*} < p_2^{i*}$) and in which the only available discriminatory technology is of the class of incentive-compatible technologies described above, where $c_1 = 0 < c_2$. For purposes of the following proposition, I assume that c_2 is large enough that no firm with $b_2^j(p_{-j}) > b_1^j(p_{-j})$ is ever constrained in achieving its desired discriminatory prices. It follows that $c_2 > p_2^{i*} - p_1^{i*} > c_1 = 0$ for $i = A, B$, which implies that the discriminatory equilibrium is implemented. The following proposition demonstrates that a (unilateral) commitment not to discriminate arises in equilibrium if and only if best-response asymmetry holds.

Proposition 7. Assume that prior to simultaneous price setting in the product market firms may simultaneously make (credible) commitments not to discriminate in the second (price-setting) stage and that the resulting two-stage game exhibits prisoner's-dilemma payoffs (i.e., has strictly dominant strategies leading to lower equilibrium payoffs) if firms can discriminate perfectly. Then if firms are constrained to employ incentive-compatible price discrimination schemes targeting a single group for lower prices, a unilateral commitment not to discriminate arises in equilibrium if and only if best-response asymmetry holds. Further, such a commitment implements the uniform-price equilibrium.

Proof. Given a uniform price for firm $-j$, an uncommitted firm j maximizes its profits by setting $\hat{p}_2^j = b_2^j(p^{-j})$ and $\hat{p}_1^j = \hat{p}_1^j(p^{-j})$. If best-response symmetry holds, then it must be that $b_2^j(p^{-j}) > b_1^j(p^{-j})$ (since price symmetry holds and $p_2^{i*} > p_1^{i*}$). As a result, j can discriminate to achieve its best-response prices as long as $\hat{p}_2^j - \hat{p}_1^j < c_2$, which follows from the above assumption on c_2. Thus, the unconstrained firm does effectively discriminate, and the constraint on firms' price-discrimination schemes is irrelevant when best-response symmetry holds. Since the payoff structure yields a prisoner's dilemma by assumption, the hypothetically constrained firm $-j$ prefers not to commit to uniform pricing when firm j discriminates. This implies that no commitments arise in equilibrium under best-response symmetry.

But if best-response asymmetry holds, then $b_2^j(p^{-j}) < b_1^j(p^{-j})$ for some j. Even if this j is an uncommitted firm, it cannot achieve profitable discrimination when faced with a rival's uniform price. At any $p_2^j < p_1^j$, market 1 effectively faces price p_2^j: since $p_1^j + c_1 > p_2^j$, consumers in market 1 will not avail themselves of the "discount" $p_2^j - p_1^j < 0$. As a result, firm j does best by setting $p_1^j = p_2^j = b_u^j(p^{-j})$, and the constraint on feasible price-discrimination schemes effectively prohibits unilateral discrimination by this firm.

This demonstrates that the subgame following a unilateral uniform-price commitment by firm k, defined as the firm for which $b_2^k > b_1^k$, is characterized by the uniform-price equilibrium prices and payoffs. Firm k therefore prefers to commit to uniform pricing if the other firm does not commit to uniform pricing but prefers not to commit if the other firm does commit to uniform pricing. Firm $-k$ prefers not to commit to uniform pricing if the rival does not commit and is indifferent if the rival does. It follows that the unique equilibrium outcome involves only firm k committing to uniform pricing. This implements the uniform-price equilibrium in the second stage. *Q.E.D.*

When best-response asymmetry holds, the discriminatory technology may constrain one of the firms from unilaterally achieving profitable price discrimination. The firm whose ranking of the markets by best-response functions coincides with the ranking by equilibrium prices will always be able to discriminate, but the other firm may discriminate profitably only in response to the first firm's discrimination. In such situations the former firm may implement the more profitable uniform-price equilibrium through a unilateral commitment not to price discriminate.

