Lecture 4b:

Choice of Risky Assets

A Risky View of Bank's Portfolio

PORTFOLIO THEORY

- Portfolio theory works out the 'best combination' of stocks to hold in your portfolio of risky assets.
- *You like return but dislike 'risk'*
- We assume the investor is trying to 'mix' or combine stocks to get the best return relative to the overall riskiness of the chosen portfolio.
- As we shall see 'Best' has a very specific meaning.

PORTFOLIO THEORY: Quest for Optimal Risk and Return

Question 1

- What proportions of your own $100 should you put in two different stocks?
- (e.g. 'weights' = 25%, 75% which implies $25, $75)
- Different 'weights' give rise to different 'risk-return' combinations and this is the '***efficient frontier***'

Question 2

- We now allow you to borrow or lend (from the bank),
- How does this alter your choice of 'weights' and the amount you actually choose to borrow or lend?
- Latter depends on your 'love of risk'

Statistics: Some Definitions

- 'Proportions' are: $w_1 + w_2 = 1$.
- Expected Return of Portfolio

$$E(R_P) = w_1 ER_1 + w_2 ER_2$$

- Variance of Portfolio

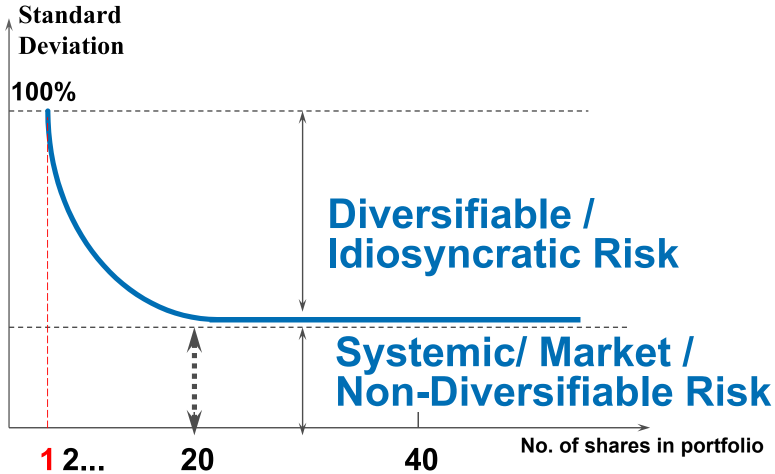
$$\sigma^2_P = w^2_1 \sigma^2_1 + w^2_2 \sigma^2_2 + 2 w_1 w_2 \sigma_{12}$$

$$\sigma^2_P = w^2_1 \sigma^2_1 + w^2_2 \sigma^2_2 + 2 w_1 w_2 (\rho \sigma_1 \sigma_2)$$

- Note $\sigma_{12} = \rho \sigma_1 \sigma_2$ from statistics where ρ is correlation

Diversification from Random Selection of Shares

Increasing the size (=n) of the portfolio (each asset has 'weight' $w_i = 1/n$)



Note: 100%=risk when holding only one asset

Can we do better than "random selection" ?

Consider 'Return' together with 'Risk'

Assumptions:

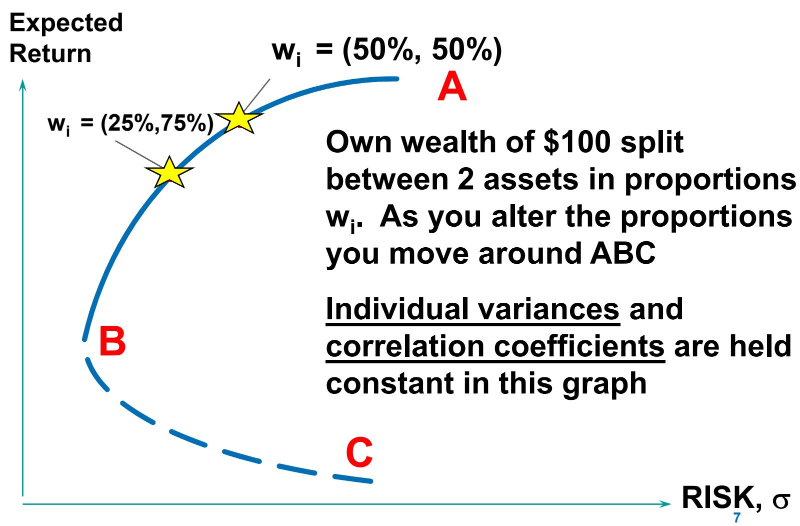
- You like return and dislike portfolio risk (variance/s.d.).
- Assume <u>everyone</u> has the same view of future returns ***ER_i*** and correlations σ_{12}, ρ_{12} .

