EE312 Macroeconomics, 1/2020 (Sec. 046401 - Sicha)
Chapter 10 (Part 1) Solow Growth Model

- **Population growth :** $N' = (1+n)N : n > -1$
- **Consumers :** $Y = C + S, \quad S = sY, \quad C = (1-s)Y$
- **Firm: The Neoclassical Production Function :** $Y = zF(K, N)$

  1. Constant returns to scale. $F(\lambda K, \lambda N) = \lambda F(K, N)$; for all $\lambda > 0$
  2. Positive and diminishing returns to private inputs: For all $K > 0$ and $L > 0$, F exhibits positive and diminishing marginal products with respective to each input.
  3. Inada conditions: The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity.

- **Per worker production function:** $y = zf(k)$
- **Law of motion for aggregate capital :** $K' = (1-d)K + I$
- **Equilibrium Output**

  In this economy, there are two markets in the current period; consumption goods are traded for current labor, consumption goods are traded for captial

  Consumers save by accumulating capital. At equilibrium, $S = I$ so that $Y = C + I$.

  Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving).

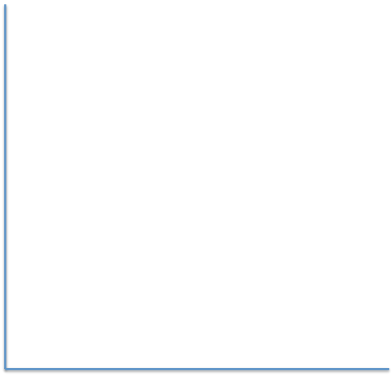
  $\frac{K'}{N} = ...$

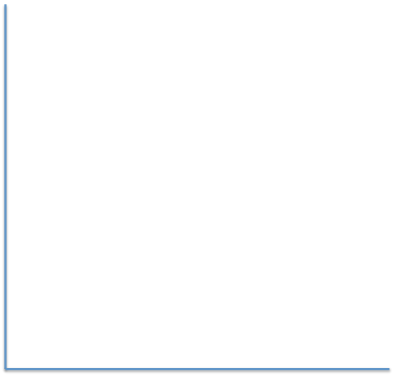
- **The law of motion for capital per worker:** $k' = ...$
- **Steady state condition** :

  steady state investment = steady state saving

  =

  .





- $Y* = y * N$, $K^* = k^* N$, $C^* = c^* N$: $Y = C + S$, $c = (1 - s)Y$
- $z \uparrow$

  $\Rightarrow$ ..... rotates .....

  $\Rightarrow$ k^*..... from k_1^* to k_2^*, y^* ..... , c^*.....

  $\Rightarrow$ In transition to the new growth path,

  - k^*..... , y^* ..... , c^*..... . Growth rate of per capita variables 0
  - Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... n.

  $\Rightarrow$When the economy has arrived the new growth path

  - k^*....., therefore, y^*, c^*..... . Growth rate of per capita variables 0
  - Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... n.

- $n \uparrow$ from n_1 to n_2

  $\Rightarrow$ ..... rotates .....

  $\Rightarrow$ k^*..... , y^* ..... , c^*.....

  $\Rightarrow$ In transition to the new growth path,

  - k^*..... from k_1^* to k_2^*, y^* ..... , c^*..... .
  - Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... n_2.

  $\Rightarrow$When the economy has arrived the new growth path

  - k^*....., therefore, y^*, c^*..... g_k, g_y, g_c.....0
  - Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... which is ..... n_1.

  $\Rightarrow$Notice that the level of per worker variables decrease but the growth rate of aggregate variable increases.

  - **Intuition** : As n increases, k^* decreases. As capital per worker falls, output per worker falls. The reason is that when labor force grows at a higher rate, the current labor force faces a harder task in bulding capital for the next period's consumers, who are a proportionately a largeer group. Thus, output per worker and capital per worker decreases at steady state.
  - This shows that the higher growth in aggregate income neednot be associated, in the long run, with higher income per worker.
  - consistent with the empirical fact: the higher population growth, the lower output per worker. High population growth corresponds with low living standards.

- $d \uparrow$

  $\Rightarrow$ ..... rotates .....

  $\Rightarrow$ k^*..... , y^* ..... , c^*.....

