

- ① Suppose the demand curve is $Q(p) = p^\epsilon$, what is the elasticity of demand? If marginal cost is \$1 and $\epsilon = -2$, what is the profit-maximizing price?

$$\epsilon_{Q,p} = \frac{\% \Delta Q}{\% \Delta p} = \frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q}$$

$$Q = p^{-2}$$

$$\frac{\Delta Q}{\Delta p} = -2 p^{-3}$$

$$\epsilon_{Q,p} = \frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q} = -2 p^{-3} \times \frac{p}{p^{-2}} = -2 p^{-3} \times p^{1+2} = -2 \#$$

substitute $Q = \frac{1}{4}$ in $Q = p^{-2}$

$$p^{-2} = \frac{1}{4}$$

$$p = 2 \#$$

the maximizing price is 2 \$ #

Maximum profit ; $MR = MC$

$$\frac{1}{2} Q^{-\frac{1}{2}} = 1$$

$$Q^{-\frac{1}{2}} = 2$$

$$Q = \frac{1}{4}$$

$$TR = P \cdot Q$$

$$TR = Q^{-\frac{1}{2}} \cdot Q = Q^{\frac{1}{2}}$$

$$MR = \frac{1}{2} Q^{-\frac{1}{2}}$$

$$p = -Q + 10$$

- ② Suppose the demand curve for corn is $Q(p) = 10 - p$. Suppose that one firm owns all five units of corn in the world and has zero marginal cost. Does a monopoly sell less output than would be sold in a competitive market in which 100 firms each own 0.05 units?

$$TR = Q(-Q + 10)$$

$$TR = -Q^2 + 10Q$$

$$MR = -2Q + 10$$

$$MR = MC$$

$$-2Q + 10 = 0$$

$$Q = 5$$

Monopoly should sell at $Q = 5$

Total outputs in competitive market is $100(0.05) = 5$

Both market have sell outputs at the same level (5 units).

8. Output is homogenous and the demand curve is

$$P = 448 - Q.$$

There are two firms with identical costs given by $C = q_i^2$ where q_i is the production of firm i .
The marginal cost of firm i is $MC_i(q_i) = 2q_i$.

- Find the Cournot equilibrium firm outputs.
- Find the Stackelberg equilibrium firm outputs.

Cournot equilibrium

$$\begin{aligned} \text{Firm 1 ; } \pi_{cn} &= P \cdot q_1 - q_1^2 \\ \pi_{cn} &= (448 - q_1 - q_2) \cdot q_1 - q_1^2 \\ \pi_{cn} &= 448q_1 - q_1^2 - q_1q_2 - q_1^2 \\ \frac{\partial \pi_{cn}}{\partial q_1} &= 448 - 2q_1 - q_2 = 0 \\ -4q_1 + 448 &= q_2 \end{aligned}$$

$$\begin{aligned} q_1 &= -4(-4q_1 + 448) + 448 \\ q_1 &= 16q_1 - 1792 + 448 \\ q_1 &= 16q_1 - 1344 \\ -15q_1 &= -1344 \\ q_1 &= 89.6 \text{ \#} \end{aligned}$$

$$\begin{aligned} q_2 &= -4(-4q_1 + 448) + 448 \\ q_2 &= -336 + 448 \\ q_2 &= 89.6 \text{ \#} \end{aligned}$$

$$\begin{aligned} \text{Firm 2 ; } \pi &= P \cdot q_2 - q_2^2 \\ \pi &= (448 - q_2 - q_1) q_2 - q_2^2 \\ \pi &= 448q_2 - q_2^2 - q_1q_2 - q_2^2 \\ \frac{\partial \pi}{\partial q_2} &= 448 - 2q_2 - q_1 - 2q_2 = 0 \\ -4q_2 &= q_1 - 448 \\ q_1 &= -4q_2 + 448 \end{aligned}$$

$\therefore$ both firm 1 and 2 will produce at the same output (89.6) #

Stackelberg Equilibrium

we assume firm 1 is a leader

$$\begin{aligned} \text{Firm 2 ; } \pi &= P \cdot q_2 - q_2^2 \\ \pi &= (448 - q_2 - q_1) q_2 - q_2^2 \\ \pi &= 448q_2 - q_2^2 - q_1q_2 - q_2^2 \\ \frac{\partial \pi}{\partial q_2} &= 448 - 2q_2 - q_1 - 2q_2 = 0 \\ -4q_2 &= q_1 - 448 \\ q_1 &= -4q_2 + 448 \\ q_2 &= \frac{-q_1 + 448}{4} \end{aligned}$$

$$\begin{aligned} \text{Firm 1 : } \pi &= (448 - q_1 - 112 + \frac{1}{4} q_1) q_1 - q_1^2 \\ &= (336 - \frac{3}{4} q_1) q_1 - q_1^2 \\ &= 336q_1 - \frac{3}{4} q_1^2 - q_1^2 \\ &= 336q_1 - \frac{7}{4} q_1^2 \\ \frac{\partial \pi}{\partial q_1} &= 336 - \frac{7}{2} q_1 = 0 \end{aligned}$$

$$\begin{aligned} q_1 &= 96 \\ q_2 &= 112 - \frac{1}{4} (96) \\ q_2 &= 88 \end{aligned}$$

$\therefore$ firm 1 will produce at $q_1 = 96$
firm 2 will produce at $q_2 = 88$

4 (write about 0.5 page) Find 1 example of an industry that has a dominant firm. Describe what this industry is, which firm is the dominant firm, which firms are fringe firms (name the ones that you

The cosmetic industry describes the industry that manufactures and distributes cosmetic products. These include color cosmetics, like foundation and mascara, skincare such as moisturisers and cleansers, haircare such as shampoos, conditioners and hair colours, and toiletries such as bubble bath and soap. The manufacturing industry is dominated by a number of multinational corporations that originated in the early 20th century, but the distribution and sale of cosmetics is spread among a wide range of different businesses.

The dominant firm of this industry is L'Oreal because these firms have high market share.

With worldwide sales, L'Oreal's market sales is over \$30.8 billion in 2021.

That is why L'Oreal is the world's large cosmetics and beauty company. As you can see

nowadays, there are so many branches of L'Oreal and most people know this brand so it can get high sales.

The fringe firm is Chanel. The total cosmetic sales is \$3.7 billion which is less than the sale of L'Oreal's