

Wantitee T.
610464089



EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

Student	Y_i	X_i
1	2.8	63
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1.1 from $\hat{\beta}_2 = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n \sum_{i=1}^n X_i^2 - (\sum_{i=1}^n X_i)^2}$

$$\hat{\beta}_2 = \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641}$$

$$= \frac{139.5}{4095} = 0.034$$

On average, if student score increase by 1 their gpa is increasing 0.034 holding all other factors that might effect the GPA constant

$$\begin{aligned}
 \text{from } \hat{\beta}_1 &= \bar{Y} - \hat{\beta}_2 \bar{X} \\
 &= \left(\frac{25.7}{8}\right) - (0.034) \left(\frac{621}{8}\right) \\
 &= 3.2125 - 2.63925 \\
 &= 0.57325
 \end{aligned}$$

If the score is 0, lowest gpa that will earn equal to 0.57325

$$1.2 \text{ find } \bar{Y} \text{ \& } \hat{U} \text{ \& } \sum_{i=1}^n \hat{U}_i = 0$$

$$\begin{aligned}
 \text{from } \hat{U}_i &= Y_i - \hat{Y}_i \\
 &= Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)
 \end{aligned}$$

$$\text{So, } \hat{Y} = Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$$

$$\begin{aligned}
 \hat{Y}_1 &= (0.57 + 0.034(63)) = 2.712 \\
 \hat{Y}_2 &= (0.57 + 0.034(72)) = 3.018 \\
 \hat{Y}_3 &= (0.57 + 0.034(78)) = 3.222 \\
 \hat{Y}_4 &= (0.57 + 0.034(81)) = 3.324 \\
 \hat{Y}_5 &= (0.57 + 0.034(87)) = 3.528 \\
 \hat{Y}_6 &= (0.57 + 0.034(75)) = 3.12 \\
 \hat{Y}_7 &= (0.57 + 0.034(75)) = 3.12 \\
 \hat{Y}_8 &= (0.57 + 0.034(90)) = 3.63
 \end{aligned}$$

then $\hat{u}_i = y_i - \hat{y}_i$

$$\hat{u}_1 = 2.8 - 2.712 = 0.088$$

$$\hat{u}_2 = 3.4 - 3.018 = 0.382$$

$$\hat{u}_3 = 3 - 3.222 = -0.222$$

$$\hat{u}_4 = 3.5 - 3.324 = 0.176$$

$$\hat{u}_5 = 3.6 - 3.528 = 0.072$$

$$\hat{u}_6 = 3 - 3.12 = -0.12$$

$$\hat{u}_7 = 2.7 - 3.12 = -0.42$$

$$\hat{u}_8 = 3.7 - 3.63 = 0.07$$

Lastly $\sum_{i=1}^n \hat{u}_i = 0$

$$\sum_{i=1}^8 (0.088 + 0.382 - 0.222 + 0.176 + 0.072 - 0.12 - 0.42 + 0.07)$$

$$\approx 0.026 \approx 0$$

1.3 find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_1)$, $\text{var}(\hat{\beta}_2)$

$$\text{var}(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \sigma_u^2$$

$$= \frac{(0.088)^2 + (0.382)^2 + (-0.222)^2 + (0.176)^2 + (0.072)^2 + (-0.12)^2 + (-0.42)^2 + (0.07)^2}{8-2}$$

$$= \frac{0.439}{6} \approx 0.07333$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sigma_u^2 \sum x_i^2}{n \sum x_i^2} = \frac{\sigma_u^2 \sum (x_i - \bar{x})^2}{n \sum (x_i - \bar{x})^2}$$

$$= \frac{0.0733 (48717)}{8 (500.624)}$$

$$\approx \frac{3571}{4005} \approx 0.892$$

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma_u^2}{\sum x_i^2}$$

$$= \frac{0.0733}{(500.624)}$$

$$= 0.00014$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y ?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

NOTE to myself

$$SRF : \hat{\beta}_1 + \hat{\beta}_2 X_i = Y_i$$
$$Y_i - \hat{Y}_i = \hat{u}_i$$

"Practice makes Perfect."

