

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{X} = \frac{621}{8} = 77.625$$

$$\bar{Y} = \frac{29.9}{8} = 3.7375$$

$$\sum X_i Y_i = 2612.4$$

$$\sum X_i^2 = 48714$$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2}$$

$$= \frac{2612.4 - 8(77.625)(3.7375)}{48714 - 8(77.625)^2} = \frac{2612.4 - 2944.9625}{48714 - 48203.125} = \frac{31}{910} = 0.0341 \checkmark$$

$$\therefore \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 3.7375 - 0.0341(77.625)$$

$$= 0.5655 \checkmark$$

1.2 For each observation i , find \hat{Y}_i and \hat{u}_i . Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$i = 1 \quad ; \quad \hat{Y}_1 = 2.714$$

$$\hat{u}_1 = 2.8 - 2.714 = 0.086$$

$$i = 2 \quad ; \quad \hat{Y}_2 = 3.021$$

$$\hat{u}_2 = 3.4 - 3.021 = 0.379$$

$$i = 3 \quad ; \quad \hat{Y}_3 = 3.225$$

$$\hat{u}_3 = 3.0 - 3.225 = -0.225$$

$$i = 4 \quad ; \quad \hat{Y}_4 = 3.327$$

$$\hat{u}_4 = 3.5 - 3.327 = 0.173$$

$$i = 5 \quad ; \quad \hat{Y}_5 = 3.532$$

$$\hat{u}_5 = 3.6 - 3.532 = 0.068$$

$$i = 6 \quad ; \quad \hat{Y}_6 = 3.123$$

$$\hat{u}_6 = 3.0 - 3.123 = -0.123$$

$$i = 7 \quad ; \quad \hat{Y}_7 = 3.123$$

$$\hat{u}_7 = 2.7 - 3.123 = -0.423$$

$$i = 8 \quad ; \quad \hat{Y}_8 = 3.634$$

$$\hat{u}_8 = 3.7 - 3.634 = 0.066$$

$$\sum_{i=1}^8 \hat{u}_i = 0 \checkmark$$

1.3 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_0)$, $\text{var}(\hat{\beta}_1)$

$$\text{Var}(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{9-2} = 0.07245 \checkmark$$

$$\text{Var}(\hat{\beta}_0) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{0.07245}{911.875} = 0.000141 \checkmark$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} (\hat{\sigma}^2) = \frac{48717(0.07245)}{8(911.875)} = 0.8625 \checkmark$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$$\bar{X} = 20$$

$$\bar{Y} = 9.1$$

$$\sum X_i Y_i = 2214$$

$$\sum X_i^2 = 4440$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim \text{NIID}(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2}$$

$$= \frac{2214 - 10(20)(9.1)}{4440 - 10(20^2)} = \frac{2214 - 1820}{4440 - 4000} = 0.895 \checkmark$$

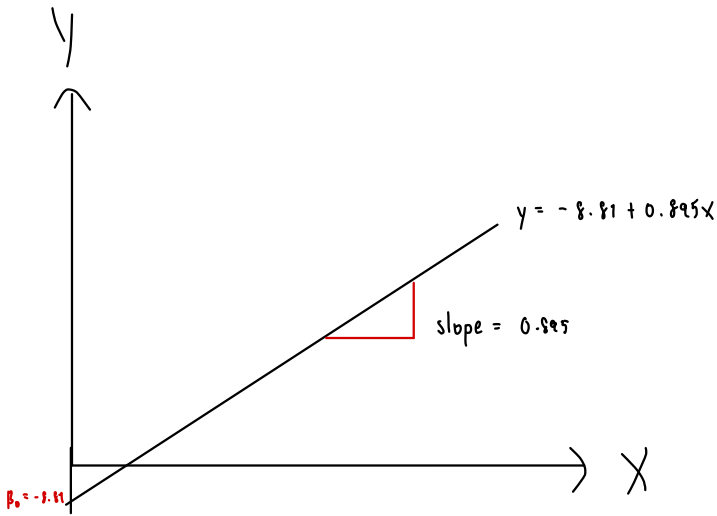
$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 9.1 - 0.895(20) = -8.81 \checkmark$$

