Chapter 6 part II

Extensions of the Two-Variable Linear Regression Model

Functional Forms of Regression Models

- The log-linear model
- Semilog models
- Reciprocal models
- The logarithmic reciprocal model

The log-linear model

The exponential regression model:

$$Y_i = \beta_1 X_i^{\beta_2} e^{u_i}$$

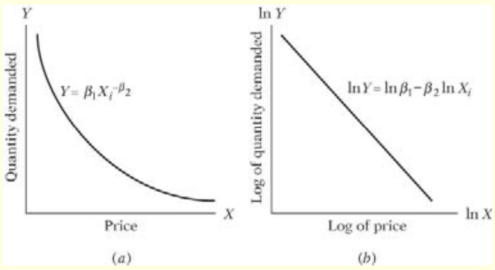
Which may be expressed alternatively as

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i$$

$$\ln Y_i = \alpha_1 + \beta_2 \ln X_i + u_i$$

$$where\ \alpha = \ln \beta_1$$

The log-linear model



The log-linear model

- The slope coefficient β_2 measures the elasticity of Y with respect to X, that is, the percentage change in Y for a given percentage change in X
- Example Price elasticity of demand

- The model assumes that the elasticity coefficient between Y and X remains constant throughout

Percentage change vs. Percentage point change

Example –The unemployment rate

The unemployment rate of 6%, if this rate goes to 8%, we say that **the percentage point change in the unemployment rate is 2**

The percentage change in the unemployment rate is (8-6)/6, or about 33%

Example

Expenditure on durable goods in relation to total personal consumption expenditure

- We wish to find the elasticity of expenditure on durable goods with respect to total personal consumption expenditure.

TABLE 6.3
Total Personal Expenditure and Categories (Billions of chained [2000] dollars; quarterly data at seasonally adjusted annual rates)

Sources: Department of Commerce, Bureau of Economic Analysis. *Economic Report of the President*, 2007, Table B-17, p. 347.

Year or quarter	EXPSERVICES	EXPDUR	EXPNONDUR	PCEXP
2003-I	4,143.3	971.4	2,072.5	7,184.9
2003-II	4,161.3	1,009.8	2,084.2	7,249.3
2003-III	4,190.7	1,049.6	2,123.0	7,352.9
2003-IV	4,220.2	1,051.4	2,132.5	7,394.3
2004-I	4,268.2	1,067.0	2,155.3	7,479.8
2004-II	4,308.4	1,071.4	2,164.3	7,534.4
2004-III	4,341.5	1,093.9	2,184.0	7,607.1
2004-IV	4,377.4	1,110.3	2,213.1	7,687.1
2005-I	4,395.3	1,116.8	2,241.5	7,739.4
2005-II	4,420.0	1,150.8	2,268.4	7,819.8
2005-III	4,454.5	1,175.9	2,287.6	7,895.3
2005-IV	4,476.7	1,137.9	2,309.6	7,910.2
2006-I	4,494.5	1,190.5	2,342.8	8,003.8
2006-II	4,535.4	1,190.3	2,351.1	8,055.0
2006-III	4,566.6	1,208.8	2,360.1	8,111.2

Note: See Table B-2 for data for total personal consumption expenditures for 1959–1989.
EXPSERVICES = expenditure on services, billions of 2000 dollars.
EXPDUR = expenditure on durable goods, billions of 2000 dollars.
EXPNONDUR = expenditure on nondurable goods, billions of 2000 dollars.
PCEXP = total personal consumption expenditure, billions of 2000 dollars.

(Continued)

$$\widehat{\ln EXDUR_t} = -7.5417 + 1.6266 \ln PCEX_t$$

$$se = (0.7161) \qquad (0.0800)$$

$$t = (-10.5309) \quad (20.3152) \qquad r^2 = 0.9695$$

The elasticity of EXPDUR with respect to PCEX is about 1.63, suggesting that if total personal expenditure goes up by 1 percent, on average, the expenditure on durable goods goes up by about 1.63 percent

Semilog Models

- Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

- Lin-Log model

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

The Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\beta_2 = \frac{\text{relative change in regressand}}{\text{absolute change in regressor}}$$

β_2 *is known as the semielasticity of Y with respect to X*

If we multiply the relative change in Y by 100, it will give the percentage change in Y for an absolute change in X

$100*\beta_2$ *is known as the semielasticity of Y with respect to X*

Example

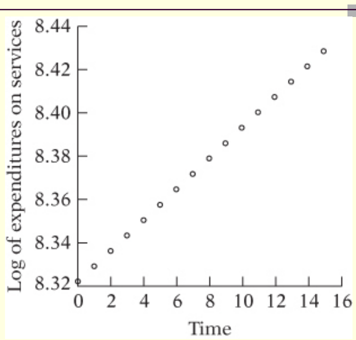
- We want to find out the growth rate of personal consumption expenditure on services for the data
- The regression results over time (t) are as follows:

$$\widehat{\ln EXS_t} = 8.3226 + 0.00705t$$
$$se = (0.0016) \quad (0.00018)$$
$$t = (5201.625) \quad (39.1667) \quad r^2 = 0.9919$$