For concreteness, consider the example of Section 2. When the low-quality firm is unable to achieve profitable price discrimination unilaterally, the high-quality firm sees its decision whether to price discriminate as a choice between all-out competition and the uniform-price equilibrium, since the low-quality firm will discriminate only if the high-quality firm does so as well. As a result, the high-quality firm commits to uniform pricing, the low-quality firm sets a uniform price because it is unable to discriminate effectively when faced with the high-quality firm's uniform price, and the uniform-price equilibrium is implemented. This implies that the conventional retailers (and not the discounters) or the leading brands (and not the private labels) may implement the uniform-price equilibrium by adopting commitments not to discriminate. This seems broadly consistent with the observed use of such policies.

5. Conclusion

■ Competitive price discrimination may intensify competition by giving firms more weapons with which to wage their war. Allowing firms to set market-specific prices through discrimination breaks the cross-market profit implications of aggressive price moves that may restrain price competition when firms are limited to uniform pricing. Thus, firms may price more aggressively in some markets when permitted to discriminate; if firms differ in which markets they target for this aggressive pricing and competitive reactions are strong, prices in all markets may fall.

Clearly, this logic applies more broadly than only to the two-firm, two-market model considered here. Since price discrimination's effect on prices is composed of the direct effect of relaxing the uniform-price constraint and the indirect and complementary competitive response to that effect, what is critical is the sign of the direct effect. Regardless of whether there are more firms than consumer groups or vice versa, a necessary condition for an unambiguous fall (rise) in prices is that *some* firm has a direct incentive to lower (raise) prices in *each* market. As long as not every firm regards the same group as its strongest market, there is the potential for unilateral incentives toward more aggressive behavior in each individual market for some firm, and if the strategic complementarity of such aggressive pricing is strong enough, prices may fall in every market.

When competitive price discrimination intensifies competition and leads to lower firm profits, firms may wish to refrain from this aggressive behavior. Committing to be less aggressive through commonly observed commitments not to price discriminate may be effective in softening price competition and raising firm profits. In the case of two examples given earlier, this might take the form of a retailer eliminating sales through an EDLP policy or a consumer goods manufacturer eliminating coupons. In either case, the conventional retailer or the branded-product company effectively cedes the most price-sensitive segment of the market to the discount retailer or private-label product, expecting the reduction in the intensity of competition to lead to higher prices from the rival, and therefore higher profits in the remaining, less price-sensitive segments of the market.

Importantly, price discrimination leads to all-out competition only if the discount retailer or the private label is so disadvantaged with respect to the most quality-sensitive market segment that, at the equilibrium uniform prices, it would like to offer that segment a discount. When this holds, the discount retailer or the private-label company cannot discriminate when faced with the rival's uniform price; sales would be attended or coupons clipped by precisely the wrong consumer segment. As a result, a commitment to uniform pricing by the conventional retailer or the branded-product company alone suffices to implement the uniform-price equilibrium.

Despite the progress made in clarifying the logic of the price and welfare effects of competitive price discrimination, conditions on demand that generate the various regimes studied here remain elusive. While I argue that best-response asymmetry is a plausible condition and give examples when it holds and when price discrimination leads to all-out competition, a more complete characterization of demand conditions that generate these results is beyond the scope of this article. Research elucidating either the conditions in which best-response asymmetry holds or in which uniform-price best responses lie near the single-market best responses (so that unambiguous price and welfare effects obtain) would substantially further our understanding of competitive price discrimination.

Appendix

■ Proofs of Propositions 1, 3, 4, and 6 follow.

Proof of Proposition 1. Since $\pi_j^i(p^i, p^{-i})$ is concave for $j = 1, 2$, $[d\pi_u^i(p^i, p^{-i})]/dp^i$ is positive at $p^i = \min_j b_j^i(p^{-i})$ and negative at $p^i = \max_j b_j^i(p^{-i})$. Since $d\pi_u^i/dp^i$ is continuous, this implies $\min_j b_j^i(p^{-i}) < b_u^i(p^{-i}) < \max_j b_j^i(p^{-i})$. Therefore, the graph of b_u^i lies entirely in U^i, and any intersection of the graphs of b_u^i and b_u^{-i} (which defines the set P_u^*) must lie in U. The existence of some $p_u^* \in P_u^*$ follows from the usual fixed-point argument, since the best-response functions b_u^i are continuous, and since attention can be restricted to the compact, convex set $[0, \hat{p}] \times [0, \hat{p}]$, where $\hat{p} = \max_j \max_i \{\hat{p}^i | b_j^{-i}(p^i) = p^i\}$. *Q.E.D.*

