Extensions of the Two-Variable Linear Regression Model

ee325 2/2011 (Ajarn Kaewkwan Tangtipongkul)

Regression through the origin

$$Y_i = \beta_2 X_i + u_i$$

Regression through the origin

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}$$

$$\mathrm{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum X_i^2}, \textit{where } \sigma^2 = \frac{\sum \hat{u}_i^2}{n-1}$$

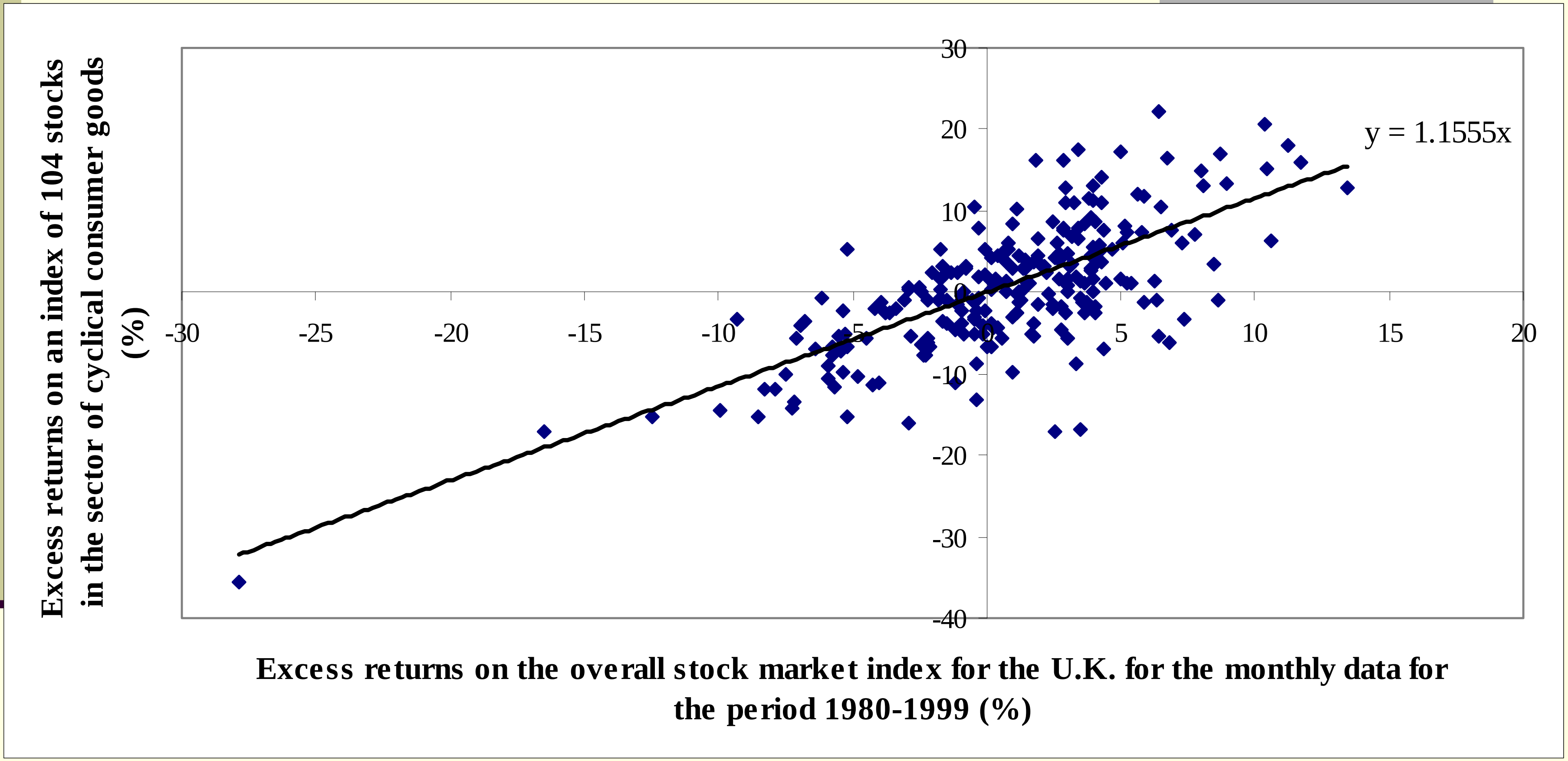
R-squared for Regression through Origin Model

$$raw\ r^2 = \frac{(\sum X_i Y_i)^2}{\sum X_i^2 \sum Y_i^2}$$

Example

Table 6.1 (P.151) gives data on excess returns on an index on 104 stocks in the sector of cyclical consumer goods and excess returns on the overall stock market index for the U.K. for the monthly data for the period 1980-1999, for a total of 240 observations.

Excess returns refers to return in excess of return on a riskless asset.


Excess returns on an index of 104 stocks in the sector of cyclical consumer goods (%)
30
20
10
0
-10
-20
-30
-40
-30
-25
-20
-15
-10
-5
0
5
10
15
20
y = 1.1555x
Excess returns on the overall stock market index for the U.K. for the monthly data for the period 1980-1999 (%)

The slope coefficient is highly significant

If the excess market rate goes up by 1 percentage point, the excess return on the index of consumer goods sector goes up by about 1.15 percentage points.

If a Beta coefficient is greater than 1, such a security is said to be volatile; it moves more than proportionately with the overall stock market index

Scaling and Units of Measurement

Consider the data given in Table 6.2, which refers to U.S. gross private domestic investment (GPDI) and gross domestic product (GDP) in billions as well as millions of (chained) 2000 dollars.

