

## **Topic 2 Part 2**

# **Consumer Choice (Chapter 4)**

# Review – Corner Solution

- Generally, people consume **positive amounts of X and Y**.
- In this case, such bundle is said to be an **interior** solution.
  
- But, some people choose to consume **ONLY one good**.
- In this case, such bundle is said to be a **corner** solution.

# Review – Corner Solution

- When the optimality condition “ $MU_x/P_x = MU_y/P_y$ ” holds, we will have an interior solution.
- This is because this condition implies the bundle where the IC and BL are **tangent (their slopes are equal)**.
- This in turn implies that such bundle is **affordable** and gives the **highest utility**.

# Review – Corner Solution

- Suppose that  $MU_x/P_x > MU_y/P_y$ .
- This means spending one dollar on X will give higher MU.
- Hence, we should consume more X.
  
- Note that when X increase,  $MU_x$  falls, equating  $MU_x/P_x = MU_y/P_y$ , and the optimal bundle is reached.
  
- **What if there is NO bundle such that  $MU_x/P_x = MU_y/P_y$ ?**

# Review – Corner Solution

- Suppose that  $MU_x/P_x > MU_y/P_y$  for all affordable bundles.
- This means that, **for every bundle we can buy**, \$1 spent on X will ALWAYS give higher MU than \$1 spent on Y.
- Therefore, we should spend money on X, i.e. we should consume only X.
- This is what a corner solution means.

# Review – Corner Solution



## LEARNING-BY-DOING EXERCISE 4.3

### Finding a Corner Point Solution

David is considering his purchases of food ( $x$ ) and clothing ( $y$ ). He has the utility function  $U(x, y) = xy + 10x$ , with marginal utilities  $MU_x = y + 10$  and  $MU_y = x$ . His income is  $I = 10$ . He faces a price of food  $P_x = \$1$  and a price of clothing  $P_y = \$2$ .

If the basic  
 $MU_x/MU_y = P_x/P_y$   
files to

**Problem** What is David's optimal basket?

These two equations

Recall the **INSENSIBLE** interior solution of  $X = 15$  and  $Y = -2.5$ .

**Hence, an interior solution is ruled out.**

**But how can we find the corner solution?**

# Review – Corner Solution

## Finding the Corner Solution

- To do so, we need to compare  $MU_x/P_x$  and  $MU_y/P_y$ .
- If  $MU_x/P_x > MU_y/P_y$ , we consume only X.
- If  $MU_x/P_x < MU_y/P_y$ , we consume only Y.

# Review – Corner Solution

From the example, we have

$$MU_x = y + 10, MU_y = x, P_x = \$1, P_y = \$2.$$

We compare

$$\begin{array}{ccc} & > \text{ OR } < & \\ \frac{MU_x}{P_x} & \boxed{\phantom{00}} & \frac{MU_y}{P_y} \\ \frac{y+10}{1} & \boxed{\phantom{00}} & \frac{x}{2} \\ & & \\ 2y + 20 & \boxed{\phantom{00}} & x \end{array}$$

# Review – Corner Solution

We compare

$$2y + 20 \square x$$

From the BL:  $10 = 1X + 2Y$ , we know that  $Y \leq 5$  and  $X \leq 10$ .

Hence, we can conclude that  **$2y + 20 > x$** .

That is,  **$MU_x/P_x > MU_y/P_y$** , and we consume only X.

That is, the corner solution where  $X = 10$  and  $Y = 0$ .

# Choice with Composite Goods

- In previous analysis, a consumer has two choices: good X and good Y.
- We will introduce a **Composite Good**.
- Suppose that **good Y represents a composite good, then good Y includes every other good except good X.**

