



# B.E. International Program

Faculty of Economics, Thammasat University



## EE 320 Introductory Mathematical Economics

Semester 1/2015

### In-Class Exercise 4

Given the following functions,

- (i) Determine whether it is a convex or concave function
- (ii) Determine  $(x^*, y^*)$  that minimizes (or maximizes) the value of  $z(x, y)$ .

1.  $z = f(x, y) = 3x^2 - xy + 2y^2 - 4x - 7y + 12$

Ans.

FONC:

$$f_x = 6x - y - 4 = 0 \quad -- (1)$$

$$f_y = -x + 4y - 7 = 0 \quad -- (2)$$

$$\Rightarrow (x^*, y^*) = (1, 2).$$

SOSC:

$$f_{xx} = 6; f_{yy} = 4; f_{xy} = f_{yx} = -1$$

$$H = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$$

$$\Rightarrow |H_1| = 6 > 0 \ \& \ |H_2| = |H| = 24 - 1 = 23 > 0.$$

Hence,  $f(x, y)$  is a convex function.

Since  $f(x, y)$  is a convex function,  $z^*$  is a minimum.

$$2. z = f(x, y) = 60x + 34y - 4xy - 6x^2 - 3y^2 + 5$$

Ans.

FONC:

$$f_x = 60 - 4y - 12x = 0 \quad \text{-- (1)}$$

$$f_y = 34 - 4x - 6y = 0 \quad \text{-- (2)}$$

$$\Rightarrow (x^*, y^*) = (4, 3).$$

SOSC:

$$f_{xx} = -12; f_{yy} = -6; f_{xy} = f_{yx} = -4$$

$$H = \begin{bmatrix} -12 & -4 \\ -4 & -6 \end{bmatrix}$$

$$\Rightarrow |H_1| = -12 < 0 \text{ \& } |H_2| = |H| = 72 - 16 = 56 > 0.$$

Hence,  $f(x, y)$  is a concave function.

Since  $f(x, y)$  is a concave function,  $z^*$  is a maximum.