Suppose in the regression of GDPI on GDP one researcher uses data in billions of dollars but another expresses data in millions of dollars.

- **Will the regression results be the same in both cases?**
- **Do the units in which the regressand and regressor are measured make any difference in the regression results?**

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

$$where\ Y = GDPI\ \ and\ \ X = GDP$$

$$Y_i^* = w_1 Y_i$$

$$X_i^* = w_2 X_i$$

Where w_1 and w_2 are constants, call the **Scale factors**

$$Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i + \hat{u}_i$$

$$where\ Y_i^* = w_1 Y_i, X_i^* = w_2 X_i, and\ \hat{u}_i^* = w_1 \hat{u}_i$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\hat{\beta}_1^* = \bar{Y}^* - \hat{\beta}_2^* \bar{X}^*$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$\hat{\beta}_2^* = \frac{\sum x_i^* y_i^*}{\sum x_i^{*2}}$$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2$$

$$\operatorname{var}(\hat{\beta}_1^*) = \frac{\sum X_i^{*2}}{n \sum x_i^{*2}} \sigma^{*2}$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

$$\operatorname{var}(\hat{\beta}_2^*) = \frac{\sigma^{*2}}{\sum x_i^{*2}}$$

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

$$\hat{\sigma}^{*2} = \frac{\sum \hat{u}_i^{*2}}{n-2}$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1$$

$$\hat{\sigma}^{*2} = w_1^2\hat{\sigma}^2$$

$$\text{var}(\hat{\beta}_1^*) = w_1^2 \,\text{var}(\hat{\beta}_1)$$

$$\text{var}(\hat{\beta}_2^*) = \left(\frac{w_1}{w_2}\right)^2 \text{var}(\hat{\beta}_2)$$

$$r_{xy}^2 = r_{x^*y^*}^2$$

Example

Gross Private Domestic Investment and GDP, United States, 1990-2005

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

$$where\ Y_i = GPDI\ \ and\ \ X_i = GDP$$

TABLE 6.2
Gross Private Domestic Investment and GDP, United States, 1990–2005 (Billions of chained [2000] dollars, except as noted; quarterly data at seasonally adjusted annual rates)

Source: *Economic Report of the President,* 2007, Table B-2, p. 328.

Year	GPDIBL	GPDIM	GDPB	GDPM
1990	886.6	886,600.0	7,112.5	7,112,500.0
1991	829.1	829,100.0	7,100.5	7,100,500.0
1992	878.3	878,300.0	7,336.6	7,336,600.0
1993	953.5	953,500.0	7,532.7	7,532,700.0
1994	1,042.3	1,042,300.0	7,835.5	7,835,500.0
1995	1,109.6	1,109,600.0	8,031.7	8,031,700.0
1996	1,209.2	1,209,200.0	8,328.9	8,328,900.0
1997	1,320.6	1,320,600.0	8,703.5	8,703,500.0
1998	1,455.0	1,455,000.0	9,066.9	9,066,900.0
1999	1,576.3	1,576,300.0	9,470.3	9,470,300.0
2000	1,679.0	1,679,000.0	9,817.0	9,817,000.0
2001	1,629.4	1,629,400.0	9,890.7	9,890,700.0
2002	1,544.6	1,544,600.0	10,048.8	10,048,800.0
2003	1,596.9	1,596,900.0	10,301.0	10,301,000.0
2004	1,713.9	1,713,900.0	10,703.5	10,703,500.0
2005	1,842.0	1,842,000.0	11,048.6	11,048,600.0

Note: GPDIBL = gross private domestic investment, billions of 2000 dollars.
GPDIM = gross private domestic investments, millions of 2000 dollars.
GDPB = gross domestic product, billions of 2000 dollars.
GDPM = gross domestic product, millions of 2000 dollars.

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GPDIM = Gross private domestic investment, millions of 2000 dollars
GDPB = Gross domestic product, billions of 2000 dollars
GDPM= Gross domestic product, millions of 2000 dollars

Both GDPI and GDP in billions of dollars:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars ⟶ millions of dollars

GDP in billions of dollars ⟶ millions of dollars

$$w_1 = 1000$$

$$w_2 = 1000$$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1 = 1000 * -926.090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2} \right) \hat{\beta}_2 = \left(\frac{1000}{1000} \right) 0.2535 = 0.2535$$

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$$r^2 = 0.9648$$

GDPI in millions of dollars → billions of dollars

GDP in millions of dollars → billions of dollars

$$w_1 = \frac{1}{1000}$$

$$w_2 = \frac{1}{1000}$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = \frac{1}{1000} * -926,090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{\frac{1}{1000}}{\frac{1}{1000}}\right)0.2535 = 0.2535$$

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GDPI in billions of dollars → billions of dollars

GDP in billions of dollars → millions of dollars

$$w_1 = 1$$

$$w_2 = 1000$$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1 = 1 * -926.090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2} \right) \hat{\beta}_2 = \left(\frac{1}{1000} \right) 0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535 GDP_t$$

$$se = (116.358) \quad (0.0000129)$$

$$r^2 = 0.9648$$

Both GDPI and GDP in millions of dollars:

$$\widehat{GPDI}_t = -926{,}090 + 0.2535 GDP_t$$

$$se = (116{,}358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in millions of dollars → billions of dollars

GDP in millions of dollars → millions of dollars

$$w_1 = \frac{1}{1000}$$

$$w_2 = 1$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = \frac{1}{1000} * -926,090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{\frac{1}{1000}}{1}\right) 0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535 GDP_t$$

$$se = (116.358) \quad (0.0000129)$$

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Both GDPI and GDP in billions of dollars:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars → millions of dollars