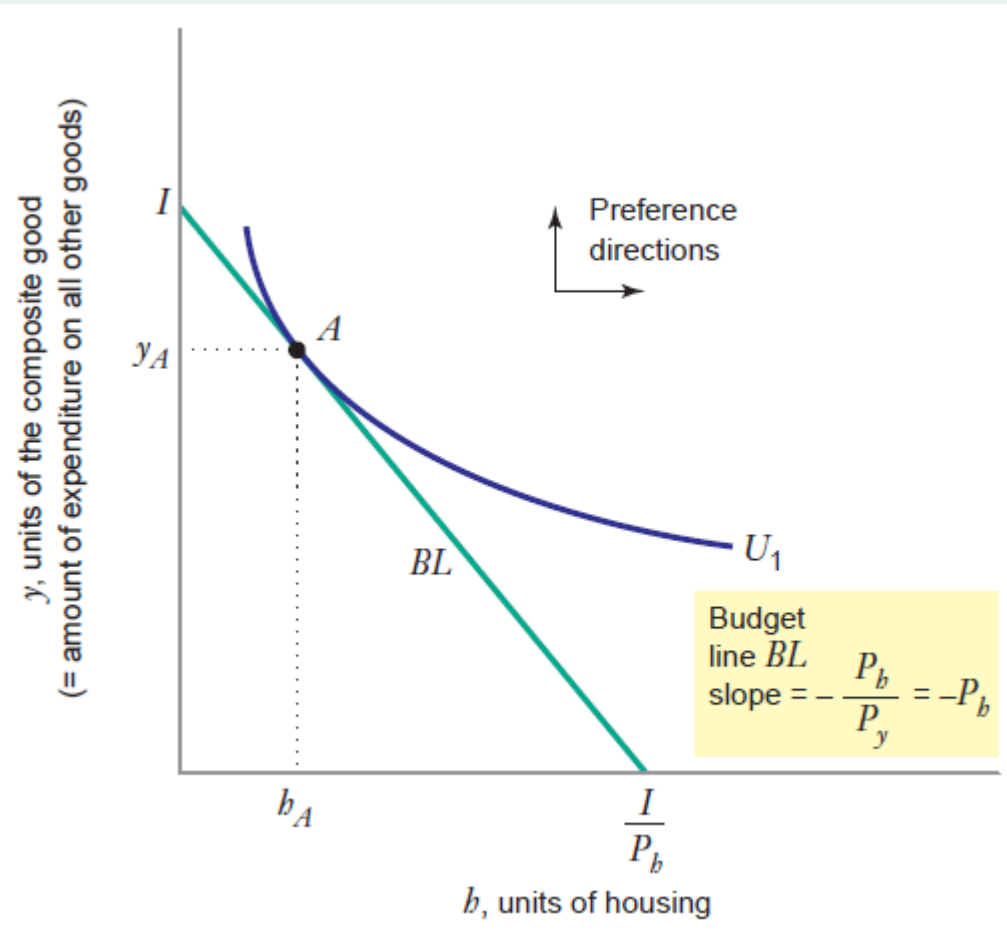
# Choice with Composite Goods

- We will plot a composite good in place of good Y, i.e. on the vertical axis.
- We assume that its price is \$1, i.e.  $P_y = 1$ .
- Hence, the slope of the budget line becomes  $-P_x/P_y = -P_x/1 = -P_x$ .

# Choice with Composite Goods

**FIGURE 4.10** Optimal Choice of Housing (with Composite Good)

The horizontal axis measures the number of units of housing  $h$ . The price of housing is  $P_h$ . If the consumer has an income of  $I$ , he could purchase at most  $I/P_h$  units of housing (the intercept of the budget line on the horizontal axis). The vertical axis measures the number of units of the composite good  $y$  (all other goods). The price of the composite good is  $P_y = 1$ . If the consumer were to spend all his income on the composite good, he could purchase  $I$  units of the composite good. Thus, the intercept of the budget line on the vertical axis is  $I$ , the level of income. The budget line  $BL$  has a slope equal to  $-P_h/P_y = -P_h$ . Given the consumer's preferences, the optimal basket is  $A$ , where the consumer purchases  $h_A$  units of housing and spends  $y_A$  dollars on other goods.



# App: Coupons and Cash Subsidies

- Governments often have programs (coupons and subsidies) aimed at helping the poor purchase more essential goods, e.g. food, housing, and education.
- **Cash Subsidies is essentially an increase in income of a consumer**, shifting his/her BL to the right.
- **Coupons/Vouchers permit consumers to buy more of a specific good only, partly shifting his/her BL to the right.**

# App: Coupons and Cash Subsidies

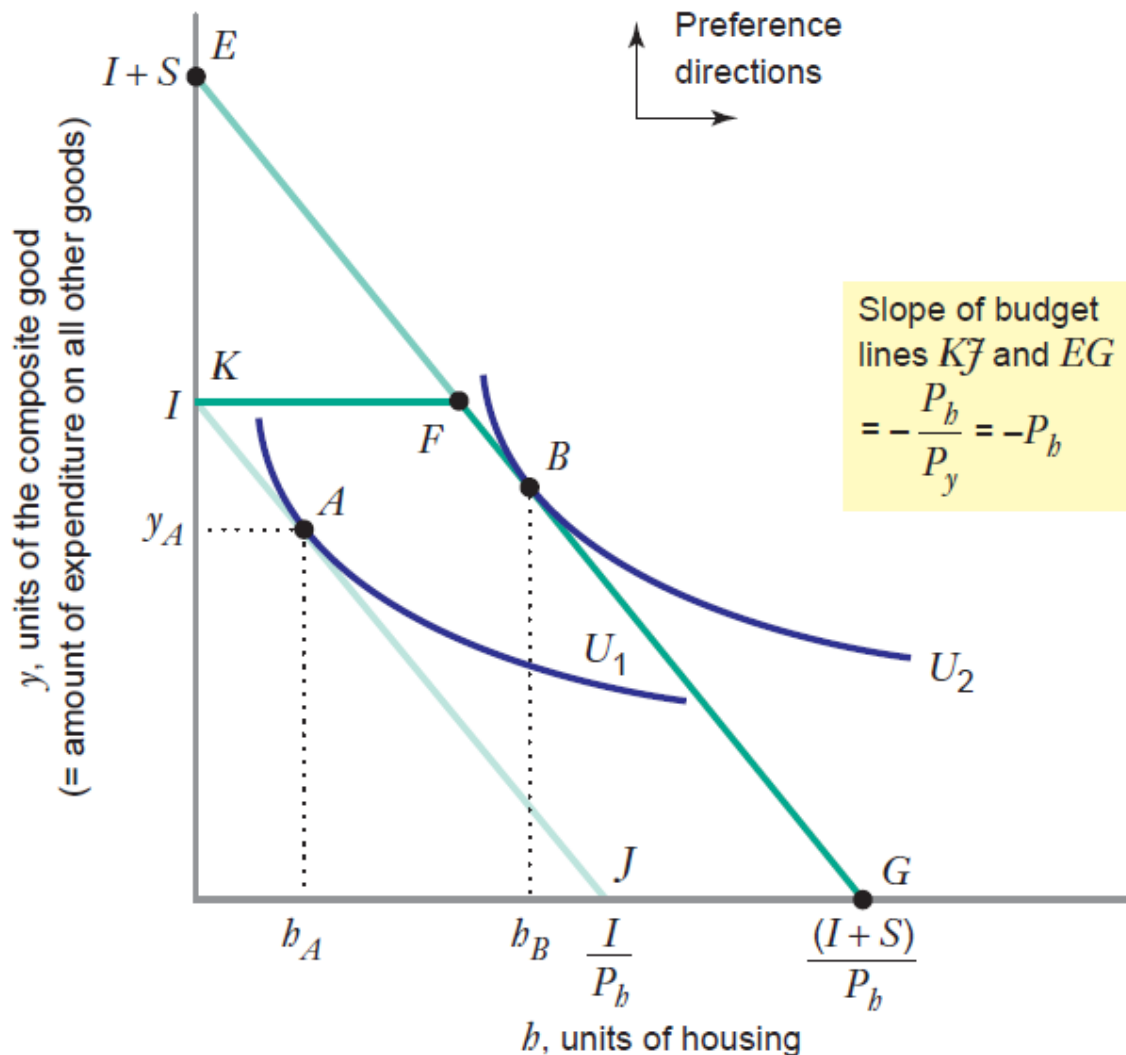
**FIGURE 4.11** Optimal Choice of Housing: Subsidy and Voucher

