Technique of Integration for One Variable Functions

Indefinite Integral (Revisions)

$\int k\,dx = kx + C$	$\int \left[f(x) \pm g(x) \right] dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C\,,\ n \neq -1$	$\int [u(x)]^n\,du = \dfrac{u^{n+1}}{n+1} + C$
$\int e^x\,dx = e^x + C$	$\int e^u\,du = e^u + C$
$\int kf(x)\,dx = k\int f(x)\,dx$	$\int \dfrac{1}{u}\,du = \ln\lvert u\rvert + C\,,\ u \neq 0$

Calculate $\int\left(2\sqrt[5]{x^4}-7x^3+10e^x-1\right)dx$

ANS: $\frac{10}{9}x^{\frac{9}{5}}-\frac{7}{4}x^4+10e^x-x+C$

Integration by Substitution (Revisions)

Find $\int(x+5)^5\,dx$.

$$\int(x+5)^5\,dx=\frac{(x+5)^6}{6}+C$$

$$\int x^2 \left(3x^3 + 7\right)^3 dx =$$

$$\frac{(3x^3 + 7)^4}{36} + C$$

$$\int \left((2t + 1)e^{t^2 + t} dt\right)$$

$$e^{t^2 + t} + C$$

$$\int \frac{7t}{5t^2 - 6} dt =$$

$$\frac{7}{10} \ln\left|5t^2 - 6\right| + C$$

Fill in the table by specifying the substitution you may choose for each integral.

Integral	**Substitution *u***
1. $\int (3x+4)^{\frac{5}{2}} dx$	
2. $\int \left(\frac{4}{3-x} \right) dx$	
3. $\int te^{2-t^2} dt$	
4. $\int t(2+t^2)^3 dt$	
5. $\int \frac{3}{(2x-5)^4} dx$	
6. $\int x^2 e^{-x^3} dx$	
7. $\int \frac{e^t}{e^t + 1} dt$	
8. $\int \frac{t+3}{\sqrt[3]{t^2+6t+5}} dt$	

Solution

Integral	Solution
1. $\int (3x+4)^{\frac{5}{2}}\,dx$	$\frac{2}{21}(3x+4)^{\frac{7}{2}}+C$
2. $\int \left(\frac{4}{3-x}\right)dx$	$-4\ln(3-x)+C$
3. $\int te^{2-t^2}\,dt$	$-\frac{1}{2}e^{2-t^2}+C$
4. $\int t(2+t^2)^3\,dt$	$\frac{1}{8}(2+t)^4+C$
5. $\int \frac{3}{(2x-5)^4}\,dx$	$-\frac{1}{2(2x-5)^3}+C$
6. $\int x^2e^{-x^3}\,dx$	$-\frac{1}{3}e^{-x^3}+C$
7. $\int \frac{e^t}{e^t+1}\,dt$	$\ln(e^t+1)+C$
8. $\int \frac{t+3}{\sqrt[3]{t^2+6t+5}}\,dt$	$\frac{3}{4}(t^2+6t+5)^{\frac{2}{3}}+C$

$$\int\left(\frac{6x^2-11x+5}{3x-1}\right)dx=$$

$$x^2-3x+\frac{2}{3}\ln|3x-1|+C$$

Definite Integral (Revision)

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx \qquad (k \text{ is a constant})$$

$$\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(t)dt \qquad \text{Any variable will give the same result.}$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

$$\int_1^5 3x^2 dx =$$

$$\int_1^2 \left(2x^3 - 1\right)^2 \left(6x^2\right) dx =$$

$$\frac{3374}{3}$$

Integration by Parts

Integration by parts technique could become very useful if the integrand is the **product of two functions.** Starting from the product rule of differentiation,

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Integrating both sides of the above equation

$$f(x)g(x) + C_1 = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$

$$\int f(x)g'(x)dx = f(x)g(x) + C_1 - \int f'(x)g(x)dx$$

Absorbing C_1 into the constant of integration for $\int f'(x)g(x)dx$. Hence,

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx}$$

$$\int xe^{x}dx$$

$$\int \frac{3x}{\sqrt{4-x}}dx$$

$$\int \frac{x}{(5x+2)^3}\,dx$$

Find $I = \int \frac{1}{x} \cdot \ln x dx$.

Ans: $I = \frac{(\ln x)^2}{2} + C$

$$\int e^{2x}x^2dx =$$

$$\frac{1}{4}\left(2x^2 - 2x + 1\right)e^{2x} + C$$

$$\int e^{2x}x^3dx =$$

$$\frac{x^3e^{2x}}{2} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$$

Find $I = \int \ln x dx$

$$I = x[\ln x - 1] + C$$

Integration by Table

- A short table of integrals is listed on pages 458 and 459 of Hoffmann and Bradley, Calculus for Business, Economics, and the Social and Life Sciences, 8^{th} edition.

$$\int \frac{1}{x(3x-6)}\,dx$$

Apply Formula 6 $\left(\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C \right)$

$$-\frac{1}{6}\ln\left|\frac{x}{3x-6}\right| + C$$

$$\int \frac{1}{6-3x^2}\,dx$$

$$\frac{1}{6\sqrt{2}}\ln\left|\frac{\sqrt{2}+x}{\sqrt{2}-x}\right| + C$$