GDP in billions of dollars → billions of dollars

$$w_1 = 1000$$

$$w_2 = 1$$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1 = 1000 * -926.090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2} \right) \hat{\beta}_2 = \left(\frac{1000}{1} \right) 0.2535 = 253.524$$

$$\widehat{GPDI}_t = -926,090 + 253.524 GDP_t$$

$$se = (116,358) \quad (12.9465)$$

$$r^2 = 0.9648$$

Both GDPI and GDP in millions of dollars:

$$\widehat{GPDI}_t = -926,090 + 0.2535 GDP_t$$

$$se = (116,358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in millions of dollars → millions of dollars

GDP in millions of dollars → billions of dollars

$$w_1 = 1$$

$$w_2 = \frac{1}{1000}$$

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1 = 1 * -926,090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2 = \left(\frac{1}{\frac{1}{1000}}\right) 0.2535 = 253.524$$

$$\widehat{GPDI}_t = -926,090 + 253.524 GDP_t$$

$$se = (116,358) \quad (12.9465)$$

$$r^2 = 0.9648$$

Functional Forms of Regression Models

- The log-linear model
- Semilog models
- Reciprocal models
- The logarithmic reciprocal model

The log-linear model

The exponential regression model:

$$Y_i = \beta_1 X_i^{\beta_2} e^{u_i}$$

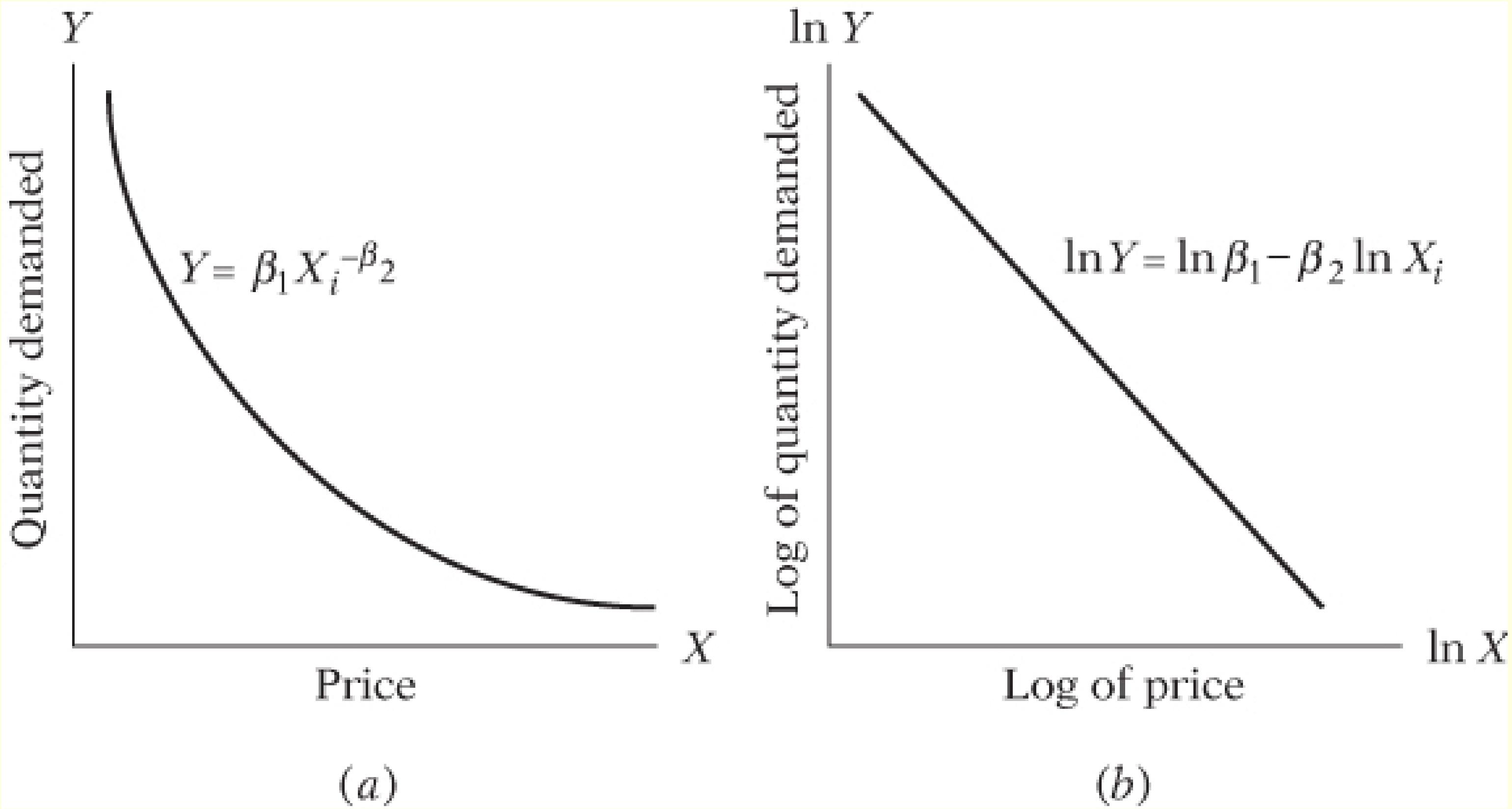
Which may be expressed alternatively as

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i$$

$$\ln Y_i = \alpha_1 + \beta_2 \ln X_i + u_i$$

$$where\ \alpha = \ln \beta_1$$

The log-linear model



$$\ln Y = \beta_1 + \beta_2 \ln X$$

$$\frac{1}{Y}\frac{dY}{dX} = \frac{\beta_2}{X}\frac{dX}{dX}$$

$$\beta_2 = \frac{X}{Y}\frac{dY}{dX} = \frac{dY/Y}{dX/X} = elasticity$$

The log-linear model

- The slope coefficient β_2 measures the elasticity of Y with respect to X, that is, the percentage change in Y for a given (small) percentage change in X