Consider two types of programs that might be implemented to increase the consumer's purchases of housing.

*Income subsidy:* If the consumer receives an income subsidy of  $S$  dollars from the government, the budget line moves from  $KJ$  to  $EG$ .

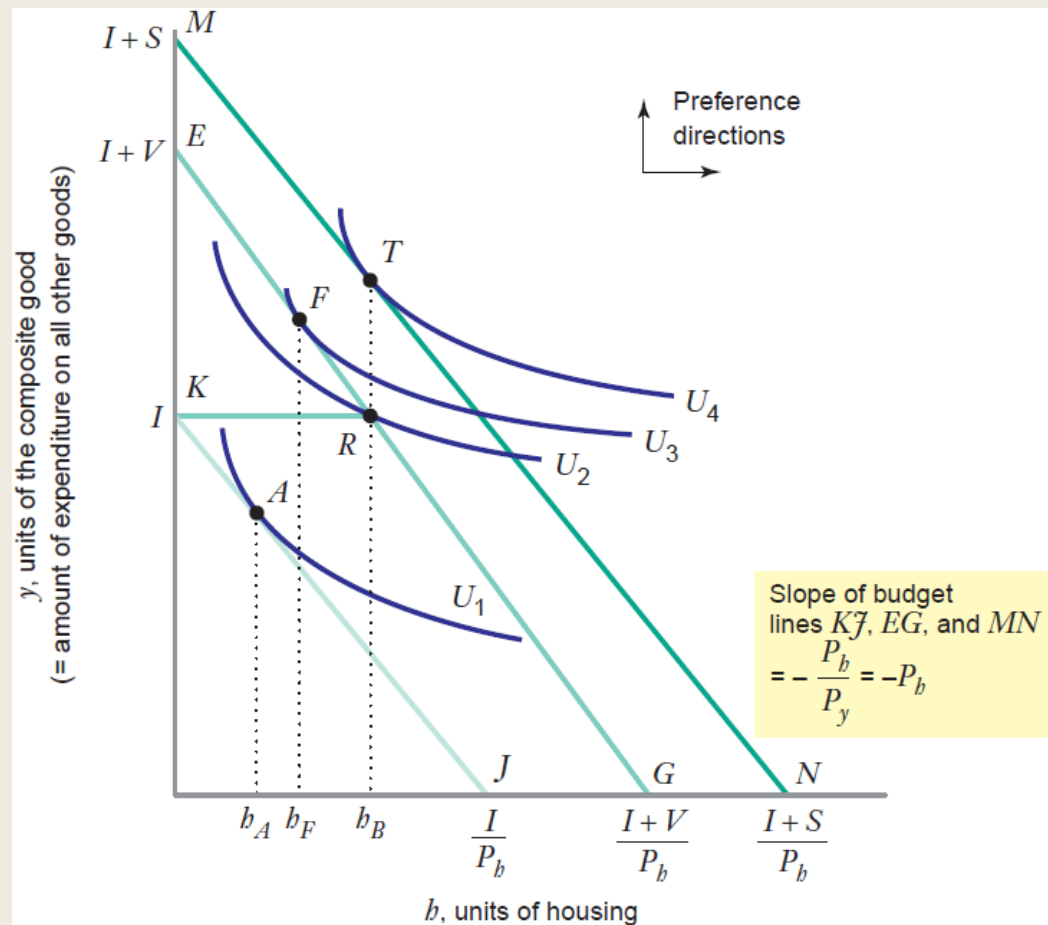
*Housing voucher:* If the government gives the consumer a voucher of  $S$  dollars that can only be spent on housing, the budget line moves from  $KJ$  to  $KFG$ .

If the consumer has the indifference map shown in the graph, he is indifferent between receiving an income subsidy of  $S$  dollars and a housing voucher worth  $S$  dollars. In either case, he will select basket  $B$ .