Over the quarterly period 2003-I to 2006-III, expenditures on services increased at the (quarterly) rate or 0.705 percent. Roughly, this is equal to an annual growth rate of 2.82 percent.

$$\widehat{\ln EXS_t} = 8.3226 + 0.00705t$$
$$se = (0.0016) \quad (0.00018)$$
$$t = (5201.625) \quad (39.1667) \quad r^2 = 0.9919$$

Since 8.3226 = log of EXS at the beginning of the study period, by taking its antilog we obtain 4115.96 (billion dollars) as the beginning value of EXS



Lin-Log models

The absolute change in Y for a percentage change in X

$$Y_i = \beta_1 + \beta_2 \ln X_i + \mu_i$$

$$\beta_2 = \frac{\textit{Change in } Y}{\textit{relative change in } X} = \frac{\Delta Y}{\Delta X / X}$$

The absolute change in Y for a percentage change in X

If $\Delta X / X$ changes by 0.01 unit or 1%, the absolute change in Y is

$$0.01 * \beta_2$$

Example

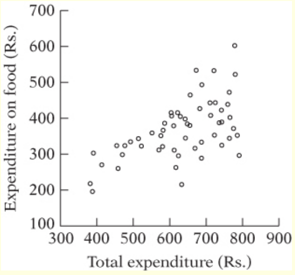
Engel expenditure – the total expenditure that is devoted to food tends to increase in arithmetic progression as total expenditure increases in geometric progression

- Food expenditure in India

$$\widehat{FoodExp_i} = -1283.912 + 257.2700 \ln TotalExp_i$$
$$t = (-4.3848)* \qquad (5.6625)*$$
$$r^2 = 0.9919$$
$$Note: *denotes\ an\ extremely\ small\ \ p\ value$$

Example

- Food expenditure in India example



- The slope coefficient of about 257 means that an increase in the total food expenditure of 1 percent, on average, leads to about 2.57 rupees increase in the expenditure on food of the 55 families included in the sample

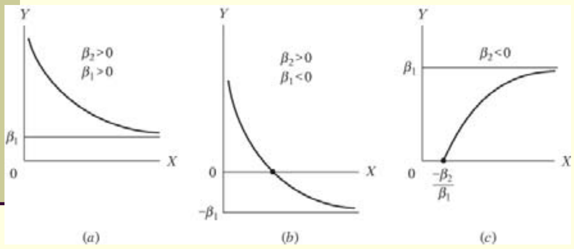
  (Note: we have divided the estimated slope coefficient by 100)

Reciprocal models

$$Y_i = \beta_1 + \beta_2 \left(\frac{1}{X_i} \right) + u_i$$

As X increases indefinitely, in term $\beta_2 \left(\frac{1}{X_i} \right)$ approaches zero and Y approaches the limiting or asymptotic value β_1

$$Y_i = \beta_1 + \beta_2 \left(\frac{1}{X_i} \right) + u_i$$



The slope

$$\frac{dY}{dX} = -\beta_2 \left(\frac{1}{X^2} \right)$$

if β_2 is positive, the slope is negative throughout

β_2 is negative, the slope is positive throughout

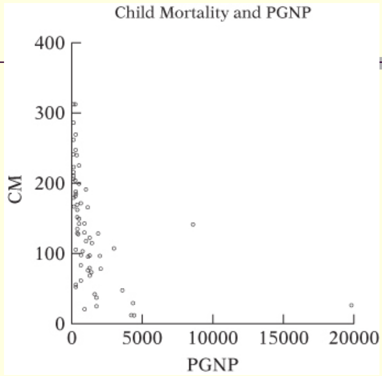
Example

- Child mortality and per capita GNP – 64 countries

$$\widehat{CM}_i = 81.79436 + 27,237.17 \left(\frac{1}{PGNP_i} \right)$$

$$se = (10.8321) \qquad (3759.999)$$

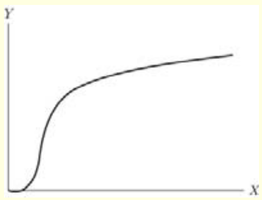
$$t = (7.5511) \qquad (7.2535) \qquad r^2 = 0.4590$$



- As per capita GNP increases indefinitely, child mortality approaches its asymptotic value of about 82 deaths per thousand.
- The positive value of the coefficient of $\left(\frac{1}{PGNP_i} \right)$ implies that the rate of change of CM with respect to PGNP is negative

The logarithmic reciprocal model

$$\ln Y_i = \beta_1 - \beta_2 \left(\frac{1}{X_i} \right) + u_i$$



The logarithmic reciprocal model

- Initially Y increases at an increasing rate and then it increases at a decreasing rate
- E.g. short run production function

  Microeconomics – if labor and capital are the inputs in a production function and if we keep the capital input constant but increase the labor input, the short-run output-labor relationship will resemble in the logarithmic reciprocal model