  $\Rightarrow$ In transition to the new growth path,

  - k^*..... from k_1^* to k_2^*, y^* ..... , c^*..... . g_k, g_y, g_c.....0
  - Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... n

  $\Rightarrow$When the economy has arrived the new growth path

  - k^*....., y^*.....c^*..... . Hence, all aggregate variables ($Y = y^* N, K = k * N, C = c^* N, S = sY^*$) grows at rate ..... n

  $\Rightarrow$**Intuition:** More of current savings is required just to keep the amount of capital per capita constant. In equilibrium output per capita and capital per capital decrease.

- $s \uparrow$

  $\Rightarrow$ ..... rotates .....

  $\Rightarrow$ k^*..... from k_1^* to k_2^*, y^* ..... , c^*.....

  $\Rightarrow$ In transition to the new growth path,

  - k^*..... , y^* ..... .[c^*.....]. g_k, g_y.....0
  - Hence, Y and K grows at rate ..... n.

  $\Rightarrow$When the economy has arrived the new growth path

  - k^*....., y^*.....c^*..... .$c^* = (1 - s_{new})y^*$. g_k, g_y, g_c.....0
  - Hence, all aggregate variables ($Y = y^*N, K = k * N, C = c^*N, S = sY^*$) grows at rate ..... n.

- golden rules : s that maximizes c^*

  $c^* \quad = \quad zf(k^*) -$ .....)

  at Steady state : $szf(k^*) =$ .....

  $c^* = zf(k^*) - (n + d)k^*$

  first order condition :

  $zf'(k^*_{golden}) =$ .....

  $MP_{k_{golden}} =$ .....

  $k^* < k^*_{golden}$; $s \uparrow \Rightarrow k$..... $\Rightarrow y$..... and c.....

  $k^* > k^*_{golden}$; $s \uparrow \Rightarrow k$..... $\Rightarrow y$..... and c.....

Note :

- "Solow's model states that investment in capital cannot drive long run growth in GDP per worker.
  - Need technological change (growth in A) to avoid diminishing returns to capital.
  - Easterly (2001) argues that "capital fundamentalism" view widely held in World Bank/IMF from 60s to 90s, despite lessons of Solow model.
  - Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run."
    (borrowed from Chapter 3 in Easterly (2001))
  - Solow-Swan, or neoclassical, growth model, implies countries converge to steady state GDP per worker (if no growth in technology)
  - Changes in savings ratio causes "level effect", but no long run growth effect higher labour force growth, ceteris paribus, implies lower GDP per worker
  - Exact outcome of Solow model does depend on precise functional forms and parameter values.
  - With standard production function, Solow model predicts economy moves to steady state because of diminishing returns to capital (assuming no growth in technology z).
  - An increase ina country's propensity to save or a decrease in the labor force growth rate imply one time increase in a country's standard of living, but there can be unbounded growth in the standard of living if and only if total factor productivity increases.

Practice Questions

1. In Solow growth model, explain the effect of a decrease in population growth rate on all endogenous variables. Explain and comment.

2. In Solow growth model, suppose that total factor productivity decreases. Analyze the effects on the economy especially consumption, capital and output per worker at steady state. Explain and comment.

3. In Solow growth model, suppose that the MPK increases for each quantity of the capital input.

   (a) Show the effects of this on the aggregate production function

   (b) Using a diagram, determine the effect on the quantity of capital per worker

   (c) Explain your results

4. In Solow growth model, suppose that depreciation rate increases. Analyze the effects on the economy especially consumption, capital and output per worker at steady state. Explain and comment.

5. Suppose that the economy is initially in a steady state and that some of the nation's capital stock is destroyed because of a natural disaster or a war.

   (a) determine the long-run effects of this on the quantity of capital per worker and on output per worker.

   (b) In the short-run does aggregate capital stock grow at a rate higher or lower than growth rate of the labor force

   (c) After World War II, growth in real GDP in Germany and Japan was very high. How do your results in parts (a) and (b) shed light on this historical experience.

6. Consider the effects of change in immigration law, which make it easier for immigrants to move to Canada (receiving work-permit and citizenship). More and more immigrants influx to the country from other troubled countries each year. Analyze the effects on the country's economy especially consumption, capital and output per worker at steady state. Explain and comment.

7. The Solow growth model predicts that poor countries should grow faster than rich countries. Comment.