2.2 Find the value of \hat{Y}_i and \hat{u}_i . Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$i=1$	$\hat{Y}_1 = 0.1455$	$\hat{u}_1 = -0.145$
$i=2$	$\hat{Y}_2 = 1.9364$	$\hat{u}_2 = 0.064$
$i=3$	$\hat{Y}_3 = 3.7273$	$\hat{u}_3 = 1.273$
$i=4$	$\hat{Y}_4 = 5.5182$	$\hat{u}_4 = 0.482$
$i=5$	$\hat{Y}_5 = 7.3091$	$\hat{u}_5 = -0.309$
$i=6$	$\hat{Y}_6 = 9.1009$	$\hat{u}_6 = -0.891$
$i=7$	$\hat{Y}_7 = 10.8918$	$\hat{u}_7 = -2.682$
$i=8$	$\hat{Y}_8 = 12.6827$	$\hat{u}_8 = 0.527$
$i=9$	$\hat{Y}_9 = 14.4736$	$\hat{u}_9 = -0.264$
$i=10$	$\hat{Y}_{10} = 16.2645$	$\hat{u}_{10} = 1.945$
		$\sum_{i=1}^{10} \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass (\bar{X}, \bar{Y}) ?



If $\bar{x} = 20$ and $\bar{y} = 9.1$ can be perfectly fit in regression function, the line pass (\bar{x}, \bar{y})

$$= y = -8.81 + 0.895(20)$$

$$= y = 9.1 \checkmark$$

\therefore The line pass (\bar{x}, \bar{y})

2.4 If $X_i = 16$, what is the predicted Y?

$$\begin{aligned} \text{If } x_i = 16; \hat{y}_i &= \beta_0 + \beta_1(16) \\ &= -8.81 + 0.895(16) = 5.51 \checkmark \end{aligned}$$

2.5 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_0)$, $\text{var}(\hat{\beta}_1)$

$$\text{Var}(\hat{u}_i) = \frac{\sum u_i^2}{n-2} = \frac{14.091}{10-2} = 1.761375 \checkmark$$

$$\text{Var}(\hat{\beta}_0) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{1.761375}{440} = 0.004003 \checkmark$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum x_i^2 (\sigma^2)}{n \sum (x_i - \bar{x})^2} = \frac{4440(1.761375)}{10(440)} = 1.7774 \checkmark$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

In order to prove that $\hat{\beta}_1$ is unbiased estimator, we have to use assumption SLR 1-4

$$\hat{\beta}_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}; \quad \text{Let } x_i = x_i - \bar{x}$$

$$k_i = \frac{u_i}{\sum x_i^2} = \frac{x_i - \bar{x}}{\sum (X_i - \bar{X})^2}$$

$$\text{Now, write as; } \hat{\beta}_1 = \sum_{i=1}^n (Y_i - \bar{Y}) k_i$$

$$= \sum_{i=1}^n (\beta_0 + \beta_1 X_i + u_i - \beta_0 - \beta_1 \bar{X}) k_i$$

$$= \sum_{i=1}^n \beta_1 (X_i - \bar{X}) k_i + \sum_{i=1}^n u_i k_i$$

$$= \beta_1 \sum_{i=1}^n x_i k_i + \sum_{i=1}^n u_i k_i$$

$$= \beta_1 \sum_{i=1}^n x_i \left(\frac{x_i}{\sum x_i^2} \right) + \sum_{i=1}^n u_i k_i$$

$$= \beta_1 \frac{\sum x_i^2}{\sum x_i^2} + \sum_{i=1}^n u_i k_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_{i=1}^n u_i k_i] = \beta_1 + E[\sum_{i=1}^n u_i k_i]$$

SLR 4: $E(u_i | X_i) = 0 \rightarrow$ This assumption takes the value of x as given (or fixed) so that we can treat x_i as a constant

$$E(\hat{\beta}_1) = \beta_1 + \sum_{i=1}^n k_i E(u_i)$$

$$= \beta_1$$

$\therefore \hat{\beta}_1$ is an unbiased estimator