# EE320 (2/2013)

## INTRODUCTORY MATHEMATICAL ECONOMICS

EQUILIBRIUM and COMPARATIVE STATIC ANALYSIS

# Topics

- Meaning of Equilibrium
- Partial Market Equilibrium: Linear Models
  - Linear models in economics
  - Break-even analysis
  - Individual and market demand & supply
  - Partial market equilibrium
- Partial Market Equilibrium: Nonlinear Models
- General Market Equilibrium
- Applications
  - Excise tax and market equilibrium
  - Macroeconomic models
- Comparative statics

# Meaning of Equilibrium

- An equilibrium is *"a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute."*

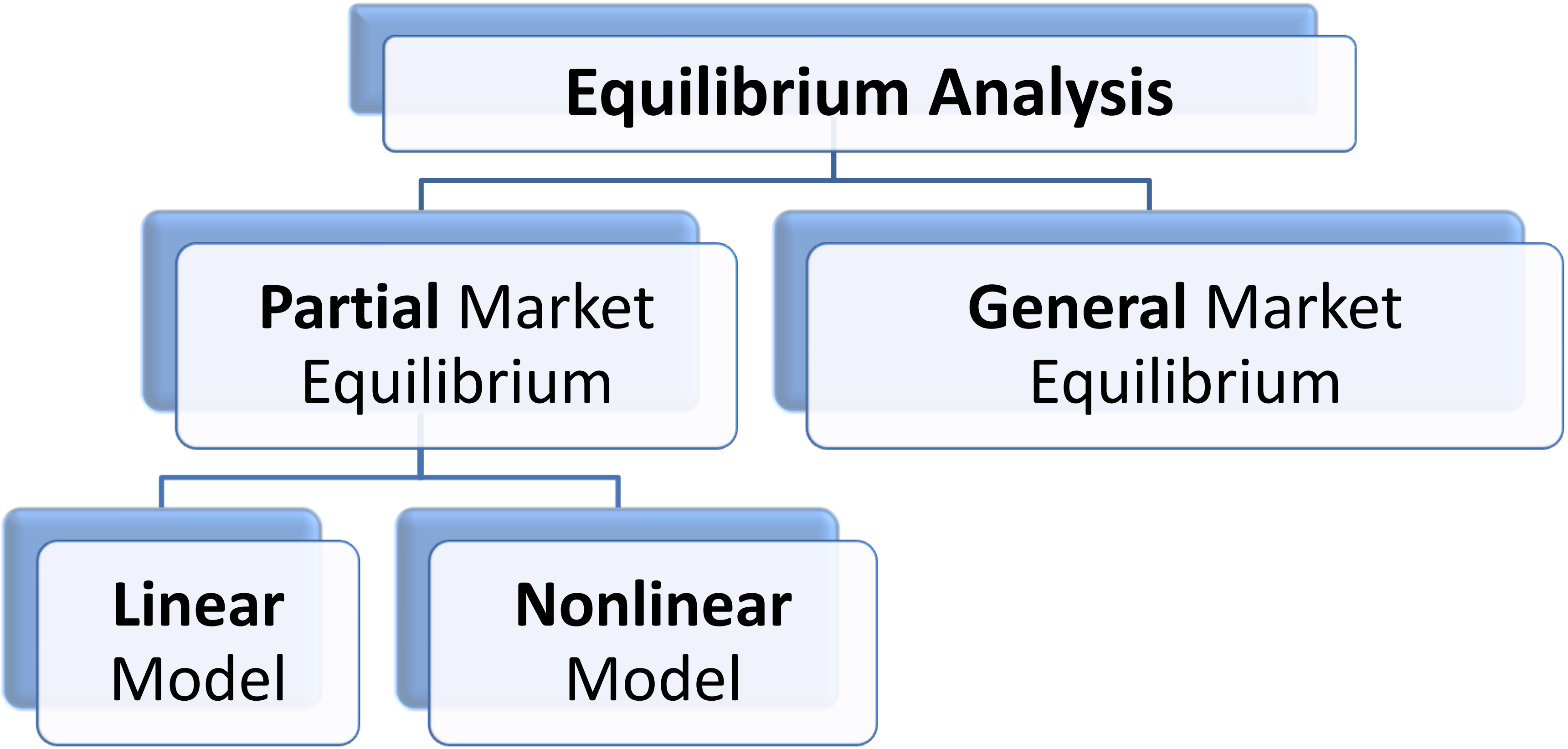
  - Machlup, F. (1958).