Percentage change vs. Percentage point change

Example –The unemployment rate

The unemployment rate of 6%, if this rate goes to 8%, we say that **the percentage point change in the unemployment rate is 2**

The percentage change in the unemployment rate is (8-6)/6, or about 33%

Example

Expenditure on durable goods in relation to total personal consumption expenditure

- We wish to find the elasticity of expenditure on durable goods with respect to total personal consumption expenditure.

TABLE 6.3

Total Personal Expenditure and Categories (Billions of chained [2000] dollars; quarterly data at seasonally adjusted annual rates)

Sources: Department of Commerce, Bureau of Economic Analysis. *Economic Report of the President,* 2007, Table B-17, p. 347.

Year or quarter	EXPSERVICES	EXPDUR	EXPNONDUR	PCEXP
2003-I	4,143.3	971.4	2,072.5	7,184.9
2003-II	4,161.3	1,009.8	2,084.2	7,249.3
2003-III	4,190.7	1,049.6	2,123.0	7,352.9
2003-IV	4,220.2	1,051.4	2,132.5	7,394.3
2004-I	4,268.2	1,067.0	2,155.3	7,479.8
2004-II	4,308.4	1,071.4	2,164.3	7,534.4
2004-III	4,341.5	1,093.9	2,184.0	7,607.1
2004-IV	4,377.4	1,110.3	2,213.1	7,687.1
2005-I	4,395.3	1,116.8	2,241.5	7,739.4
2005-II	4,420.0	1,150.8	2,268.4	7,819.8
2005-III	4,454.5	1,175.9	2,287.6	7,895.3
2005-IV	4,476.7	1,137.9	2,309.6	7,910.2
2006-I	4,494.5	1,190.5	2,342.8	8,003.8
2006-II	4,535.4	1,190.3	2,351.1	8,055.0
2006-III	4,566.6	1,208.8	2,360.1	8,111.2

Note: See Table B-2 for data for total personal consumption expenditures for 1959–1989.
EXPSERVICES = expenditure on services, billions of 2000 dollars.
EXPDUR = expenditure on durable goods, billions of 2000 dollars.
EXPNONDUR = expenditure on nondurable goods, billions of 2000 dollars.
PCEXP = total personal consumption expenditure, billions of 2000 dollars.

(*Continued*)

$$\widehat{\ln EXDUR_t} = -7.5417 + 1.6266 \ln PCEX_t$$

As these result shows , the elasticity of EXPDUR with respect to PCEX is about 1.63, suggesting that if total personal expenditure goes up by 1 percent, on average, the expenditure on durable goods goes up by about 1.63 percent

Thus, expenditure on durable goods is very responsive to changes in personal consumption expenditure. This is one reason why producers of durable goods keep a keen eye on changes in personal income and personal consumption expenditure

Semilog Models

- Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

- Lin-Log model

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

The Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\ln Y = \beta_1 + \beta_2 X$$

$$\frac{1}{Y}\frac{dY}{dX} = \beta_2$$

$$\beta_2 = \frac{dY/Y}{dX}$$

$$\beta_2 = \frac{relative\ change\ in\ regressand}{absolute\ change\ in\ regressor}$$

β_2 *is known as the semielasticity of* Y *with respect to* X

$100 * \beta_2$ *is known as the semielasticity of* Y *with respect to* X

Example

- We want to find out the growth rate of personal consumption expenditure on services for the data
- The regression results over time (t) are as follows:

