chapter 6 (Q1-Q5) 6204641341 Pimchanok S.

1\) 1. Two individuals agree at date 0 to a forward contract that matures at date 2. The contract is written on an underlying asset that pays a dividend at date 1 equal to D_1. Let f_2 be the date 2 random payoff (profit) to the individual who is the long party in the forward contract. Also let m_{0i} be the stochastic discount factor over the period from dates 0 to i where $i = 1, 2$, and let $E_0[\cdot]$ be the expectations operator at date 0. What is the value of $E_0[m_{02} f_2]$? Explain your answer.

Let S_i = the price of underlying asset at date i

D_0 = dividend at date 0

$$\Rightarrow S_0 = E_0[m_{01} D_1] + E_0[m_{02} S_2]$$

$$= D_0 + E_0[m_{02} S_2]$$

Let F_{02} = forward price & (Long position) Payoff $(f_2) = S_2 - F_{02}$

From the stochastic discount factor approach,

$$E_0[m_{02} f_2] = E_0[m_{02}(S_2 - F_{02})]$$

$$= E_0[m_{02} S_2] - E[m_{02} F_{02}] \quad ; \quad E[m_{02} F_{02}] = E[m_{02}] F_{02} = R_f^{-2} F_{02}$$

Thus, $E_0[m_{02} f_2] = E[m_{02} S_2] - E[m_{02} F_{02}]$

$$= S_0 - D_0 - R_f^{-2} F_{02}$$

∴ We know that no arbitrage means that $F_{02} = R_f^2 (S_0 - D_0)$ is satisfied, implying that $E_0[m_{02} f_2] = 0$ #

2)

2. Assume that there is an economy populated by infinitely-lived representative individuals who maximize the lifetime utility function

$$E_0\left[\sum_{t=0}^{\infty} -\delta^t e^{-ac_t}\right]$$

where c_t is consumption at date t and $a > 0$, $0 < \delta < 1$. The economy is a Lucas (1978) endowment economy having multiple risky assets paying date t dividends that total d_t per capita. Write down an expression for the equilibrium per capita price of the market portfolio in terms of the assets' future dividends.

$$p_0 = E_0\left[\sum^{\infty} \frac{u_c(c_t, t)}{u_c(c_0, 0)} d_t\right]$$

: $u(c_t, t) = -\delta^t e^{-ac_t} \rightarrow u_c(c_t, t) = a\delta^t e^{-ac_t}$

: $c_t = d_t$ Then, $p_0 = E_0\left[\sum^{\infty} \delta^t e^{-a(d_t - d_0)} d_t\right]$ ※

3)

3. For the Lucas model with labor income, show that assumptions (6.25) and (6.26) lead to the pricing relationship (6.27) and (6.28).

$$\Rightarrow \quad p_t = E_t\left[\sum^{\infty} \delta^j \left(\frac{c^{\gamma}_{t+j}}{c^{*}_{j}}\right)^{\gamma-1} d_{t+j}\right]$$

$$\frac{p_t}{d_t} = E_t\left[\sum^{\infty} \delta^j \left(\frac{c^{\gamma}_{t+j}}{c^{*}_{j}}\right)^{\gamma-1} \left(\frac{d_{t+j}}{d_t}\right)\right]$$

$$= E_t\left[\sum^{\infty} \delta^j e^{(\gamma-1)\ln(c_{t+j}/c_t) + \ln(d_{t+j}/d_t)}\right]$$

$$\Rightarrow \quad \ln(c_{t+j}/c_t) = j\mu_c + \sigma_c \sum^{j} \eta_{t+j}$$

$$\ln(d_{t+j}/d_t) = j\mu_c + \sigma \sum^{j} \varepsilon_{t+j}$$

Then, $$\frac{p_t}{d_t} = E_t\left[\sum^{\infty} \delta^j e^{(\gamma-1)(j\mu_c + \sigma_c \sum^{j} \eta_{t+j}) + j\mu_c + \sigma \sum^{j} \varepsilon_{t+j}}\right]$$

$$\frac{p_t}{d_t} = \sum^{\infty} \delta^j e^{[(\gamma-1)\mu_c+\mu_d]} e^{\frac{1}{2}(1-\gamma)^2\sigma_c^2+\sigma_d^2-2(1-\gamma)\sigma_c\sigma_d\rho]}$$