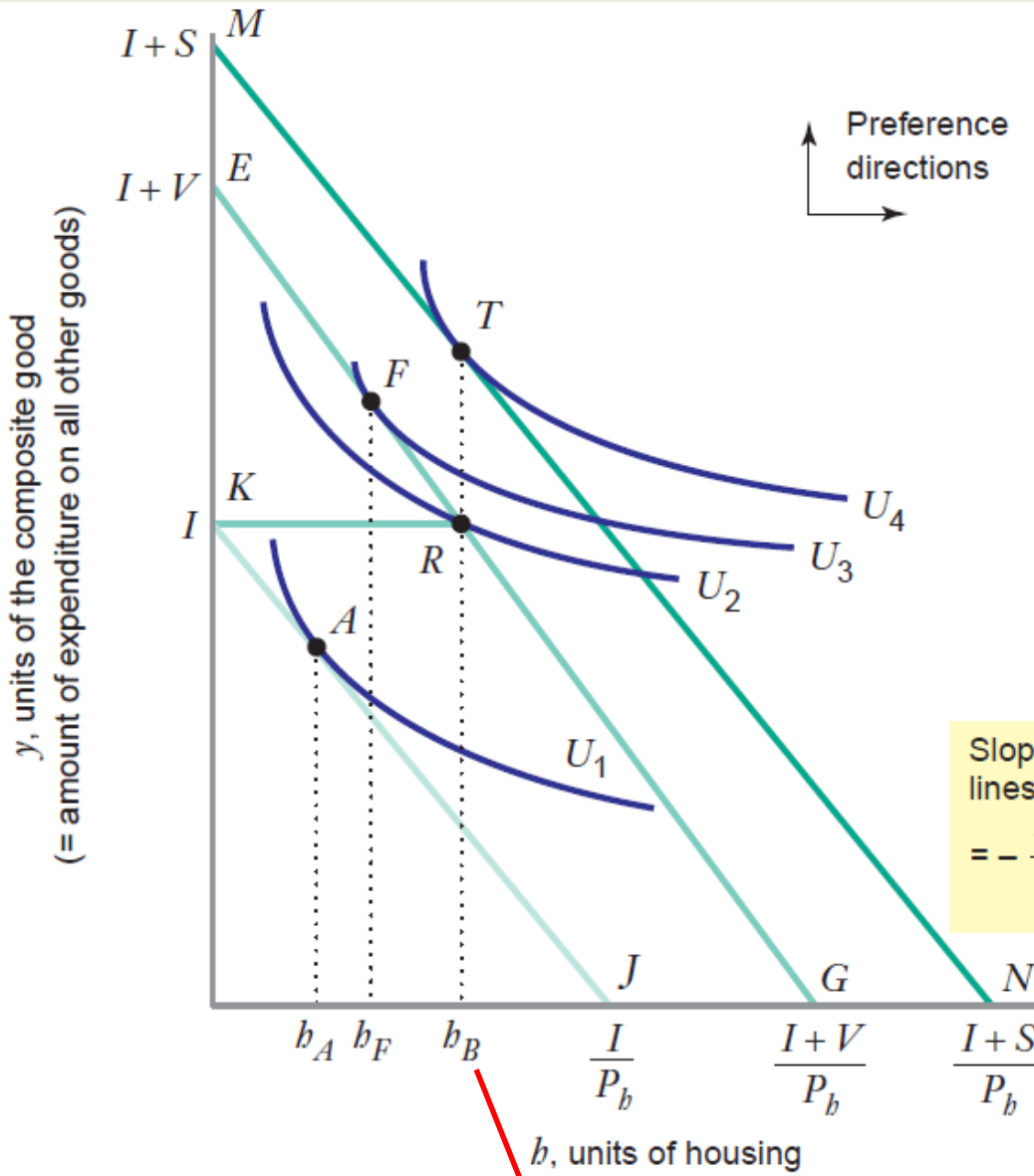
**FIGURE 4.12**  
Optimal Choice of Housing: Subsidy and Voucher

If a consumer has an income  $I$ , he will choose  $h_A$  units of housing. The government could induce him to choose  $h_B$  units of housing with either of the following two programs:



- Give him an income subsidy of  $S$  dollars, moving the budget line to  $MN$ . The consumer chooses basket  $T$ .
- Give him a housing voucher worth  $V$  dollars that can be spent only on housing, moving the budget line to  $KRG$ . The consumer chooses basket  $R$ .

Since basket  $T$  lies on a higher indifference curve than basket  $R$ , a consumer with the preferences in the graph would prefer an income subsidy of  $S$  dollars over a housing voucher worth  $V$  dollars. However, the government might choose the voucher program because it would cost less. To induce the consumer to choose  $h_B$  units of housing, the government must spend  $(S - V)$  dollars more if it chooses the cash subsidy program instead of the voucher program.



A is the original bundle.

T is the bundle when the govt gives a cash subsidy of S.

F is the bundle when the govt gives a cash subsidy of V.

R is a bundle when the govt gives a housing voucher of V.

Slope of budget lines  $KJ$ ,  $EG$ , and  $MN$   
 $= -\frac{P_b}{P_y} = -P_b$

F gives higher utility than R. But, at F, the consumer buys less housing.

Suppose the govt wants  $h_B$  units of housing to be consumed. It can save money of  $S-V$  if it chooses a voucher program instead of a cash subsidy.

# App: Coupons and Cash Subsidies

## Key Points

- Since a cash subsidy allows a consumer to buy whatever he wants and whatever he needs, this program tends to give higher utility.
- Coupons/Vouchers will save cost for the government, and are effective when the government wants to increase the consumption of a specific good.