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Note: See Table B-2 for data for total personal consumption expenditures for 1959–1989.
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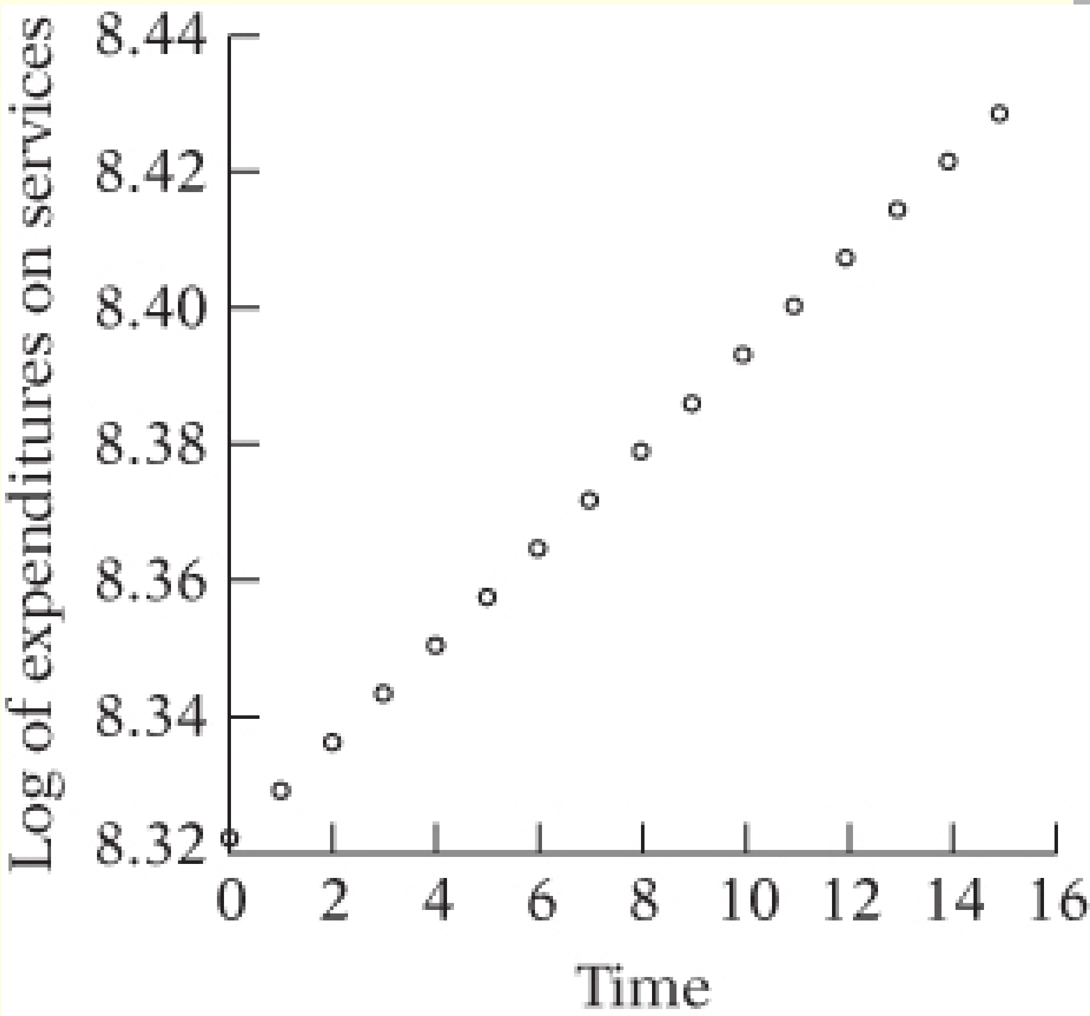
PCEXP = total personal consumption expenditure, billions of 2000 dollars.

(*Continued*)



Log of expenditures on services
8.44
8.42
8.40
8.38
8.36
8.34
8.32
0
2
4
6
8
10
12
14
16
Time

$$\widehat{\ln EXS_t} = 8.3226 + 0.00705t$$

Over the quarterly period 2003-I to 2006-III, expenditures on services increased at the (quarterly) rate of 0.705 percent. Roughly, this is equal to an annual growth rate of 2.82 percent

Lin-Log models

The absolute change in Y for a percentage change in X

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

$$Y = \beta_1 + \beta_2 \ln X$$

$$\frac{dY}{dX} = \beta_2 \frac{1}{X}$$

$$\beta_2 = \frac{dY}{dX / X}$$

$$\beta_2 = \frac{Change\ in\ Y}{relative\ change\ in\ X}$$

The absolute change in Y for a percentage change in X

If $\Delta X / X$ changes by 0.01 unit or 1%, the absolute change in Y is

$$0.01 * \beta_2$$

Example

- Food expenditure in India example

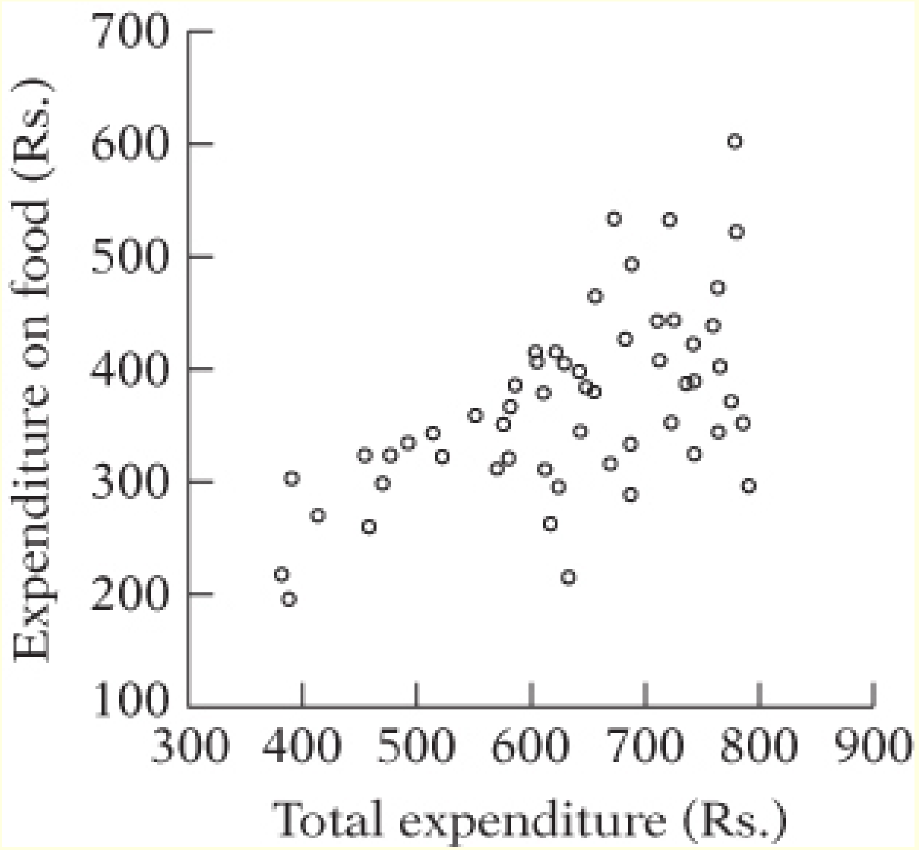


TABLE 2.8 Food and Total Expenditure (Rupees)