Proof of Proposition 3. For any $\hat{p} \in P_u^*$, $b_1^i(\hat{p}^{-i}) < b_u^i(\hat{p}^{-i}) = \hat{p}^i$ by Proposition 1. Taking the inverse of b_1^{-i} of both sides of firm $-i$'s analogous expression yields $\hat{p}^i < [b_1^{-i}]^{-1}(\hat{p}^{-i})$. Together, these imply $b_1^i(\hat{p}^{-i}) < \hat{p}^i < [b_1^{-i}]^{-1}(\hat{p}^{-i})$ for any $\hat{p}^i \in P_u^*$. Let $\hat{p}^{-i} = p_1^{-i*} - \delta$, for any $\delta \geq 0$. Since $b_1^{i\prime} < 1$ by Assumptions 2 and 3, $b_1^i(\hat{p}^{-i}) \geq p_1^{i*} - \delta \geq [b_1^{-i}]^{-1}(\hat{p}^{-i})$, which establishes a contradiction to the prior inequality and proves that $p_u^* \in P_u^*$ implies $p_u^* > p_1^{i*}$ for $i = A, B$. An analogous argument proves the remaining inequality. *Q.E.D.*

Proof of Proposition 4. Let subscripts W and S denote a firm's weak and strong market respectively.

Claim 1. There exists a price vector $\tilde{p}$ such that $\tilde{p}^i = b_W^i(b_W^{-i}(\tilde{p}^i))$. This follows from the usual fixed-point argument.

Claim 2. For $i = A, B$ and $k = 1, 2$, $\tilde{p}^i < p_k^i$. By definition, $b_S^{-i}(\tilde{p}^i) > b_W^{-i}(\tilde{p}^i) = \tilde{p}^{-i} = [b_W^i]^{-1}(\tilde{p}^i)$. Also by definition, p_W^i satisfies $b_S^{-i}(p_W^i) = [b_W^i]^{-1}(p_W^i)$. Since $b_S^{-i\prime} < 1 < [b_W^i]^{-1\prime}$, the above inequality is satisfied only if $p_W^i > \tilde{p}^i$. Further, this implies $p_S^{-i} > \tilde{p}^{-i}$, since $b_S^{-i\prime} > 0$.

Claim 3. $\hat{p} = \tilde{p} + (\epsilon, \epsilon) \in U$ for small $\epsilon > 0$. To show $b_W^i(\hat{p}^{-i}) < \hat{p}^i$, note that $b_W^{i\prime} < 1$ implies $b_W^i(\tilde{p}^{-i} + \epsilon) < \tilde{p}^i + \epsilon = \hat{p}^i$. That $b_S^i(\hat{p}^{-i}) > \hat{p}^i$ follows from the fact that b_S^i is continuous and $b_S^i(\tilde{p}^{-i}) > b_W^i(\tilde{p}^{-i}) = \tilde{p}^i$.

Together, Claims 1–3 prove the existence of $\hat{p} \in U$ such that $\hat{p}^i < p_j^{i*}$. With Proposition 2, this proves the proposition for $\hat{p}_j < p_j^{*i}$. An analogous argument proves the result for $\hat{p}_j > p_j^{*i}$. *Q.E.D.*

Proof of Proposition 6. Without loss of generality, assume that $b_2^B > b_1^B$ and that $b_1^A > b_2^A$. Consider the limiting case as $b_1^A(p) \to b_2^A(p)$ at all p, and note that market 2 equilibrium prices p_2^* do not change as b_1^A varies. Proposition 4 established the existence of $\tilde{p}$ such that $\tilde{p}^i = b_W^i(b_W^{-i}(\tilde{p}^i))$ and that $\tilde{p}^i < p_k^{i*}$ for $i = A, B$ and $k = 1, 2$. Since market 1 is B's weak market and market 2 is A's weak market, $p_1^* \to \tilde{p}$ as $b_1^A(p) \to b_2^A(p)$. So, for b_1^A near b_2^A, $p_1^{i*} < p_2^{i*}$ for $i = A, B$. *Q.E.D.*

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