- In essence, an equilibrium for a specified model is a situation characterized by a lack of tendency to change.
- Due to the lack of tendency to change, the analysis of equilibrium is referred to as statics.

# Equilibrium Analysis

- However, an equilibrium does *not* necessarily constitute a desirable or an ideal state of affairs; i.e., there are both good and bad equilibria.
- "Desirable" equilibrium is referred to as "goal" equilibrium (to be discussed in optimization problems).
- "Non-goal" equilibrium, which is discussed in this topic, is a result from a process of interaction and adjustment of economic forces.
  - Equilibrium in a market under given demand and supply
  - Equilibrium of national income given conditions of consumption and investment

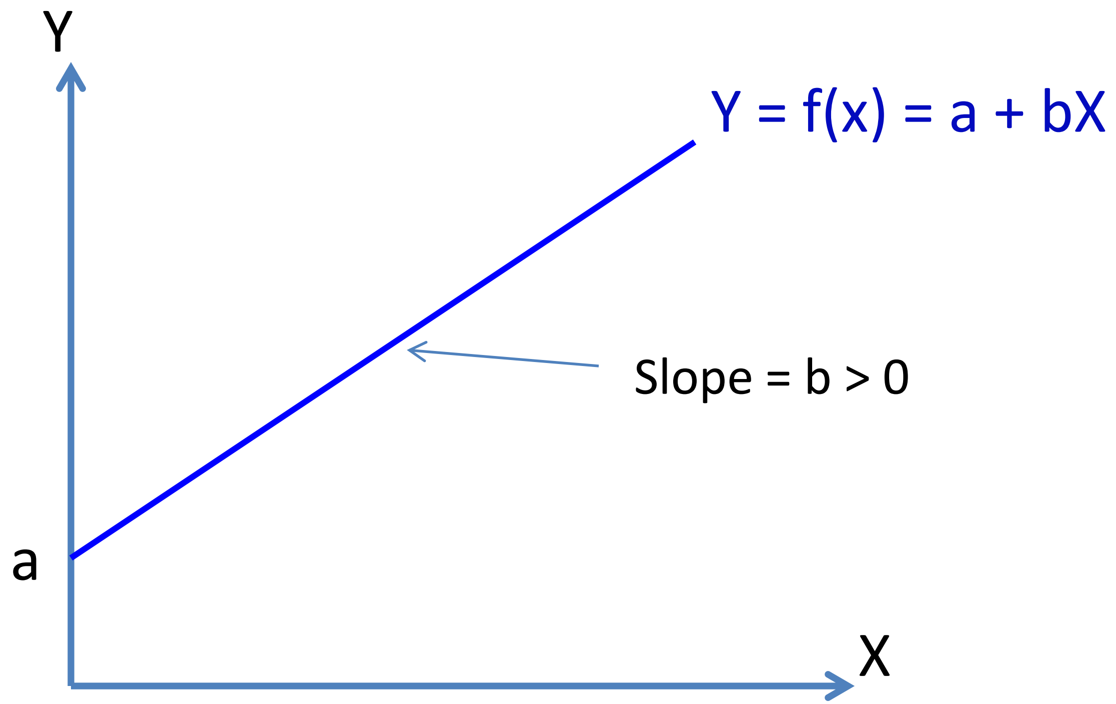
# Types of Market Equilibrium Analysis



# PARTIAL MARKET EQUILIBRIUM:
# LINEAR MODELS

# Linear Models in Economics

- General form of a linear model: Y = a + bX



# Example: Cost Function (1)

- Theory: Cost of a firm consists of fixed costs and variable costs.
- In a mathematical model:

$$TC = TFC + TVC$$

$$TC(Q) = TFC + TVC(Q) \rightarrow TC = f(Q)$$

- Assumption: Total cost function has a linear relationship.

$$TC(Q) = f(Q) = TFC + cQ,$$

where c is a constant.

