# Solution to Price Discrimination Practice Problems.



a) $Q = q^r + q = 18 - 5p + 10 - 2p$

$= 28 - 7p \quad$ or $\quad P = 4 - \frac{Q}{7}$

Then, use this demand curve (for 2 person) to find price. (using the 1-person average demand would get you the same answer)

$$\pi^{mono} = \text{total revenue} - \text{total cost}$$
$$= PQ - mcQ$$
$$= (P - mc)Q \qquad ; \; mc = 2$$
$$= (\underbrace{4 - \frac{Q}{7}}_{P} - 2)Q = 2Q - \frac{Q^2}{7}$$

First order condition (F.O.C.): $0 = \frac{d\pi^{mono}}{dQ}$,

$$0 = 2 - \frac{2Q}{7}$$

$Q = 7$ #, plug this back into demand curve

$$P = 4 - \frac{Q}{7} = 4 - \frac{7}{7} = 3 \;\#$$

b) $\boxed{\pi^{under}}$ $= \text{total revenue} - \text{total cost}$

$$= Pq^r - mcq^r$$
$$= (P - mc)q^r$$
$$= \left(\frac{18 - q^r}{5} - 2\right) q^r$$
$$= \left(\frac{8 - q^r}{5}\right) q^r$$

$q^r = 18 - 5p$

$P = \frac{18 - q^r}{5}$

②

FOC : $0 = \frac{d\pi^{under}}{dq^r}$

$= \left(\frac{8-q^r}{5}\right)\times 1 + q^r\left(\frac{-1}{5}\right)$

$= \frac{8-2q^r}{5}$

$q^r = 4$ # $\Rightarrow P = \frac{18-q^r}{5} = 2.8$ #

$\boxed{\pi^{over}}$

$= Pq - mcq$

$= (P-mc)q$

$= \left(\frac{10-q}{2} - 2\right)q$

$= \left(\frac{6-q}{2}\right)q$

| $q = 10-2p$ |
|---|
| $P = \frac{10-q}{2}$ |

FOC: $0 = \frac{d\pi^{over}}{dq}$

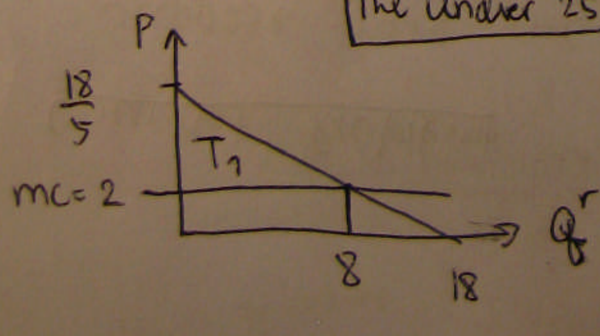
$= \left(\frac{6-q}{2}\right)\times 1 + q\left(\frac{-1}{2}\right)$

$= \frac{6-2q}{2}$

$q = 3$ # $\Rightarrow P = \frac{10-q}{2} = 3.5$ #

c) Separate cover charge for each group.

$\boxed{\text{The under 25}}$




$q^r = 18 - 5P$

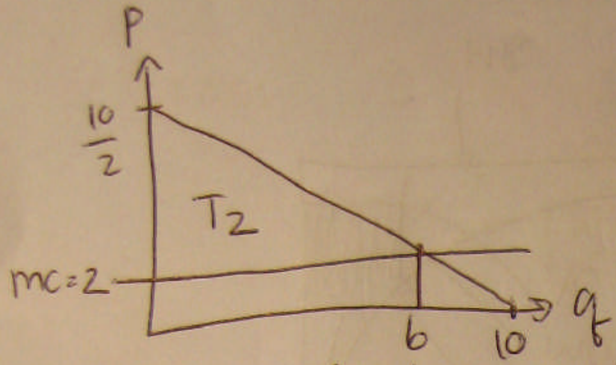
$P = \frac{18}{5} - \frac{1}{5}q^r$

cover charge $= T_1$

price/unit $= mc = 2\$$ #

$T_1 = \frac{1}{2}\times \text{height} \times \text{base} = \frac{1}{2}\times\left(\frac{18}{5}-2\right)\times 8 = 6.4$ #

The over 25




$q = 10 - 2p$

$P = \frac{10}{2} - \frac{q}{2}$

cover charge $= T_2$

price/unit $= mc = \$2$ #

$T_2 = \frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2}\left(\frac{10}{2} - 2\right) \times 6 = \$9$ #

d) charge cover charge $= T_1 = \$6.4$

price per drink $= mc = \$2$

if cover charge exceeds \$6.4, the under 25 will not buy because the cover charge exceeds their surplus.

e) before midnight's linear pricing = same price for all units with no cover charge

$Q = \frac{2}{7} q^r + \frac{7}{7} q$

only $\frac{2}{7}$ of the under. all of the over 25

$= \frac{2}{7}(18 - 5p) + \frac{7}{7}(10 - 2p)$

$= \frac{36 - 10P + 70 - 14P}{7} = \frac{106 - 24P}{7}$

$Q = \frac{106 - 24P}{7} \Rightarrow P = \frac{106 - 7Q}{24}$

$\pi^{\text{before}}$ = total revenue − total cost

$= PQ - mcQ$

$= (P - mc)Q$

$$= \left(\frac{106 - 7Q}{24} - 2\right) Q$$

FOC: $0 = \frac{d\pi^{before}}{dQ}$

$$= \left(\frac{106 - 7Q - 46}{24}\right) + Q\left(\frac{-7}{24}\right)$$

$$= 60 - 14Q$$

$$Q = 4.28 \Rightarrow P = \frac{106 - 7 \times Q}{24} \approx 3.17 \#$$

AFTER Midnight ⇒ only the under 25 stays.

so, from part a) the linear (uniform) monopoly price is \$ 2.8 / unit for the under 25.

Compare profits

in part d) nightclub charges \$ 6.4 as a cover charge per person. Then it charges \$ 2 per unit to cover the cost.

Thus profit / person = \$ 6.4.

in part e) $\pi^{before} = (P - mc)Q$

$= (3.17 - 2)\,4.28$

$= \$5$

but this is for $\frac{2}{7} + \frac{7}{7} = \frac{9}{7}$ person $>$ 1 person.

Thus $\pi^{before}$ per person $= 5 \times \frac{7}{9} \approx 3.88$/person

$\pi^{After} = (P - mc)Q$

$= (2.8 - 2)\,4$

$= \$3.2$ / person.

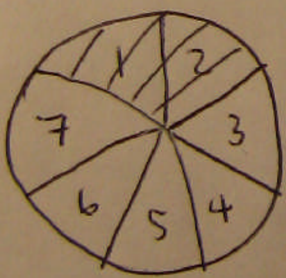
Thus, average $\pi$ $= 3.88 \times \frac{9}{14} + 3.2 \times \frac{5}{14}$

$= 2.49 + 1.14$

$= \$3.63$ / person.

why $\frac{9}{14}$ and $\frac{5}{14}$?

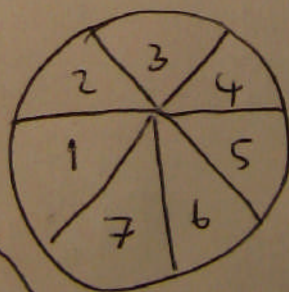
under 25




$\frac{2}{7}$ before midnight

$\frac{5}{7}$ after midnight

over 25




all before midnight

There are 14 portions of customers