<u>2-Stage Decision Process</u>

STAGE 1

- Use only "own wealth" of $100 and work out the risk-return combinations which are open to you by distributing this $100 in different combinations (proportions, w_i) in the *all* available stocks. This gives the **<u>"efficient frontier"</u>**

Efficient Frontier: Composite of Risky Assets



Own wealth of $100 split between 2 assets in proportions w_i. As you alter the proportions you move around ABC

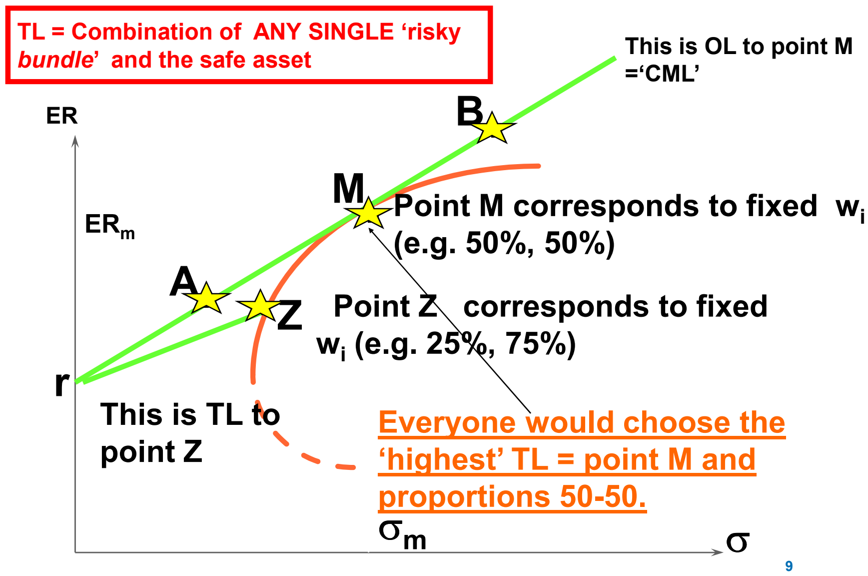
<u>Individual variances</u> and <u>correlation coefficients</u> are held constant in this graph

Borrowing and Lending, 'safe rate'= r

STAGE 2

- You are now allowed to borrow and lend at risk free rate, r while still investing in any SINGLE 'risky bundle' on the efficient frontier .
- For each SINGLE risky bundle, this gives a new set of risk-return combinations ="transformation line, TL" ~ which is a 'straight line'
- Each risky asset bundle has its 'own' TL
- You can move along this TL by altering your borrowing/lending

Opportunity Line(s)/Locus



Capital Market Lines: Some Properties

NO BORROWING OR LENDING (ONLY USE OWN $100)

- You are then at point M

LEND SOME OF $100 (e.g lend $90 at r and $10 in risky bundle)

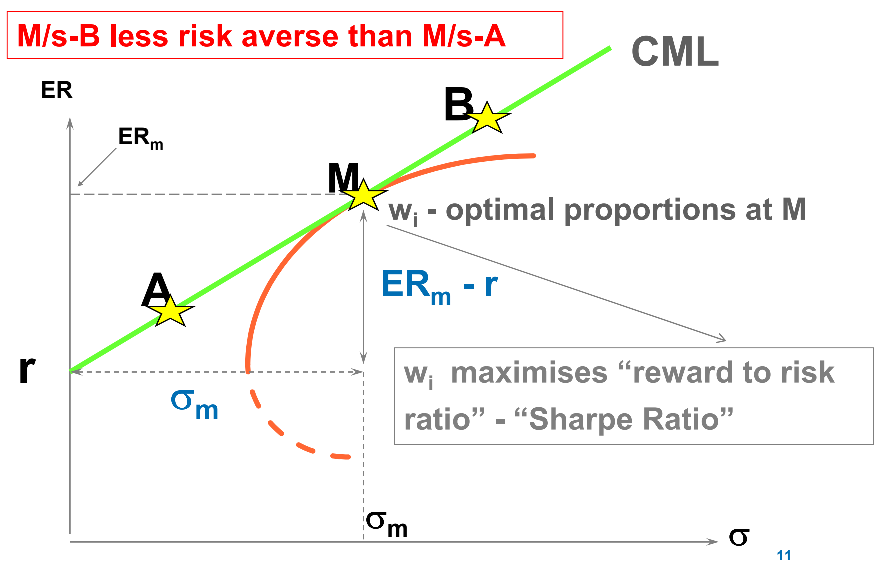
- You are then at point like A

BORROW (say $50) and put all $150 in risky assets

- You are then at point like B

- Surprisingly the proportions at A and B are the same as at M (i.e. 50%,50%) - but the $ amounts are NOT the same! (Tricky !)