- With this theoretical model and given data, we can approximate the cost function.
  - Example: Suppose TC = \$2,000 when Q = 40 units, and TC = \$2,800 when Q =80 units. What is the cost function? What is the total cost if Q =100 units?

# Example: Cost Function (2)

- Answer: 1) TC = 1200 + 20Q.

  2) When Q = 100, TC = $3,200

# Breakeven Analysis

- The breakeven level for a product is a level at which total revenue (TR) equals total costs (TC); i.e., normal profit (π) is 0.
- Theoretical model:

$$\pi \equiv TR - TC = 0$$

$$\text{where } TR = PQ$$

$$TC = a + cQ$$

- Empirical model:

Suppose the 2nd-year BE students want to raise a fund by selling whistles. The price of a whistle purchased from a factory is 15 baht, and the cost of travelling to the factory and other fixed cost is 1000 baht. The students can sell whistles at 25 baht per piece. How many whistles should these students sell in order to breakeven?

# Illustration of a Breakeven Analysis

# Individual and Market Demand

- Theory - Demand for a good is determined by many factors:

  $Q^x_d = f(P^x, P^y, I, \text{taste, etc.})$

- Individual demand for a good as a function of its price:

  $Q_d = f(P)$, ceteris paribus

  $Q_d = a - bP$, where $b > 0$.

- Market demand:

  $Q_{Md} = Q_{d1} + Q_{d2} + \ldots + Q_{dn}$

  (Suppose that there are n buyers.)

- Example:

  Suppose $Q_{d1} = 250 – P$ and $Q_{d2} = 500 – 2P$. What does a market demand function look like?

# Illustration of a Market Demand Function

# Individual and Market Supply

- Individual supply of a good as a function of its price:

$$Q_s = f(P), \quad \text{ceteris paribus}$$

$$Q_s = c + dP, \quad \text{where } d > 0.$$

- Market supply:

$$Q_{Ms} = Q_{s1} + Q_{s2} + \ldots + Q_{sq}$$

(Suppose that there are q sellers.)

- Example:

Suppose that there are 2 sellers, and each seller *i* has the same supply function: $Q_{si} = -50 + 2P$. What does the market supply function look like?

# Illustration of a Market Supply Function

# Some Notations:

## System of Equations /Simultaneous Equations

- Systems of equations are two or more equations containing common multiple variables.

<u>Example</u>: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

  - Note that any value of P makes this system correct.

- Simultaneous equations are two or more equations in which the common variables are jointly determined.

<u>Example</u>: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

$Q_d = Q_s$ - (3)

  - This system contains simultaneous equations because there is a unique value of P* that must be true in all equations.
  - Equation (3) specifies the equilibrium condition in the market.

# Partial Market Equilibrium: A Linear Model

- Partial market equilibrium analysis deals with price determination in an isolated market. That is, only one commodity is being considered.
- Equilibrium occurs in the market *if and only* if the excess demand is zero (i.e. if and only if the market is clear).
- Model:
  - One conditional equation: $Q_d = Q_s$.
  - Two behavioral equations:

$$Q_d = a - bP, \quad (a, b > 0)$$
$$Q_s = -c + dP, \quad (c, d > 0)$$

  - *a, b, c,* and *d* are parameters, whereas $Q_d$, $Q_s$, and $P$ are endogenous variables.
  - ➢ Solve the model for the 3 endogenous variables

# Solution by Elimination of Variables

- Show that $P^* = \frac{a+c}{b+d}$ and $Q^* = \frac{ad-bc}{b+d}$.

# PARTIAL MARKET EQUILIBRIUM:
# NONLINEAR MODELS

# Example: A Nonlinear Model

- Suppose that a market demand function is nonlinear. The model can be rewritten as:

$$Q_d = a - bP^2, \quad (a, b > 0)$$
$$Q_s = -c + dP, \quad (c, d > 0)$$
$$Q_d = Q_s.$$

➔ $bP^2 + dP - a - c = 0$

- Numerical example: $Q_d = 4 - P^2$ and $Q_s = 4P - 1$. What are the equilibrium price and quantity in this model?