(2.1)

$$\bar{X} = 20 \quad \bar{Y} = 9.1$$

X_i	Y_i	X_i	Y_i	X_i^2	$X_i Y_i$
10	0	-10	-9.1	100	91
12	2	-8	-7.1	64	56.8
14	5	-6	-4.1	36	24.6
16	6	-4	-3.1	16	12.4
18	7	-2	-2.1	4	4.2
22	10	2	0.9	4	1.8
24	10	4	0.9	16	3.6
26	15	6	5.9	36	35.4
28	16	8	6.9	64	55.2
30	20	10	10.9	100	109

$$\sum X^2 = 440 \quad \sum X_i Y_i = 394$$

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2} = \frac{394}{440} = 0.8954$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8954)(20) = -8.808$$

2.2. $\hat{y}_i$ & $\hat{u}_i$ & $\sum u_i = 0$

from $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

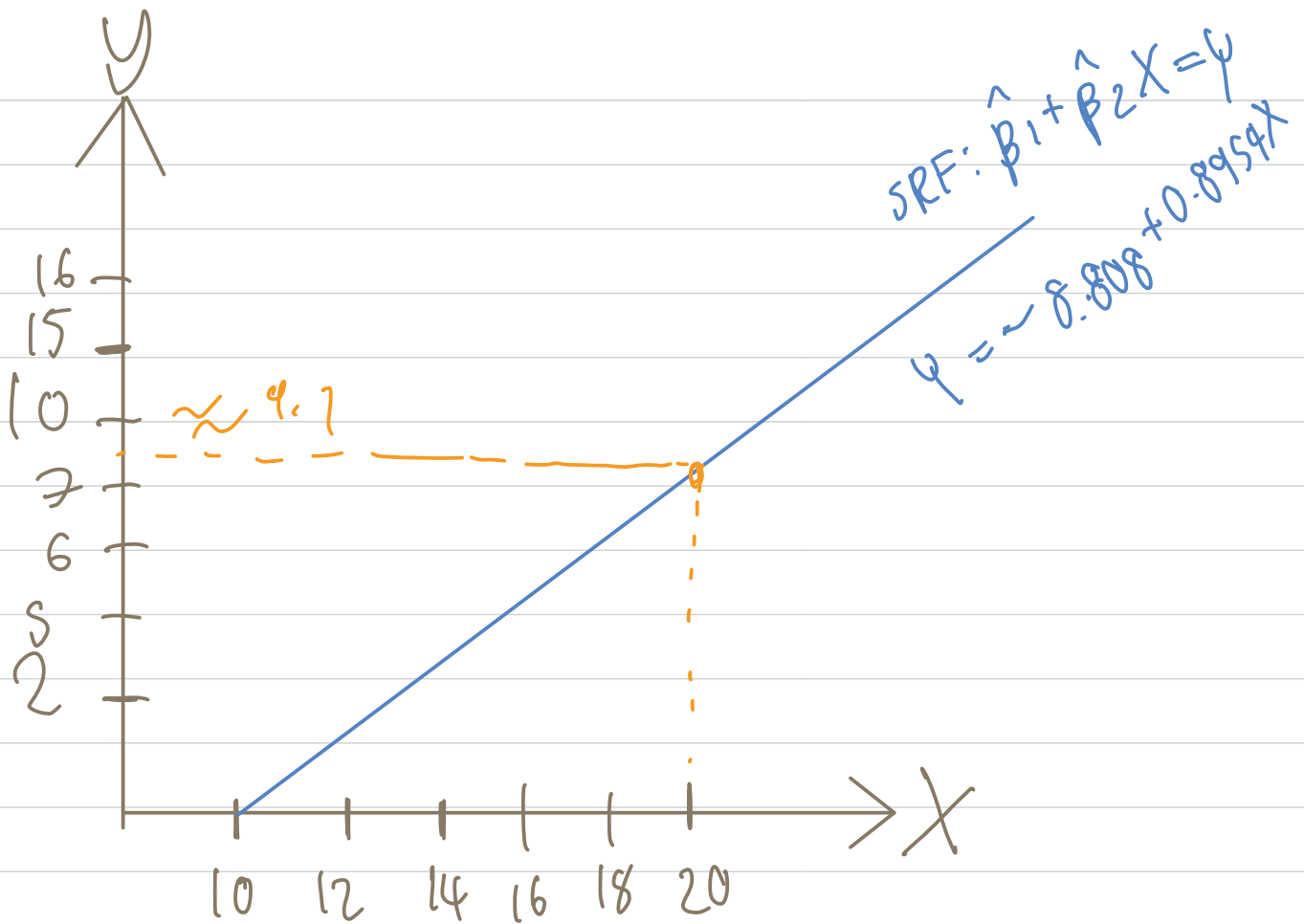
$$\begin{aligned}\hat{y}_1 &= (-8.808 + 0.8954(10)) = 0.145 \\ \hat{y}_2 &= (-8.808 + 0.8954(12)) = 1.936 \\ \hat{y}_3 &= (-8.808 + 0.8954(14)) = 3.727 \\ \hat{y}_4 &= (-8.808 + 0.8954(16)) = 5.518 \\ \hat{y}_5 &= (-8.808 + 0.8954(18)) = 7.309 \\ \hat{y}_6 &= (-8.808 + 0.8954(22)) = 10.891 \\ \hat{y}_7 &= (-8.808 + 0.8954(24)) = 12.682 \\ \hat{y}_8 &= (-8.808 + 0.8954(26)) = 14.473 \\ \hat{y}_9 &= (-8.808 + 0.8954(28)) = 16.264 \\ \hat{y}_{10} &= (-8.808 + 0.8954(30)) = 18.055\end{aligned}$$

