

Long Run Production - production in the time frame long enough to change every input.  $\Rightarrow$  no fixed cost.

i.e. in LR, if  $Q=0$ , Total cost = 0.

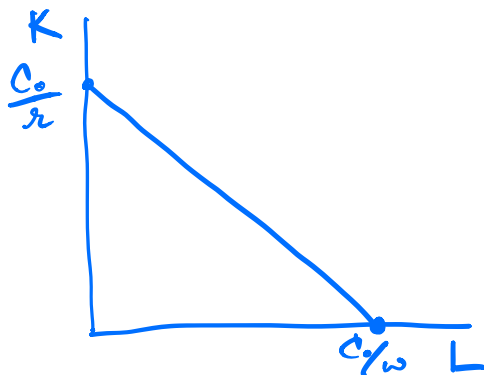
There are two problems in LR that each firm faces.

①  $\max Q = \text{output}$   
under the constraint  
of given cost =  $C_0$  }  $\Leftrightarrow$   $\max Q = f(L, K)$   
s.t.  $\omega L + rK = C_0$

②  $\min \text{Cost} = C$   
under the constraint  
of attaining output  $Q_0$  }  $\Leftrightarrow$   $\min C = \omega L + rK$   
s.t.  $f(L, K) = Q_0$

$Q = f(L, K)$  is the production function  
 $\omega = \text{wage} - \text{price of } L$   
 $r = \text{interest} - \text{price of } K.$

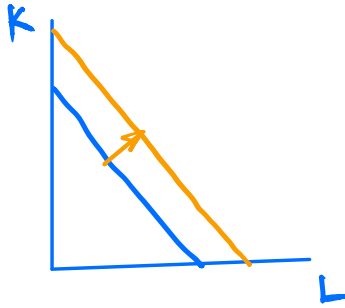
Isocost. - given  $C_0$ ,  $\omega L + rK = C_0$  is a line called Isocost because every point on this line is the quantity  $L + K$  that the firm has to pay the same cost  $C_0$ .



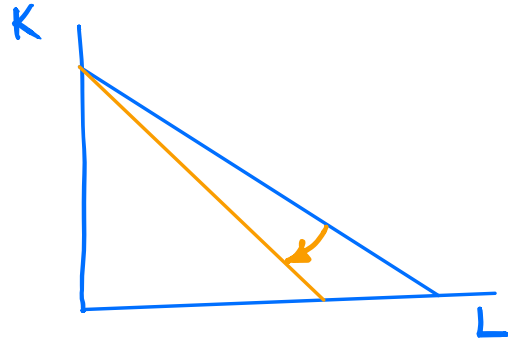
$$\omega L + rK = C_0$$
$$K=0, L = C_0/\omega$$
$$L=0, K = C_0/r$$
$$\text{slope of isocost} = -\frac{\omega}{r}$$

## Change in Isoquant.

1)  $C_0$  increases to  $C_1$



2)  $w$  increases.



Isoquant. - A curve whose every point is  $L+K$  that produce the same output  $Q$ .

## Properties of Isoquant.

- 1) Each point of  $(L, K)$ , there is exactly 1 Isoquant passing through it  $\rightarrow$  There are infinite number of isoquants.
- 2) Isoquants can't intersect or be tangent to each other.
- 3) Higher Isoquant means higher output.
- 4) Isoquant always has negative slopes.

$$\text{slope of Isoquant} = - \frac{MP_L}{MP_K}$$

= Marginal Rate of Technical Substitution  
(MRTS)

5) Diminishing MRTS as we increase L



## Production Equilibrium.

1) Maximize Q

$$\max Q = f(L, K)$$

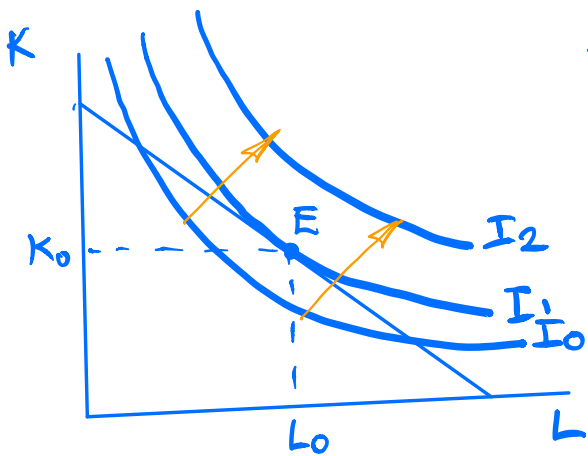
$$\text{St. } \omega L + rK = C_0$$

The equilibrium is at  $E = (L_0, K_0)$

because

1) E is on Isocost given  $\omega L_0 + rK_0 = C_0$

2)  $I_1$  is the highest Isoquant attainable given  $C_0$  and we have  $I_1$  tangent Isocost.