# App: Join a Club (Membership)

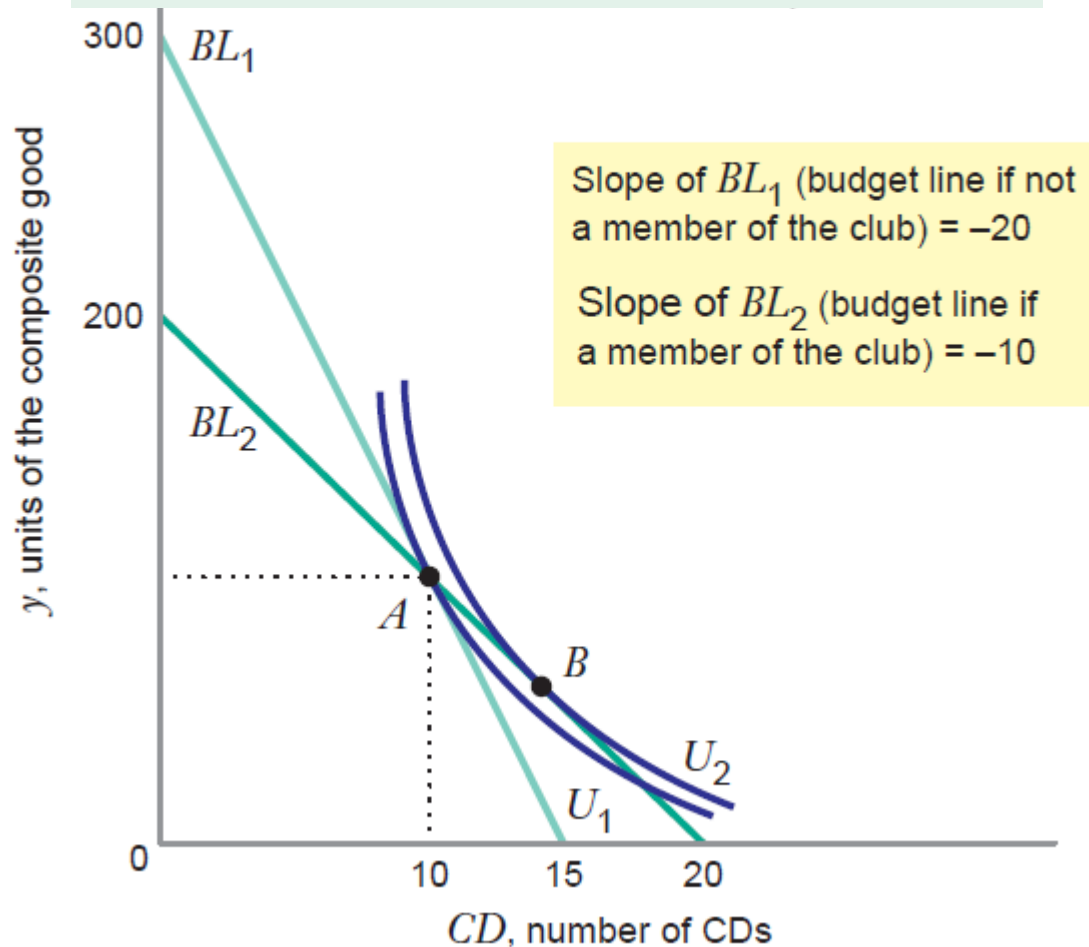
- A consumer can pay a membership fee to join a club (e.g. restaurants), and gets some discount (e.g. on food).
- Suppose that a consumer has income of \$300.
- With \$300, he can buy the maximum amount of 300 units of Y (composite good).
- He can buy X for 20\$ each. With \$300, he can buy the maximum amount of 15 units of X.

# App: Join a Club (Membership)

- He can pay a membership fee of \$100 to get the discounted price of X from \$20 to \$10 each.
- Now, he has \$200 to spend on Y. **This decreases the Y-intercept of the budget line.**
- Since X is cheaper, he can buy more X (up to 20 units). **This increases the X-intercept of the budget line.**

### FIGURE 4.13 Joining a Club

If the consumer does not belong to the CD club, his budget line is  $BL_1$  and his optimal basket is  $A$ , with utility  $U_1$ . If he joins the club, his budget line is  $BL_2$  and his optimal basket is  $B$ , with utility  $U_2$ . The consumer will be better off joining the club (i.e., will achieve a higher level of utility) and will buy more CDs.



#### Before joining a club...

At  $A$  (original optimal bundle), we have  $MU_x/P_x = MU_y/P_y$ .

That is,  $MU_x/20 = MU_y$ .

#### After joining a club...

$P_x$  falls from \$20 to \$10.

