

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\begin{aligned} \text{a. } \sum_{i=1}^5 (a + bx_i) &= (a + bx_1) + (a + bx_2) + (a + bx_3) + (a + bx_4) + (a + bx_5) \\ &= 5a + b \sum_{i=1}^5 x_i = 5a + b(x_1 + x_2 + x_3 + x_4 + x_5) \end{aligned}$$

$$\text{b. } \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$\text{c. } \sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = \frac{2310}{6} = 385$$

$$\text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 [2x+2+3] = \sum_{x=1}^2 2x+5 = 2(1) + 2(2) + 5 = 11$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$\begin{aligned} f(x) &= P(X=x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1 \\ 8b &= 1 \\ b &= 1/8 \end{aligned}$$

- b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= 1 - P(X > 2) \\ &= 1 - P(X=3) - P(X=4) \\ &= 1 - (0.5)(0.125) - (0.25)(0.125) = 0.90625 \end{aligned}$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$\begin{aligned} P(-2 \leq X \leq 3) &= 1 - P(X=4) \\ &= 1 - (0.25)(0.125) \\ &= 0.96875 \end{aligned}$$

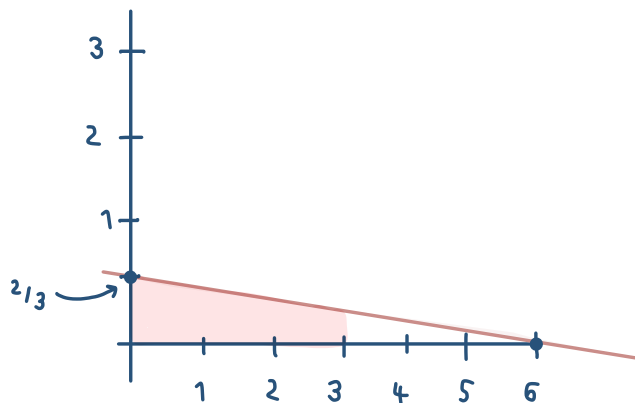
- d. Find the answer for $P(X \geq 1)$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X=0) - P(X=-1) - P(X=-2) \\ &= 1 - (2.25)(0.125) - (1)(0.125) - (0.5)(0.125) \\ &= 0.53125 \end{aligned}$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$ $(x, y) = (6, \frac{2}{3})$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_1^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9} \right] = -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9} = \frac{16}{18} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(X \geq 2) &= \int_2^3 f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_2^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9} \right] = -\frac{9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9} = \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{x^3}{27} + \frac{6x^2}{18} \right]_0^3 \\ &= -\frac{(3)^3}{27} + \frac{6(3)^2}{18} \\ &= -\frac{27}{27} + \frac{6 \cdot 9}{18} \\ &= -1 + 3 \\ &= 2 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

x / y	1	2	3	4	5	6	
0	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
1	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	1

- b. Find the marginal probability distribution function (PDF) of X

The marginal probability of x $p(x=x)$ is $\frac{1}{6}$

- c. Find the marginal probability distribution function (PDF) of Y

$$p(Y=y) = \frac{1}{2}$$

- d. Find the conditional probability distribution function (PDF) of

X given Y is equal to 1

x	1	2	3	4	5	6
$p(x=x y=1)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

- e. Find the expected value of X given Y is equal to 1

$$\begin{aligned} E(X | Y=1) &= \sum x_i p(x=x_i | Y=1) = \frac{\sum x_i p(x=x_i, Y=1)}{p(Y=1)} = \frac{1}{p(Y=1)} \sum x_i p(x=x_i, Y=1) \\ &= \frac{1}{0.5} \left[(1 \cdot \frac{1}{12}) + (2 \cdot \frac{1}{12}) + (3 \cdot \frac{1}{12}) + (4 \cdot \frac{1}{12}) + (5 \cdot \frac{1}{12}) + (6 \cdot \frac{1}{12}) \right] = \frac{7}{2} \end{aligned}$$

- f. Find the variance of X given Y is equal to 1

$$\begin{aligned} \text{var}(X | Y=1) &= \sum (x - E(X | Y=1))^2 \cdot p(x | Y=1) \\ &= \left[\left(1 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(2 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(3 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(4 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] \\ &\quad + \left[\left(5 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] + \left[\left(6 - \frac{7}{2}\right)^2 \cdot \frac{1}{6} \right] \\ &= \frac{10}{3} \end{aligned}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^3 x_i\right) \\ &= \frac{1}{N} E(x_1 + x_2 + x_3) \\ &= \frac{1}{N} [E(x_1) + E(x_2) + E(x_3)] \\ &= \frac{1}{3} [\mu_x + \mu_x + \mu_x] \\ &= \frac{1}{3} \cdot 3\mu_x \\ E(\bar{X}) &= \mu_x \end{aligned}$$

$$\begin{aligned} \text{var}(\bar{X}) &= \text{var}\left(\frac{1}{N} \sum_{i=1}^3 x_i\right) \\ &= \frac{1}{N^2} \text{var}(x_1 + x_2 + x_3) \\ &= \frac{1}{N^2} [\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + 2\text{cov}(x_1, x_2) + 2\text{cov}(x_1, x_3) + 2\text{cov}(x_2, x_3)] \\ &= \frac{1}{3^2} \left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 \right] \\ &= \frac{1}{3^2} \cdot \frac{9}{4}\sigma_x^2 \\ &= \frac{\sigma_x^2}{4} \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^4 x_i\right) \\ &= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4) \\ &= \frac{1}{N} [E(x_1) + E(x_2) + E(x_3) + E(x_4)] \\ &= \frac{1}{4} [\mu_x + \mu_x + \mu_x + \mu_x] \\ &= \frac{1}{4} \cdot 4\mu_x \\ &= \mu_x \end{aligned}$$

$$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^4 x_i\right)$$

$$= \frac{1}{N^2} \text{var}(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{N^2} [\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + \text{var}(x_4)]$$

$$= \frac{1}{4^2} [\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2] = \frac{1}{16} \cdot 4\sigma_x^2$$

$$= \frac{\sigma_x^2}{4} = 0.25\sigma_x^2$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\tilde{x} = \frac{1}{4} (0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$E(\tilde{x}) = E\left(\frac{1}{N} \sum_{i=1}^4 x_i\right)$$

$$= \frac{1}{4} E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$= \frac{1}{4} [E(0.5x_1) + E(x_2) + E(0.5x_3) + E(2x_4)]$$

$$= \frac{1}{4} (0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4} \cdot 4\mu_x$$

$$= \mu_x$$

$\therefore \tilde{x}$ is unbiased estimator of μ .

$$\text{var}(\tilde{x}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^4 x_i\right)$$

$$= \frac{1}{4^2} \text{var}(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$= \frac{1}{16} [\text{var}(0.5x_1) + \text{var}(x_2) + \text{var}(0.5x_3) + \text{var}(2x_4)]$$

$$= \frac{1}{16} [0.25\text{var}(x_1) + \text{var}(x_2) + 0.25\text{var}(x_3) + 4\text{var}(x_4)]$$

$$= \frac{1}{16} [0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2]$$

$$= \frac{5.5\sigma_x^2}{16}$$

$$= 0.34\sigma_x^2$$

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$\bar{x}$ is the better estimator for μ_x

because its variance is smaller than

the $\tilde{x}$'s.