Observation	Food Expenditure	Total Expenditure	Observation	Food Expenditure	Total Expenditure
1	217.0000	382.0000	29	390.0000	655.0000
2	196.0000	388.0000	30	385.0000	662.0000
3	303.0000	391.0000	31	470.0000	663.0000
4	270.0000	415.0000	32	322.0000	677.0000
5	325.0000	456.0000	33	540.0000	680.0000
6	260.0000	460.0000	34	433.0000	690.0000
7	300.0000	472.0000	35	295.0000	695.0000
8	325.0000	478.0000	36	340.0000	695.0000
9	336.0000	494.0000	37	500.0000	695.0000
10	345.0000	516.0000	38	450.0000	720.0000
11	325.0000	525.0000	39	415.0000	721.0000
12	362.0000	554.0000	40	540.0000	730.0000
13	315.0000	575.0000	41	360.0000	731.0000
14	355.0000	579.0000	42	450.0000	733.0000
15	325.0000	585.0000	43	395.0000	745.0000
16	370.0000	586.0000	44	430.0000	751.0000
17	390.0000	590.0000	45	332.0000	752.0000
18	420.0000	608.0000	46	397.0000	752.0000
19	410.0000	610.0000	47	446.0000	769.0000
20	383.0000	616.0000	48	480.0000	773.0000
21	315.0000	618.0000	49	352.0000	773.0000
22	267.0000	623.0000	50	410.0000	775.0000
23	420.0000	627.0000	51	380.0000	785.0000
24	300.0000	630.0000	52	610.0000	788.0000
25	410.0000	635.0000	53	530.0000	790.0000
26	220.0000	640.0000	54	360.0000	795.0000
27	403.0000	648.0000	55	305.0000	801.0000
28	350.0000	650.0000			

Source: Chandan Mukherjee, Howard White, and Marc Wuyts, *Econometrics and Data Analysis for Developing Countries,* Routledge, New York, 1998, p. 457.

$$\widehat{FoodExp_i} = -1283.912 + 257.2700 \ln TotalExp_i$$

- As this figure suggests, food expenditure increases more slowly as total expenditure increases, perhaps giving credence to Engel's law

- The slope coefficient of about 257 means that an increase in the total food expenditure of 1 percent, on average, leads to about 2.57 rupees increase in the expenditure on food of the 55 families included in the sample

  (Note: we have divided the estimated slope coefficient by 100)

Reciprocal models

$$Y_i = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$$

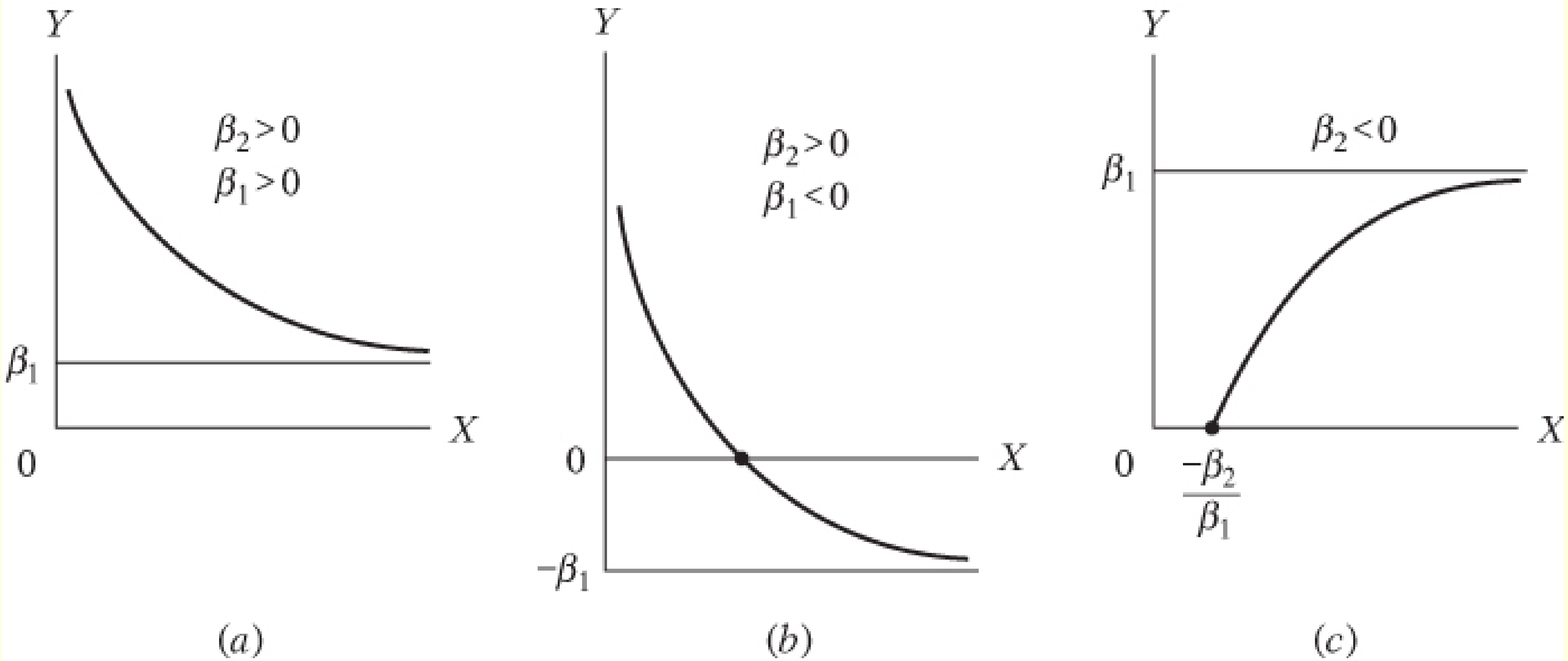
As X increases indefinitely, in term $\beta_2\left(\frac{1}{X_i}\right)$ approaches zero and Y approaches the limiting or asymptotic value β_1

