

- ① Suppose the demand curve is $Q(p) = p^\epsilon$, what is the elasticity of demand? If marginal cost is \$1 and $\epsilon = -2$, what is the profit-maximizing price?

$$\text{Since } Q = p^{-2}; \quad \frac{\Delta Q}{\Delta P} = -2p^{-3}$$

$$\begin{aligned} \text{Elasticity of Demand ; } \epsilon_{Q,P} &= \frac{\% \Delta Q}{\% \Delta P} \\ &= \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \\ &= -2p^{-3} \times \left(\frac{p}{p^{-2}} \right) \\ &= -2p^{-3} p^3 \\ &= -2 \quad \# \end{aligned}$$

$$\begin{aligned} \text{Substitute ; } Q &= \frac{1}{4} \\ p^{-2} &= \frac{1}{4} \\ p &= 2 \quad \# \end{aligned}$$

Maximum Profit ; $MR = MC$

$$\frac{1}{2} Q^{-\frac{1}{2}} = 1$$

$$Q^{-\frac{1}{2}} = 2$$

$$Q = \frac{1}{4}$$

$$\begin{aligned} \frac{dQ}{dP} \cdot \frac{P}{Q} &= -2 \cdot \frac{2}{\frac{1}{4}} \\ &= -16 \end{aligned}$$

$$\begin{aligned} MR &= \frac{dTR}{dQ} \\ TR &= P \cdot Q \\ &= Q^{-\frac{1}{2}} \cdot Q \\ &= Q^{\frac{1}{2}} \\ MR &= \frac{1}{2} Q^{-\frac{1}{2}} \end{aligned}$$

- ② Suppose the demand curve for corn is $Q(p) = 10 - p$. Suppose that one firm owns all five units of corn in the world and has zero marginal cost. Does a monopoly sell less output than would be sold in a competitive market in which 100 firms each own 0.05 units?

$$Q = 10 - P$$

$$P = 10 - Q$$

$$TR = P \cdot Q$$

$$TR = (10 - Q)Q$$

$$TR = -Q^2 + 10Q$$

$$MR = \frac{dTR}{dQ} = -2Q + 10$$

$$MR = MC$$

$$-2Q + 10 = 0$$

$$2Q = 10$$

$$Q = 5$$

At $Q=5$, should be sold by Monopoly

Number of firm = 100

$$\begin{aligned} \text{Total output sold by competitive firm} &= (0.05)(100) \\ &= 5 \end{aligned}$$

\therefore Both market sold at the same level of output.

Problem 8, Chapter 13 in Church and Ware (2000) (Church and Ware (2000) is an e-book and is available online).

8. Output is homogenous and the demand curve is

$$P = 448 - Q.$$

There are two firms with identical costs given by $C = q_i^2$ where q_i is the production of firm i . The marginal cost of firm i is $MC_i(q_i) = 2q_i$.

- (a) Find the Cournot equilibrium firm outputs.
 (b) Find the Stackelberg equilibrium firm outputs.

a) Cournot Equilibrium

$$\begin{aligned} \text{Firm 1 ; } \pi_1 &= (P \cdot q_1) - q_1^2 \\ &= (448 - q_1 - q_2) q_1 - q_1^2 \\ &= 448q_1 - q_1^2 - q_1q_2 - q_1^2 \\ \frac{\partial \pi_1}{\partial q_1} &= 448 - 2q_1 - q_2 - 2q_1 = 0 \\ & \qquad \qquad \qquad q_2 = 448 - 4q_1 \end{aligned}$$

$$\begin{aligned} \text{Firm 2 ; } \pi_2 &= P q_2 - q_2^2 \\ \pi &= (448 - q_1 - q_2) q_2 - q_2^2 \\ \pi &= 448q_2 - q_2^2 - q_1q_2 - q_2^2 \\ \frac{\partial \pi}{\partial q_2} &= 448 - 2q_2 - q_1 - 2q_2 = 0 \\ & \qquad \qquad \qquad -4q_2 = q_1 - 448 \\ & \qquad \qquad \qquad q_1 = -4q_2 + 448 \end{aligned}$$

$$\begin{aligned} q_1 &= -4(-4q_2 + 448) + 448 \\ q_1 &= 16q_2 - 1792 + 448 \\ q_1 &= 16q_2 - 1344 \\ -15q_2 &= -1344 \\ q_2 &= 89.6 \end{aligned}$$

$$\begin{aligned} q_2 &= -4(89.6) + 448 \\ q_2 &= 89.6 \end{aligned}$$

∴ Both firm produced at the same level of output which is 89.6 units

b) Stackelberg Equilibrium

→ Assume firm 1 is a leader

$$\begin{aligned} \text{Firm 2 ; } \pi &= P \cdot q_2 - q_2^2 \\ \pi &= (448 - q_1 - q_2) q_2 - q_2^2 \\ \pi &= 448q_2 - q_2^2 - q_1q_2 - q_2^2 \\ \frac{\partial \pi}{\partial q_2} &= 448 - 2q_2 - q_1 - 2q_2 = 0 \\ & \qquad \qquad \qquad -4q_2 = q_1 - 448 \\ & \qquad \qquad \qquad 448 - 4q_2 = q_1 \\ & \qquad \qquad \qquad q_2 = -\frac{q_1 - 448}{4} \end{aligned}$$

$$\begin{aligned} \text{Firm 1 ; } \pi &= (448 - q_1 - 112 + \frac{1}{4} q_1) q_1 - q_1^2 \\ &= (336 - \frac{3}{4} q_1) q_1 - q_1^2 \\ &= 336q_1 - \frac{3}{4} q_1^2 - q_1^2 \\ &= 336q_1 - \frac{7}{4} q_1^2 \\ \frac{\partial \pi}{\partial q_1} &= 336 - \frac{7}{2} q_1 = 0 \\ q_1 &= 96 \\ q_2 &= 112 - \frac{96}{4} \\ q_2 &= 88 \end{aligned}$$

- 4 (write about 0.5 page) Find 1 example of an industry that has a dominant firm. Describe what this industry is, which firm is the dominant firm, which firms are fringe firms (name the ones that you

According to a beer industry, Snow beer is the best selling beer in the world. It became a dominator of other brands of beer. The Chinese beer Snow Beer originally comes from Shenyang, Liaoning province from China. It was first founded in 1993 making it a relative young pup in the Chinese beer world. This makes Snow become a price taker. They've set not that high prices against other competitors but still maintain the profits.

For the fringe firm, I would say Heineken. Eventhough it is one of the top 10 beer firms but is still dominated by the Snow beer. Heineken has been exploiting the African region to increase their profits. The company recently acquired the South African Distell Group. They have to set the price similarly to the dominant firm in order to maintain their business.