ID Solution sketch.

Quiz#2
EE320 Introductory Mathematical Economics
Section 046403

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Question 1 (100 points) Consider the following supply and demand model:

$$\begin{aligned} Q^d &= D(P,T) \\ Q^s &= S(P,A) \end{aligned}$$

Where P is price, T is specific tax and A is level of productivity of producers. Also note that $\frac{\partial D}{\partial P} < 0$, $\frac{\partial S}{\partial P} > 0$, $\frac{\partial D}{\partial T} < 0$, and $\frac{\partial S}{\partial A} > 0$.

Assure $\frac{dA}{dT} = 0$. $\frac{dP}{dT}$ $\frac{dQ}{dA}$.

Determine the impacts of specific tax on equilibrium price (50 points) and that of productivity on equilibrium quantity (50 points).

$Q^d = D(P,T) \Rightarrow \quad dQ = D_P dP + D_T dT$

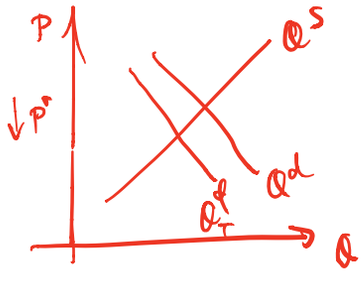
$Q^S = S(P,A) \Rightarrow \quad dQ = S_P dP + S_A dA.$

(1) $\frac{dP}{dT}$:

$$\frac{dQ}{dT} - D_P \frac{dP}{dT} = D_T$$

$$\frac{dQ}{dT} - S_P \frac{dP}{dT} - S_A \frac{dA}{dT}^{0} \Rightarrow \begin{bmatrix} 1 & -D_P \\ 1 & -S_P \end{bmatrix} \begin{bmatrix} \frac{dQ}{dT} \\ \frac{dP}{dT} \end{bmatrix} = \begin{bmatrix} D_T \\ 0 \end{bmatrix}$$

$$\frac{dP}{dT} = \frac{\begin{vmatrix} 1 & D_T \\ 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -D_P \\ 1 & -S_P \end{vmatrix}} = \frac{-D_T}{-S_P + D_P} = \frac{-(\ominus D_T)}{D_P - S_P} : \frac{\oplus}{\ominus - \oplus} < 0$$



② $\frac{dQ}{dA}$:

$$\frac{dQ}{dA} - D_P \frac{dP}{dA} = D_T \cancel{\frac{dT}{dA}}^{0}$$

$$\frac{dQ}{dA} - S_P \frac{dP}{dA} = S_A$$

$$\Rightarrow \begin{bmatrix} 1 & -D_P \\ 1 & -S_P \end{bmatrix} \begin{bmatrix} \frac{dQ}{dA} \\ \frac{dP}{dA} \end{bmatrix} = \begin{bmatrix} 0 \\ S_A \end{bmatrix}$$

$$\frac{dQ}{dA} = \frac{\begin{vmatrix} 0 & -D_P \\ S_A & -S_P \end{vmatrix}}{\begin{vmatrix} 1 & -D_P \\ 1 & -S_P \end{vmatrix}} = \frac{D_P S_A}{D_P - S_P} \qquad \frac{(+)}{(-)} > 0.$$