CML and Market Portfolio (M)



Market Portfolio = Passive Investment Strategy

- Optimal w_i maximizes "reward to risk ratio" - "Sharpe Ratio".

- At the time you choose your optimal proportions you expect to obtain a '*reward to risk ratio*' of

$$SR = (ER_m - r) / \sigma_m$$

- Note that both M/s-A and M/s-B have the same Sharpe ratio

- Of course the *realized* Sharpe ratio could be very different to what you envisaged (because your forecasts turned out to be poor).

Capital Asset Pricing Model (CAPM)

- So if you want $ER_m - r$ additional market return, you must accept σ_m additional unit of market risk.
- However, if you have a tip that stock *i* will perform great, and you want $ER_i - r$ additional return. How much more risk must you accept?
- CAPM postulates that $\frac{E(R_i) - r}{E(R_m) - r} \equiv \beta_i$

  where each stock has its own "beta", β_i.
- In other words, $R_i = r + \beta_i (R_m - r) + \varepsilon_i$
- We know from univariate OLS that

$$\beta_i = \frac{\text{cov}(R_i, R_m)}{\sigma_m^2} = \frac{\sigma_{im}}{\sigma_m} \times \frac{1}{\sigma_m}$$

Arbitrage Pricing Theory (APT)

- CAPM views that all risks can be diversified away except for the *systemic risk* from market movement (σ_m).
- APT views that there are *more than one systemic risk* that cannot be diversified away. So to hold an asset you must accept all these risks.

$$R_i = \beta_i^1(\text{factor } 1) + \beta_i^2(\text{factor } 2) + \cdots + \beta_i^k(\text{factor } k) + \epsilon_i$$

  where these factors could be default risk, inflation risk, GDP growth, term structure of yield curve, country risk etc.
- In other words, the pricing equation can be thought of as

$$E(R_i) = R_f + \beta_i^1[E(R_{\text{factor } 1}) - R_f] + \cdots + \beta_i^k[E(R_{\text{factor } k}) - R_f]$$

- Finance people tend to like the model. I don't, since it's an "*anything goes*" model.

BANKING BEHAVIOR: PORTFOLIO OF RISKY LOANS

Viewing Through A Bank's Eye

- Bank earns interest from a "risk-free" asset (holding government securities, or inter-bank deposits etc.)
- All remaining assets are risky.
- These assets are made up of low risk commercial paper and high risk loans of varying risk characteristics.
- Portfolio theory tells us how to combine assets to define an efficiency frontier.

Algebraic Model

- Like before, *ER* is expected returns on a composite of risky assets.
- We can construct Bank's *Opportunity locus* as follows:

$$E(\tilde{R}) = wE(R) + (1-w)r$$

$$\sigma_{\tilde{R}}^2 = w^2 \sigma_R^2$$

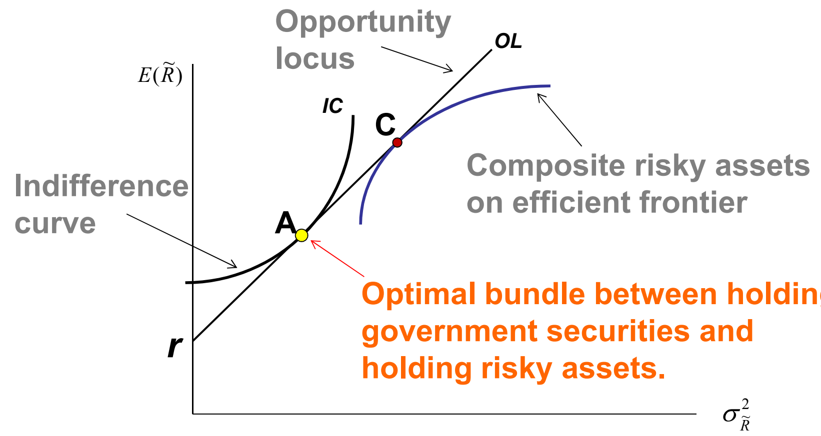
$$E(\tilde{R}) = \left(\frac{E(R) - r}{w} \right) w^2 + r$$

$$E(\tilde{R}) = \left(\frac{E(R) - r}{w \sigma_R^2} \right) \sigma_{\tilde{R}}^2 + r$$