$$Y_i = \beta_1 + \beta_2\left(\frac{1}{X}\right)$$

$$\frac{dY}{dX} = -\beta_2\left(\frac{1}{X^2}\right)$$

if β_2 is positive, the slope is negative throughout

β_2 is negative, the slope is positive throughout


Y
$\beta_2 > 0$
$\beta_1 > 0$
β_1
0
X
(a)
Y
$\beta_2 > 0$
$\beta_1 < 0$
0
X
$-\beta_1$
(b)
Y
$\beta_2 < 0$
β_1
0
$\frac{-\beta_2}{\beta_1}$
X
(c)

Example

- Child mortality and per capita GNP – 64 countries

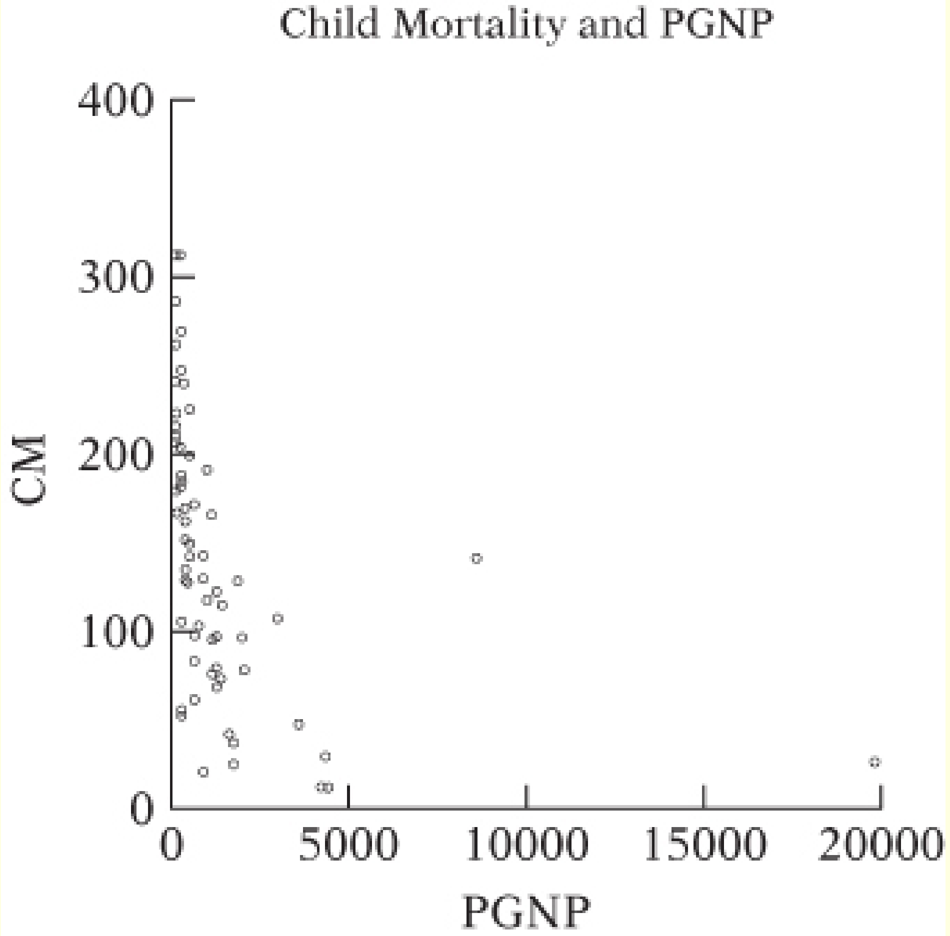


TABLE 6.4 **Fertility and Other Data for 64 Countries**

Observation	CM	FLFP	PGNP	TFR	Observation	CM	FLFP	PGNP	TFR
1	128	37	1870	6.66	33	142	50	8640	7.17
2	204	22	130	6.15	34	104	62	350	6.60
3	202	16	310	7.00	35	287	31	230	7.00
4	197	65	570	6.25	36	41	66	1620	3.91
5	96	76	2050	3.81	37	312	11	190	6.70
6	209	26	200	6.44	38	77	88	2090	4.20
7	170	45	670	6.19	39	142	22	900	5.43
8	240	29	300	5.89	40	262	22	230	6.50
9	241	11	120	5.89	41	215	12	140	6.25
10	55	55	290	2.36	42	246	9	330	7.10
11	75	87	1180	3.93	43	191	31	1010	7.10
12	129	55	900	5.99	44	182	19	300	7.00
13	24	93	1730	3.50	45	37	88	1730	3.46
14	165	31	1150	7.41	46	103	35	780	5.66
15	94	77	1160	4.21	47	67	85	1300	4.82
16	96	80	1270	5.00	48	143	78	930	5.00
17	148	30	580	5.27	49	83	85	690	4.74
18	98	69	660	5.21	50	223	33	200	8.49
19	161	43	420	6.50	51	240	19	450	6.50
20	118	47	1080	6.12	52	312	21	280	6.50
21	269	17	290	6.19	53	12	79	4430	1.69
22	189	35	270	5.05	54	52	83	270	3.25
23	126	58	560	6.16	55	79	43	1340	7.17
24	12	81	4240	1.80	56	61	88	670	3.52
25	167	29	240	4.75	57	168	28	410	6.09
26	135	65	430	4.10	58	28	95	4370	2.86
27	107	87	3020	6.66	59	121	41	1310	4.88
28	72	63	1420	7.28	60	115	62	1470	3.89
29	128	49	420	8.12	61	186	45	300	6.90
30	27	63	19830	5.23	62	47	85	3630	4.10
31	152	84	420	5.79	63	178	45	220	6.09
32	224	23	530	6.50	64	142	67	560	7.20