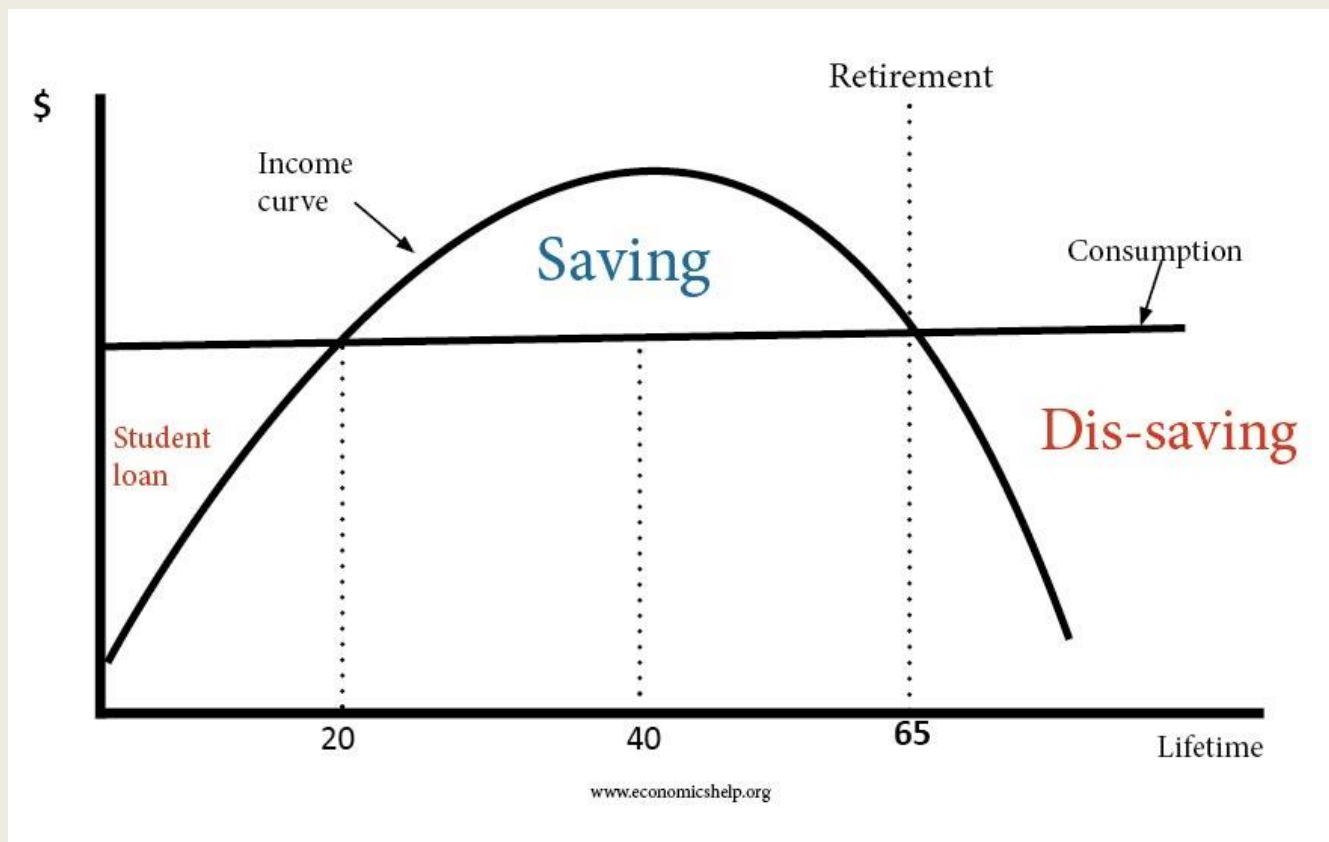
At  $A$ , now we have  **$MU_x/10 > MU_y$** .

In this case, **we should buy more X**, i.e. move to  $B$  (new optimal bundle).

This will give us higher utility.

# App: Borrowing and Lending

We want to “smooth” consumption because we don’t want to be too poor when young and old. We want to have stable income over life. This is why people borrow and save/lend.



# App: Borrowing and Lending

- Using composite goods, we can modify the model of consumer choice to allow for borrowing and lending.
- In the first year, our income is  $I_1$ . In the second year, our income is  $I_2$ . Assume that  $I_2 > I_1$ .
- Hence, we can buy  $I_1$  units of composite goods in the first year, and  $I_2$  units of composite goods in the second year.
- That is, we consume more in the second year.

# App: Borrowing and Lending

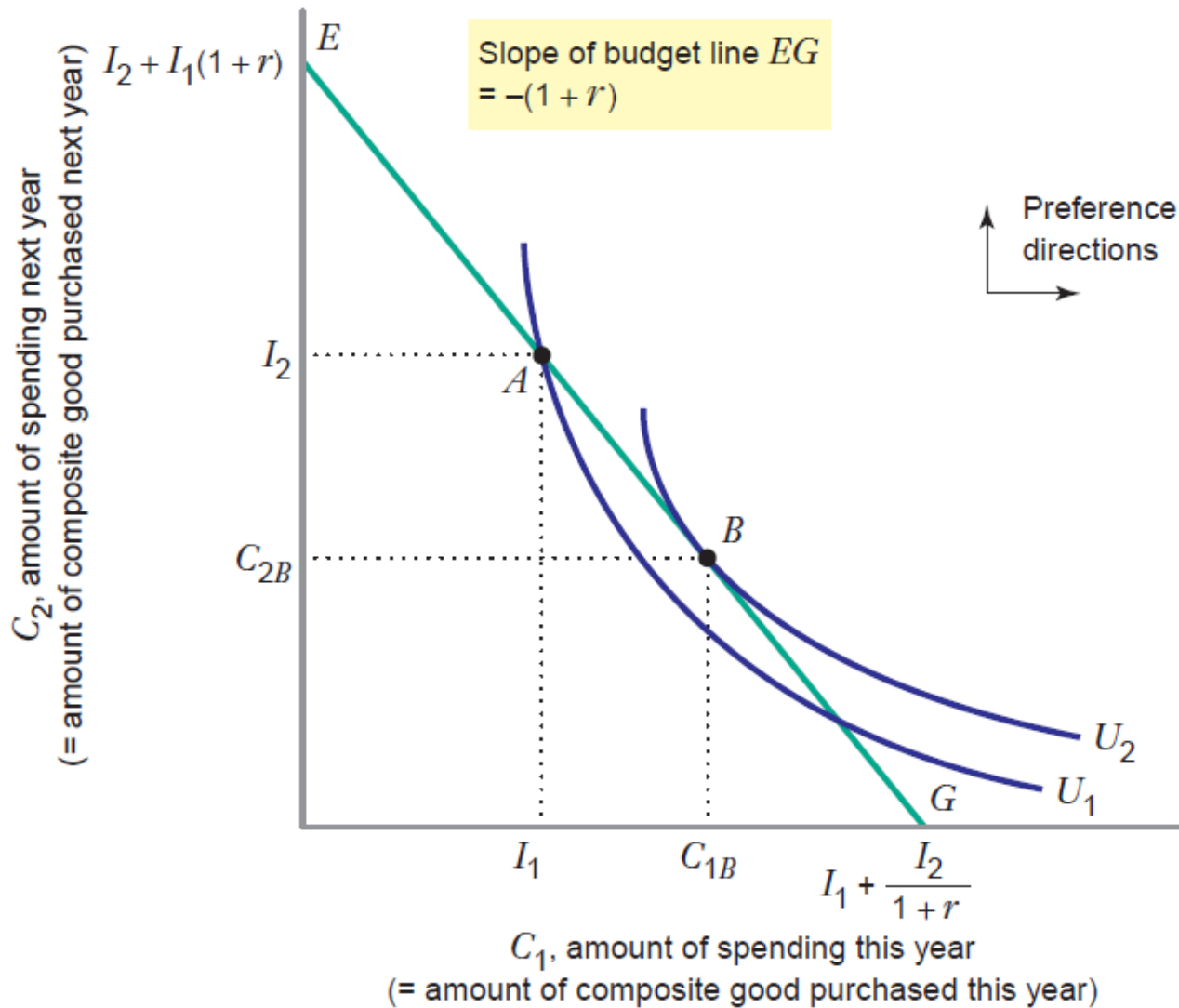
- Note that  $I_2 > I_1$ . Without borrowing and lending,  $MU_{\text{first year}} > MU_{\text{second year}}$ .
- Thus, our utility is not maximized. We should consume more in the first year.
- We can smooth consumption by borrowing in the first year and paying back (with interest rate) later.
- This is so that  $MU_{\text{first year}} = MU_{\text{second year}}$ , and our utility will be maximized.