$$\hat{u}_i = y_i - \hat{y}_i$$

$$\begin{aligned}\hat{u}_1 &= 0 - 0.145 = -0.145 \\ \hat{u}_2 &= 2 - 1.936 = 0.064 \\ \hat{u}_3 &= 5 - 3.727 = 1.273 \\ \hat{u}_4 &= 6 - 5.518 = 0.482 \\ \hat{u}_5 &= 7 - 7.309 = -0.309 \\ \hat{u}_6 &= 10 - 10.891 = -0.891 \\ \hat{u}_7 &= 10 - 12.682 = -2.682 \\ \hat{u}_8 &= 15 - 14.473 = 0.527 \\ \hat{u}_9 &= 16 - 16.264 = -0.264 \\ \hat{u}_{10} &= 20 - 18.055 = 1.945\end{aligned}$$

$$\sum_{i=1}^{10} \hat{u}_i = 0 \rightarrow -0.145 + 0.064 + 1.273 + 0.482 - 0.309 - 0.891 - 2.682 - 0.527 - 0.244 + 1.945 = 0.018 \approx 0$$

2.3



2.4 if $X_i = 18$ $\hat{Y}$ estimate

$$\begin{aligned}
 \hat{Y} &= (-8.808) + 0.8954(X) \\
 &= (-8.808) + (0.8954(18)) \\
 &= 7.31
 \end{aligned}$$

2.5 find $\text{Var}(\hat{u}_i)$, $\text{Var}(\hat{\beta}_1)$, $\text{Var}(\hat{\beta}_2)$

$$\text{Var}(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{\sigma_u^2}{2}$$

$$= \frac{14.08075}{(10-2)} = 1.760$$

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma_u^2}{\sum X_i^2}$$

$$= \frac{1.760}{440} = 0.004$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sigma_u^2 \sum X_i^2}{n \sum X_i^2}$$

$$= \frac{1.760 (440)}{10 (440)} = 1.760$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim \text{NIID}(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\begin{aligned}\hat{\beta}_1 &= \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2} \\ &= \bar{Y} - \hat{\beta}_2 \bar{X}\end{aligned}$$

proof that is unbiased estimator

$$\begin{aligned}E(\hat{\beta}_1) &= E(\bar{Y} - \hat{\beta}_2 \bar{X}) \\ &= E(\bar{Y}) - \hat{\beta}_2 E(\bar{X}) \\ &= \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2 \\ \therefore E(\hat{\beta}_1) &= (\beta_1)\end{aligned}$$