Note: CM = Child mortality, the number of deaths of children under age 5 in a year per 1000 live births.
FLFP = Female literacy rate, percent.
PGNP = per capita GNP in 1980.
TFR = total fertility rate, 1980–1985, the average number of children born to a woman, using age-specific fertility rates for a given year.

Source: Chandan Mukherjee, Howard White, and Marc Whyte, *Econometrics and Data Analysis for Developing Countries,* Routledge, London, 1998, p. 456.

$$\widehat{CM}_i = 81.7944 + 27,237.17\left(\frac{1}{PGNP_i}\right)$$

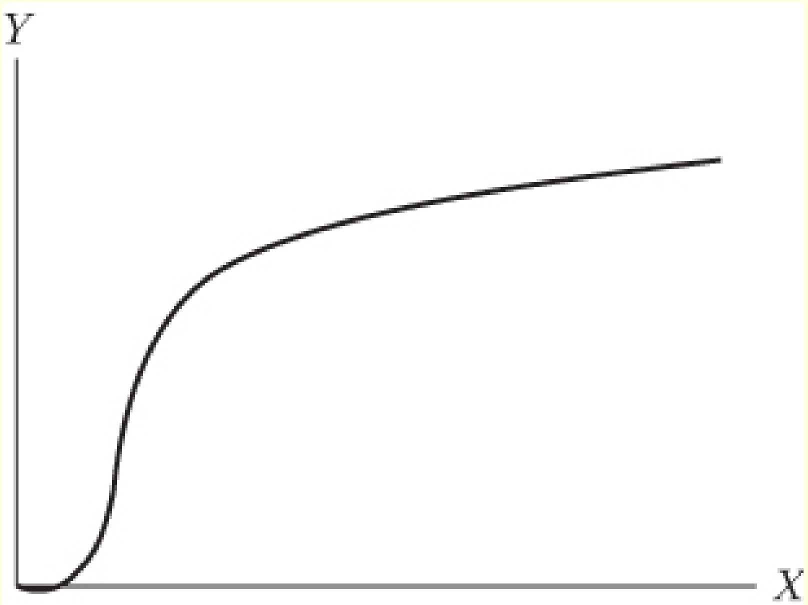
- As per capita GNP increases, one would expect child mortality to decrease because people can afford to spend more on health care, assuming all other factors remain constant

- The relationship is not a straight line one: As per capita GNP increases, initially there is a dramatic drop in child mortality but the drop tapers of as per capita GNP continues to increase

As per capita GNP increases indefinitely, child mortality approaches its asymptotic value of about 82 deaths per thousand

The logarithmic reciprocal model

$$\ln Y_i = \beta_1 - \beta_2 \left(\frac{1}{X_i} \right) + u_i$$



The logarithmic reciprocal model

- Initially Y increases at an increasing rate and then it increases at a decreasing rate

- E.g. short run production function

  Microeconomics – if labor and capital are the inputs in a production function and if we keep the capital input constant but increase the labor input, the short-run output-labor relationship will resemble in the logarithmic reciprocal model

Choice of Functional Form

TABLE 6.6

Model	Equation	Slope $\left(=\frac{dY}{dX}\right)$	Elasticity $\left(=\frac{dY}{dX}\frac{X}{Y}\right)$
Linear	$Y=\beta_1+\beta_2 X$	β_2	$\beta_2\left(\frac{X}{Y}\right)^*$
Log–linear	$\ln Y=\beta_1+\beta_2 \ln X$	$\beta_2\left(\frac{Y}{X}\right)$	β_2
Log–lin	$\ln Y=\beta_1+\beta_2 X$	$\beta_2 (Y)$	$\beta_2 (X)^*$
Lin–log	$Y=\beta_1+\beta_2 \ln X$	$\beta_2\left(\frac{1}{X}\right)$	$\beta_2\left(\frac{1}{Y}\right)^*$
Reciprocal	$Y=\beta_1+\beta_2\left(\frac{1}{X}\right)$	$-\beta_2\left(\frac{1}{X^2}\right)$	$-\beta_2\left(\frac{1}{XY}\right)^*$
Log reciprocal	$\ln Y=\beta_1-\beta_2\left(\frac{1}{X}\right)$	$\beta_2\left(\frac{Y}{X^2}\right)$	$\beta_2\left(\frac{1}{X}\right)^*$

Note: * indicates that the elasticity is variable, depending on the value taken by X or Y or both. When no X and Y values are specified, in practice, very often these elasticities are measured at the mean values of these variables, namely, $\bar{X}$ and $\bar{Y}$.