EE 320 Introductory Mathematical Economics

Semester 2/2012

Problem Set 4

Matrix Algebra and Applications

1. Given

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}; \qquad B = \begin{bmatrix} 5 & 1 \\ -2 & 6 \end{bmatrix}$$

a) Find *AB* and *BA*.
b) Show that *(AB)′* = *B′A′*.

2. Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}; \qquad B = \begin{bmatrix} 0 & 3 & 2 \\ 4 & 1 & 2 \end{bmatrix}; \qquad C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 2 & 0 \end{bmatrix}$$

a) Show that *(A+B)′* = *A′+B′*.
b) Show that *(AC)B* = *A(CB)*.
c) Show that *C(A + B)* = *CA* + *CB*.

3. Write the following systems as matrix equations:

a)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ x_1 - x_2 + x_3 &= 5 \\ x_1 - x_2 + 2x_3 &= 15 \end{aligned}$$

b)
$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 0 \\ x_1 + x_2 - x_3 &= 0 \end{aligned}$$

(Questions 4 – 12 are from Sydsaeter and Hammond, 2008)

4. For what values of *a* is $\begin{bmatrix} a & a^2 - 1 & -3 \\ a + 1 & 2 & a^2 + 4 \\ -3 & 4a & -1 \end{bmatrix}$ symmetric?

5. An *n x n* matrix ***P*** is said to be <u>orthogonal</u> if ***P′P*** = ***I****$_n$*.

a) For $\lambda = \pm 1/\sqrt{2}$, show that $\boldsymbol{P} = \begin{bmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{bmatrix}$ is orthogonal.
b) Show that the 2 x 2 matrix $\begin{bmatrix} p & -q \\ q & p \end{bmatrix}$ is orthogonal if and only if $p^2 + q^2 = 1$.

6. Find the following determinants.

a) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

c) $\begin{vmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{vmatrix}$

d) $\begin{vmatrix} a & 0 & b \\ 0 & e & 0 \\ c & 0 & d \end{vmatrix}$

7. Let$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{bmatrix}$.

Calculate AB, |A|, |B|, |A|·|B| and |AB|.

8. Prove that the inverse of $\begin{bmatrix} 1 & 1 & -3 \\ 2 & 1 & -3 \\ 2 & 2 & 1 \end{bmatrix}$ is $\begin{bmatrix} -1 & 1 & 0 \\ 8/7 & -1 & 3/7 \\ -2/7 & 0 & 1/7 \end{bmatrix}$.

9. Use Cramer's rule to solve the following systems of equations. Check your answers.

a)
$$\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ x_1 + x_2 - x_3 &= 0 \\ -x_1 - x_2 - x_3 &= -6 \end{aligned}$$

b)
$$\begin{aligned} x_1 - x_2 &= 0 \\ x_1 + 3x_2 + 2x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

10. Consider the simple macro model described by the three equations

(i) $Y = C + A_0$ (ii) $C = a + b(Y-T)$ (iii) $T = d + tY$

where Y is income, C is consumption, T is tax revenue, A_0 is the constant (exogenous) autonomous expenditure, and *a, b, d,* and *t* are all positive parameters. Find the equilibrium values of the endogenous variables *Y, C,* and *T* by writing the equations in matrix form and applying Cramer's rule.

11. Examine for what values of the constants *a* and *b* the system of equations

$$\begin{aligned} ax + y &= 3 \\ x + z &= 2 \\ y + az + bu &= 6 \end{aligned}$$

$y + u = 1$

has a unique solution in the unknown *x*, *y*, *z*, and *u*. Find the unique solution (expressed in terms of *a* and *b*).

12. Consider and economy divided into an agricultural sector (A) and an industrial sector (I). To produce one unit in sector A requires 1/6 unit from A and 1/4 unit from I. To produce in sector I requires 1/4 unit from A and 1/4 unit from I. Suppose final demands in each of the two sectors are 60 units.

a) Write down the Leontif system for this economy.
b) Find the number of units that has to be produced in each sector in order to meet the final demands.

(Questions 13 – 14 are from Dowling, 2012)

13. Use matrix inversion to solve for the unknowns in the system of linear equations given below.

$2x_1 + 4x_2 - 3x_3 = 12$

$3x_1 - 5x_2 + 2x_3 = 13$

$-x_1 + 3x_2 + 2x_3 = 17$

14. Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.