$$E(\tilde{R}) = \theta \sigma_{\tilde{R}}^2 + r$$

Portfolio Allocation

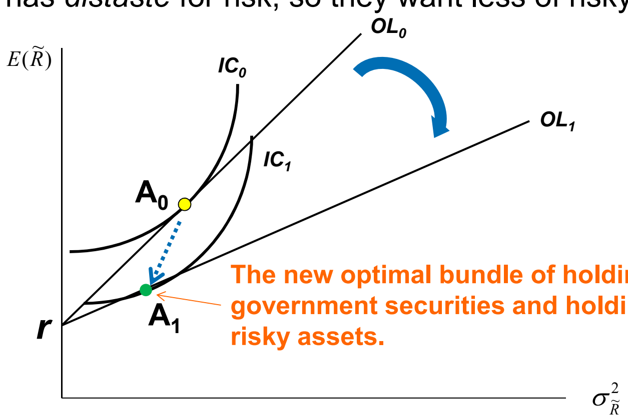
- **Depending on *risk appetite/preference* of the bank, banks may choose to go *all risky loans* (point *C*) or somewhere in between.**



where the bank having preference: $E(U(\pi)) = E(\pi) - \frac{1}{2}\gamma\sigma_{\pi}^2$

Increase in Riskiness of Loans

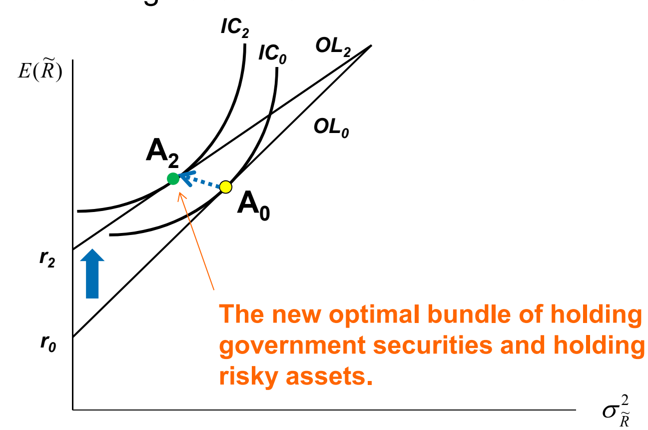
- For any *E(R)*, σ_R increases. That is, for the same expected return, risk increases.
- Bank has *distaste* for risk, so they want less of risky assets.



Take Away: When systemic risk increases, expect more securities/liquidity holding, and less of risky loans.

Increase in (Securities') Interest Rate

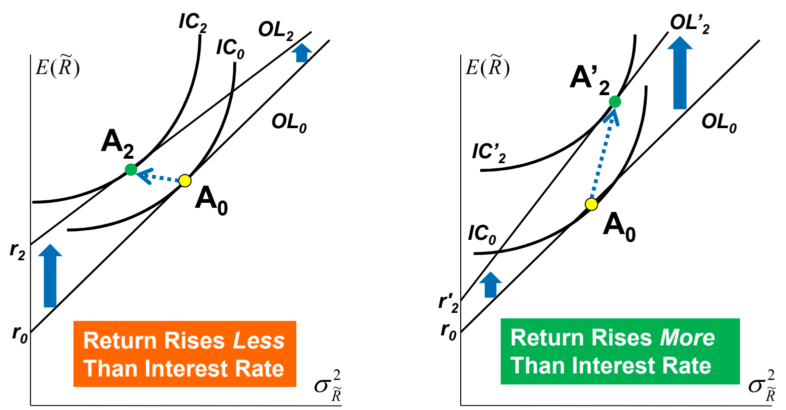
- When interest rate increases (*ceteris paribus*), the relative return of holding securities versus loans increase.



Take Away: When policy rate or interest rate on government securities rises, bank *may* shift towards holding more securities.

Increase in ***Interest Rates*** Together

- When interest rates move up together, but to varying degree.
- In that case, it depends, which went up more.



Take Away: If interest rates move together, return on loan portfolio will also move with interest rate. Bank may do either.

Issues with Portfolio Model for Banks

- Results are sensitive to utility function of bank's profit.
- How do we find bank's utility function? Is it set by shareholders or management?
- Cannot accommodate the customer loan relations, a **tenet** in banking relationship and businesses.
- Returns on loans are unlikely to look like stock returns, nor are risks on loans look like stock's standard deviations
  - For example, what do you think the risk of a default on loan should look like? What about delinquency?

Benefits from Portfolio Model for Banks

- Easiest model to understand risk-return trade-off.
- Intuitions can be combined with previous models to get broader understand of banks.