Choice of Functional Form

TABLE 6.6

Model	Equation	Slope $\left(=\frac{dY}{dX}\right)$	Elasticity $\left(=\frac{dY}{dX}\frac{X}{Y}\right)$
Linear	$Y=\beta_1+\beta_2 X$	β_2	$\beta_2\left(\frac{X}{Y}\right)^*$
Log–linear	$\ln Y=\beta_1+\beta_2\ln X$	$\beta_2\left(\frac{Y}{X}\right)$	β_2
Log–lin	$\ln Y=\beta_1+\beta_2 X$	$\beta_2(Y)$	$\beta_2(X)^*$
Lin–log	$Y=\beta_1+\beta_2\ln X$	$\beta_2\left(\frac{1}{X}\right)$	$\beta_2\left(\frac{1}{Y}\right)^*$
Reciprocal	$Y=\beta_1+\beta_2\left(\frac{1}{X}\right)$	$-\beta_2\left(\frac{1}{X^2}\right)$	$-\beta_2\left(\frac{1}{XY}\right)^*$
Log reciprocal	$\ln Y=\beta_1-\beta_2\left(\frac{1}{X}\right)$	$\beta_2\left(\frac{Y}{X^2}\right)$	$\beta_2\left(\frac{1}{X}\right)^*$

Note: * indicates that the elasticity is variable, depending on the value taken by X or Y or both. When no X and Y values are specified, in practice, very often these elasticities are measured at the mean values of these variables, namely, $\bar{X}$ and $\bar{Y}$.

- The underlying theory
- Rate of change and the elasticity of the regressand with respect to the regressor
- The coefficients of the model chosen should satisfy certain a priori expectations
- Sometimes more than one model may fit a given set of data reasonably well
  - Comparing r-squared? (dependent variable of the two models must be the same)

Example

Inflation Rate and Unemployment Rate

$$(1) Linear\ \ Model\ \ \widehat{(\pi_t-\pi_{t-1})}=3.7844-0.6385UN_t$$

$$t=(4.1912)\ \ (-4.2756)$$

$$r^2=0.2935$$

$$(2) reciprocal\ \ model\ \ \widehat{(\pi_t-\pi_{t-1})}=-3.0684+17.2077\left(\frac{1}{UN_t}\right)$$

$$t=(-3.1635)\ \ (3.2886)$$

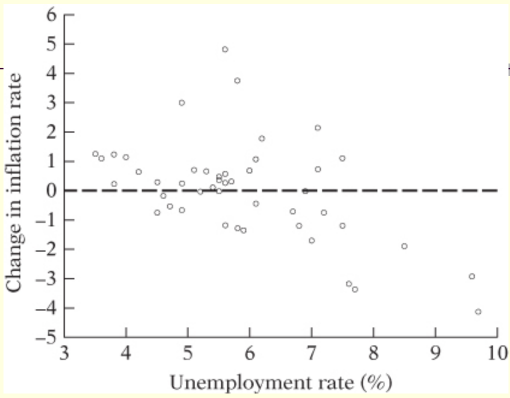
$$r^2=0.1973$$

TABLE 6.5
Inflation Rate and Unemployment Rate, United States, 1960–2006 (For all urban consumers; 1982–1984 = 100, except as noted)

Source: Economic Report of the President, 2007, Table B-60, p. 399, for CPI changes and Table B-42, p. 376, for the unemployment rate.

Year	INFLRATE	UNRATE	Year	INFLRATE	UNRATE
1960	1.718	5.5	1984	4.317	7.5
1961	1.014	6.7	1985	3.561	7.2
1962	1.003	5.5	1986	1.859	7.0
1963	1.325	5.7	1987	3.650	6.2
1964	1.307	5.2	1988	4.137	5.5
1965	1.613	4.5	1989	4.818	5.3
1966	2.857	3.8	1990	5.403	5.6
1967	3.086	3.8	1991	4.208	6.8
1968	4.192	3.6	1992	3.010	7.5
1969	5.460	3.5	1993	2.994	6.9
1970	5.722	4.9	1994	2.561	6.1
1971	4.381	5.9	1995	2.834	5.6
1972	3.210	5.6	1996	2.953	5.4
1973	6.220	4.9	1997	2.294	4.9
1974	11.036	5.6	1998	1.558	4.5
1975	9.128	8.5	1999	2.209	4.2
1976	5.762	7.7	2000	3.361	4.0
1977	6.503	7.1	2001	2.846	4.7
1978	7.591	6.1	2002	1.581	5.8
1979	11.350	5.8	2003	2.279	6.0
1980	13.499	7.1	2004	2.663	5.5
1981	10.316	7.6	2005	3.388	5.1
1982	6.161	9.7	2006	3.226	4.6
1983	3.212	9.6			

Note: The inflation rate is the percent year-to-year change in CPI. The unemployment rate is the civilian unemployment rate.



Model (1) shows that if the unemployment rate goes down by 1 percentage point, on average, the change in the inflation rate goes up by about 0.64 percentage points

Model (2) shows that even if the unemployment rate increases indefinitely, the most the change in the inflation rate will go down will be about 3.07 percentage points