$$-\frac{MP_L}{MP_K} = -\frac{\omega}{r}$$

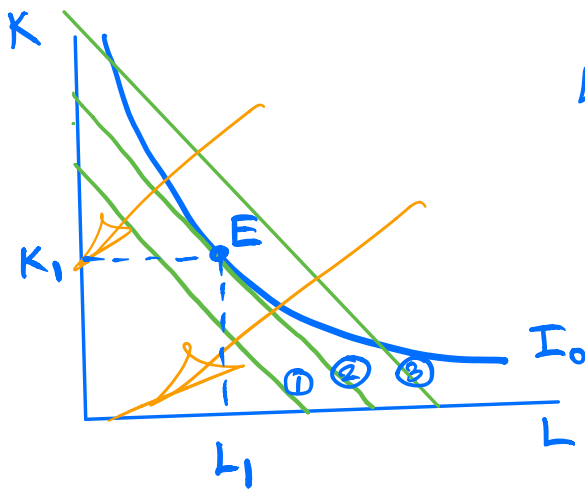
Equilibrium conditions at  $E = (L_0, K_0)$

1)  $\omega L_0 + rK_0 = C_0$  — on Isocost.

$$2) \left( \frac{MP_L}{MP_K} \right)_{at E} = \frac{\omega}{r} \Leftrightarrow \left\{ \frac{MP_L}{\omega} = \frac{MP_K}{r} \right\}$$

The last \$ spent of L gives the same output as the last \$ spent on K.

## 2) Minimizing Cost C



$$\min C = \omega L + rK$$
$$\text{St. } f(L, K) = Q_0$$

Equilibrium is at  $E = (L_1, K_1)$

because

- 1)  $L_1$  and  $K_1$  can produce  $Q_0$  because  $E$  is on  $I_0$  that gives  $Q_0$ .
- 2) Isocost 2 is the lowest isocost we can stay on and still produce  $Q_0$ .

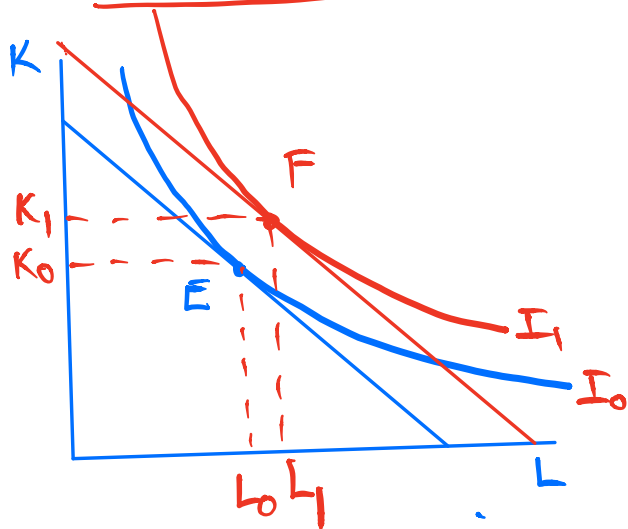
Equilibrium conditions at  $E = (L_1, K_1)$

1)  $f(L_1, K_1) = Q_0$  -  $(L_1, K_1)$  is on the given Isoquant.

2)  $\left( \frac{MP_L}{MP_K} \right)_{\text{at } E} = \frac{\omega}{r}$ .

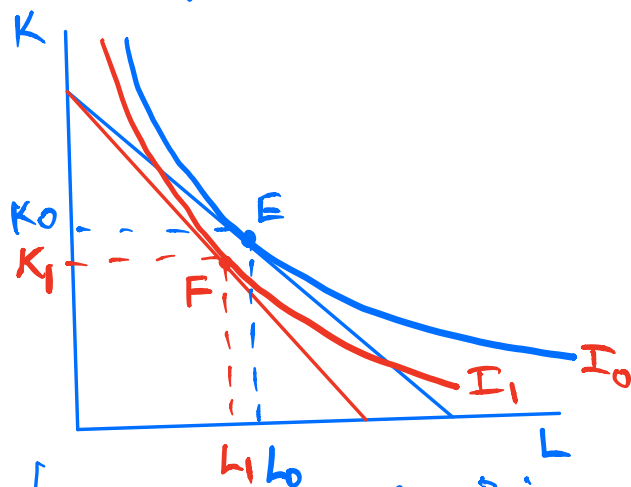
Change in Equilibrium .  $\max Q = f(L, K)$   
 St.  $\omega L + rK = C_0$

1)  $C_0$  increases to  $C_1$



Exactly the same as in higher income in consumption - but no discussion of necessary/luxury goods.

2) Higher wage

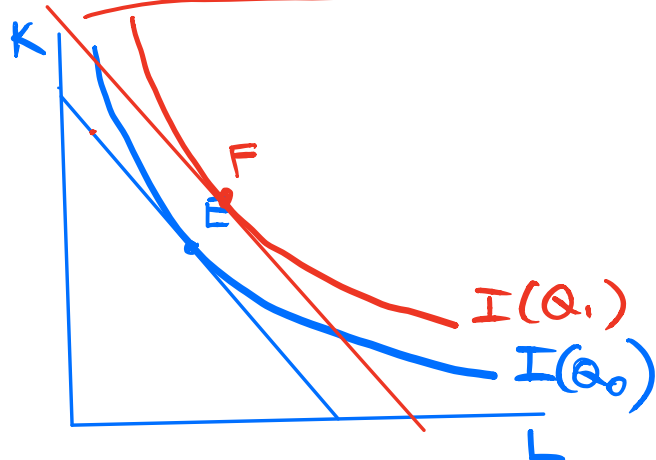


same as when  $P_x$  increases in consumption but no discussion of Subst + Income Effects.

Change in Equilibrium

$\min \text{Cost} = \omega L + rK$   
 St  $f(L, K) = Q_0$

①  $Q_0$  increases to  $Q_1$



② higher wage