# App: Borrowing and Lending

- Let  $r$  denote the interest rate.
- If we were to spend all incomes in the second period, we can buy  $l_2 + l_1 \times (1+r)$  units of composite good.
- If we were to spend all incomes in the first period, we can buy  $l_1 + l_2 / (1+r)$  units of composite good.
- Hence, the slope of budget line is

$$-\frac{l_2 + l_1 \times (1+r)}{l_1 + l_2 / (1+r)} = -(1+r)$$

# App: Borrowing and Lending

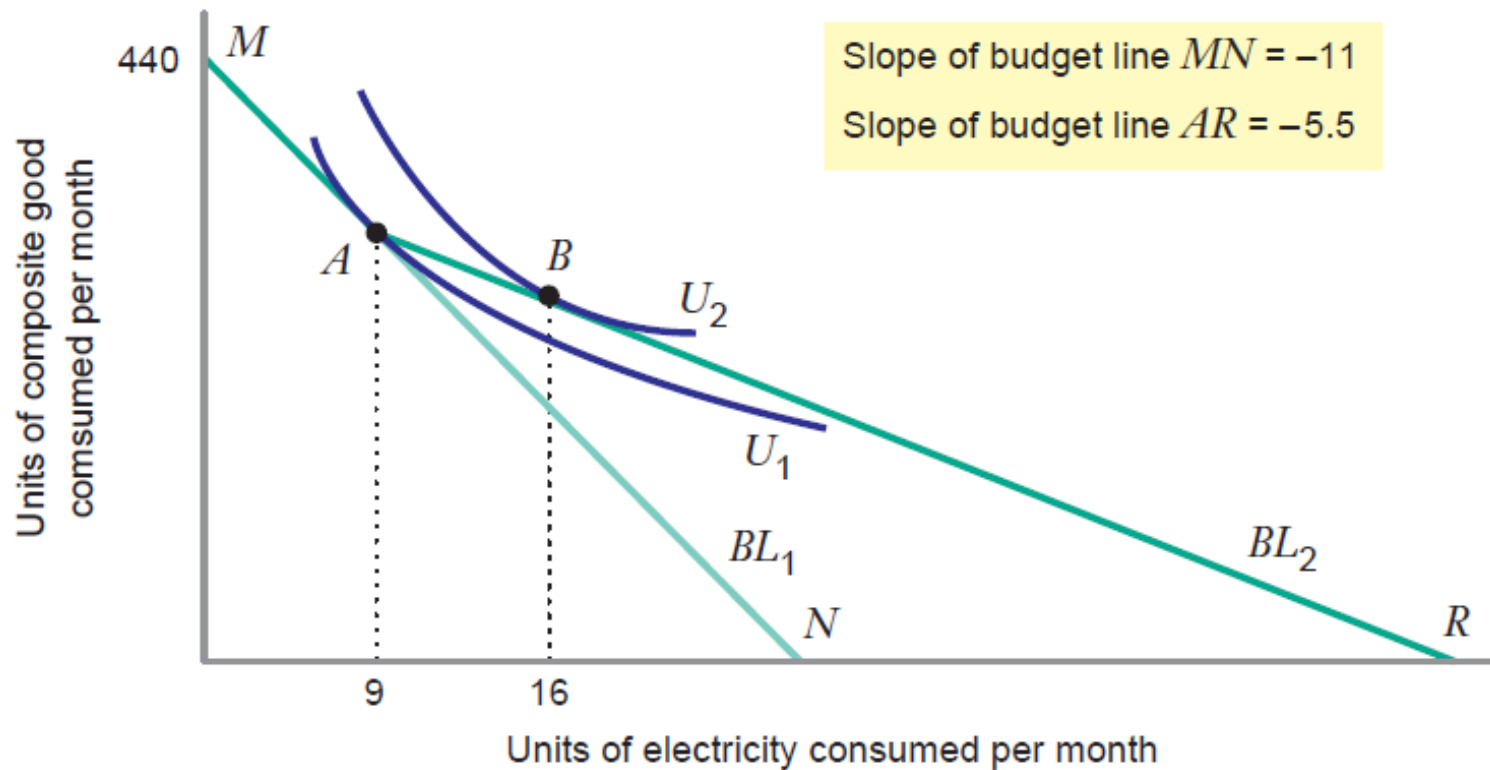


**FIGURE 4.15** Borrowing and Lending

A consumer receives income  $I_1$  this year and  $I_2$  next year. If he neither borrows nor lends, he will be at basket  $A$ . Suppose he can borrow or lend at an interest rate  $r$ . If his indifference map is as shown in the graph, he would choose basket  $B$ , borrowing  $(C_{1B} - I_1)$  from the bank this year and repaying the loan next year. Borrowing has increased his utility from  $U_1$  to  $U_2$ .