$$= \frac{1}{1-\delta e^{-(1-\gamma)\mu_c+\mu_d+\frac{1}{2}((1-\gamma)^2\sigma_c^2+\sigma_d^2)(1-\gamma)\sigma_c\sigma_d\rho}}$$

$$p_t = d_t\frac{\delta e^a}{1-\delta e^a}$$

$$; \quad a = \mu_d-(1-\gamma)\mu_c+\frac{1}{2}\left[(1-\gamma)^2\sigma_c^2+\sigma_d^2\right]-(1-\gamma)\sigma_c\sigma_d\rho$$

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4) 4. Consider a special case of the model of rational speculative bubbles discussed in this chapter. Assume that infinitely-lived investors are risk-neutral and that there is an asset paying a constant, one-period risk-free return of $R_f = \delta^{-1} > 1$. There is also an infinitely-lived risky asset with price p_t at date t. The risky asset is assumed to pay a dividend of d_t which is declared at date t and paid at the end of the period, date $t+1$. Consider the price $p_t = f_t + b_t$ where

$$f_t = \sum_{i=0}^{\infty} \frac{E_t[d_{t+i}]}{R_f^{i+1}} \tag{1}$$

and

$$b_{t+1} = \begin{cases} \frac{R_f}{q_t}b_t + e_{t+1} & \text{with probability } q_t \\ z_{t+1} & \text{with probability } 1-q_t \end{cases} \tag{2}$$

where $E_t[e_{t+1}] = E_t[z_{t+1}] = 0$ and where q_t is a random variable as of date $t-1$ but realized at date t and is uniformly distributed between 0 and 1.

a) 4.a Show whether or not $p_t = f_t + b_t$ subject to the specifications in (1) and (2) is a valid solution for the price of the risky asset.

check that (2) satisfies $E_t[b_{t+1}] = R_f b_t$

$q_t \sim$ R.V. at date $t-1$

$q_t \sim$ realized at date t

Thus, $E_t[b_{t+1}] = \frac{R_f}{q_t} b_t q_t + E_t[e_{t+1}]q_t + (1-q_t)E_t[z_{t+1}]$

$= R_f b_t$

$\therefore$ $p_t = f_t + b_t$ is valid. \#

b) 4.b Suppose that p_t is the price of a barrel of oil. If $p_t \geq p_{solar}$, then solar energy, which is in perfectly elastic supply, becomes an economically efficient perfect substitute for oil. Can a rational speculative bubble exist for the price of oil? Explain why or why not.

$$E_t[b_{t+1}] = R_t b_t \Rightarrow \lim E_t[b_{t+1}] = \begin{cases} +\infty & \text{if } b_t > 0 \\ -\infty & \text{if } b_t < 0 \end{cases}$$

∴ if the bubble exists, the bubble component must be expected to rise infinitely. Howevery, this is not reasonable if there is an upper bound for the oil's price. Thus, because p_t can't exceed p_{solar}, b_t can't also exceed $p_{solar} - p_t^*$. Hence, a bubble path exists to infinity can't exist. #

c) 4.c Suppose p_t is the price of a bond that matures at date $T < \infty$. In this context, the d_t for $t \leq T$ denotes the bond's coupon and principal payments. Can a rational speculative bubble exist for the price of this bond? Explain why or why not.

∴ A rational speculative bubble doesn't exist for the price of this bond. At maturity, $p_t = d_t$ and 0 after date T. Also, the rational price is $p_t = p_t^*$. #

5) 5. Consider an endowment economy with representative agents who maximize the following objective function:

$$\max_{C_s, \{\omega_{is}\}, \forall s, i} E_t \left[\sum_{s=t}^{T} \delta^s u(C_s) \right]$$

where $T < \infty$. Explain why a rational speculative asset price bubble could not exist in such an economy.

with a finite timeline, $p_t \neq f_t + b_t$; $b_t \neq 0$ is not true because at date T, $p_t = f_t = d_t$. Since $b_t = 0$ with certainty, the bubble $E_t[b_{t+1}] = \delta^{-1} b_t$, implying that $E_{t-1}[b_t] = E_{t-1}[0]$

$= \delta^{-1} b_{t-1}$

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