# App: Quantity Discounts

- **A Quantity Discount is offered to buyers to induce them to buy in a larger quantity.**
- Consider the example of the power company, who charges \$11 per unit of X (electricity) for the first 9 units and charges \$5.5 per unit for additional units.
- Hence, for  $X \geq 9$ , the budget line becomes flatter because more X can be bought.
- The consumer has income of \$440 to be spent on X and Y (composite goods) whose price is \$1 per unit.



**Before Quantity Discount... $P_x = 11$**

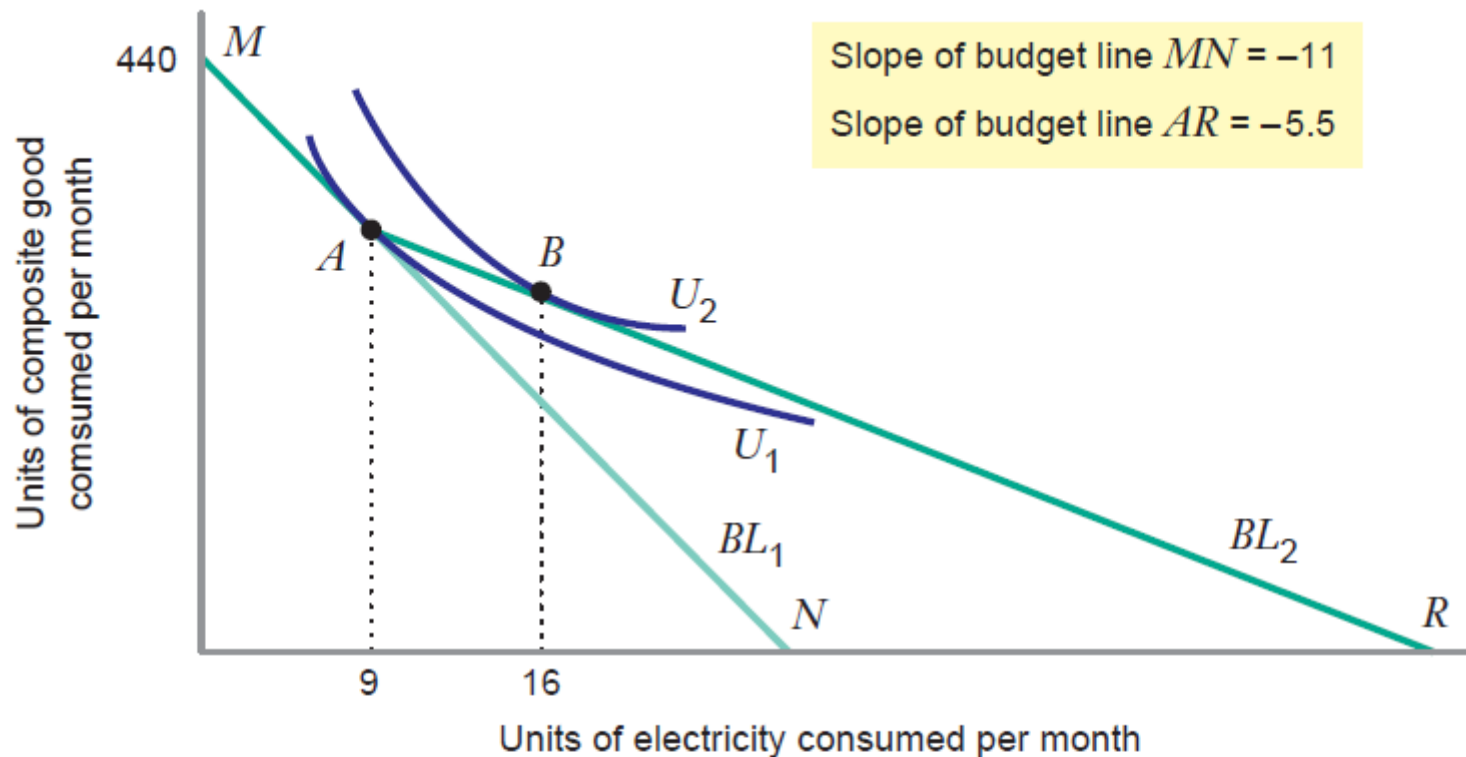
At A (original optimal bundle),  
 $MU_x/P_x = MU_y/P_y$

$$MU_x/11 = MU_y$$

**After Quantity Discount... $P_x = 5.5$**

Now, at A (original optimal bundle),  
 $MU_x/5.5 > MU_y$

Thus, the consumer should consume more X until  $MU_x/5.5 = MU_y$ ,  
i.e. move from A to B.



**FIGURE 4.17** Quantity Discount

If the electric power company sells electricity at a price of \$11 per unit, the budget line facing the consumer is  $MN$ . Given the indifference map shown in the graph, the consumer would choose basket  $A$ , with 9 units of electricity. If the supplier offers a quantity discount, charging \$11 for each of the first 9 units, but only \$5.50 per *additional* units, the budget line is now composed of two segments,  $MA$  and  $AR$ . The consumer will buy a total of 16 units of electricity (at basket  $B$ ). Thus, the quantity discount has induced her to buy 7 extra units of electricity. The figure shows that a quantity discount may enable the consumer to achieve a higher level of satisfaction.