# Graphic Solution of a Nonlinear Model

- $D = \{(P, Q) | Q = 4 - P^2 \}$
- $S = \{(P,Q) | Q = 4P - 1\}$

# Quadratic Formula

- Given a quadratic equation of the form

$$aX^2 + bX + c = 0,$$

there are 2 roots, which can be obtained from the quadratic formula:

$$X_1^*, X_2^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$, $X^*_1 \neq X^*_2$.

If $b^2 - 4ac = 0$, $X^*_1 = X^*_2$.

If $b^2 - 4ac > 0$, no real value root exists.

- Own-practice: Use the process called "completing the square" to show the 2 roots of a quadratic equation.
  - Hint: Start by dividing both sides by a.

# Quadratic Formula

# GENERAL MARKET EQUILIBRIUM

EXCISE TAX AND MARKET EQUILIBRIUM

# General Market Equilibrium

- General market equilibrium deals with simultaneous equilibria in a group of related markets, e.g. input and output markets.
  - Ex: A change in the price of an input good may affect the price of the output good, with a possible subsequent effect on the price of the input good.
- The equilibrium condition of an *n*-commodity market model involves *n* equations:

$$E_i \equiv Q_{di} - Q_{si} = 0 \qquad (i = 1, 2, \ldots, n)$$

$$Q_{di} = Q_{di}(P_1, P_2, \ldots, P_n)$$

$$Q_{si} = Q_{si}(P_1, P_2, \ldots, P_n)$$

  - If a solution exists, there will be a set of price $P^*_i$ and quantity $Q^*_i$ such that all *n* equations in the equilibrium conditions are simultaneously satisfied.

# 2-Commodity Market Model

- Example: Suppose that the demand and supply functions of a two-commodity market model are as follows:

$$Q_{d1} = 8 - 3P_1 + P_2 \quad \text{and} \quad Q_{s1} = 2 + 2P_1$$
$$Q_{d2} = 12 + 5P_1 - 2P_2 \quad \text{and} \quad Q_{s2} = 2 + 3P_2$$

Find $P^*_i$ and $Q^*_i$ (i = 1, 2).

Answer:

$$(Q_1^*, P_1^*) = (6, 2)$$
$$(Q_2^*, P_2^*) = (14, 4)$$

# Self-Practice:

# 2-Commodity Market Model (General case)

- Market for good 1:

$$Q_{d1} - Q_{s1} = 0$$
$$Q_{d1} = a_0 + a_1P_1 + a_2P_2$$
$$Q_{s1} = b_0 + b_1P_1 + b_2P_2$$

- Market for good 2:

$$Q_{d2} - Q_{s2} = 0$$
$$Q_{d2} = \alpha_0 + \alpha_1P_1 + \alpha_2P_2$$
$$Q_{d2} = \beta_0 + \beta_1P_1 + \beta_2P_2$$

- Let $c_i \equiv a_i - b_i$ and $\gamma_i \equiv \alpha_i - \beta_i$

➢ Show that $P_1^* = \dfrac{c_2\gamma_0 - c_0\gamma_2}{c_1\gamma_2 - c_2\gamma_1}$ and $P_2^* = \dfrac{c_0\gamma_1 - c_1\gamma_0}{c_1\gamma_2 - c_2\gamma_1}$

Note: See complete derivation in Chiang&Wainwright, p. 41-42.

# Self-practice (cont'd)

# APPLICATIONS OF MARKET EQUILIBRIUM

# Excise Tax and Market Equilibrium

- Excise tax (a.k.a. excise duty) is a type of tax charged on goods produced within the country, as opposed to custom duties which are charged on goods from outside the country.
- Two types of excise tax:
  1. Specific tax – tax imposed based on physical units (e.g. pieces, gallons, or individual items) of the taxed item good.
     - $P^t = P + T$, where P is price without tax.
  2. Ad-valorem tax – tax imposed based on the monetary value of the taxed item. Suppose that a *t*% ad-valorem tax is imposed.
     - $P^t = (1+t)P$ , where P is price without tax.

# Specific Tax (imposed on consumers)

- Suppose the government imposes a *T* baht specific tax on a good. Find the P*, Q*, $P^t$ (market price), and tax revenue.

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D' = P + T = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b(P + T) = -c + dP$$

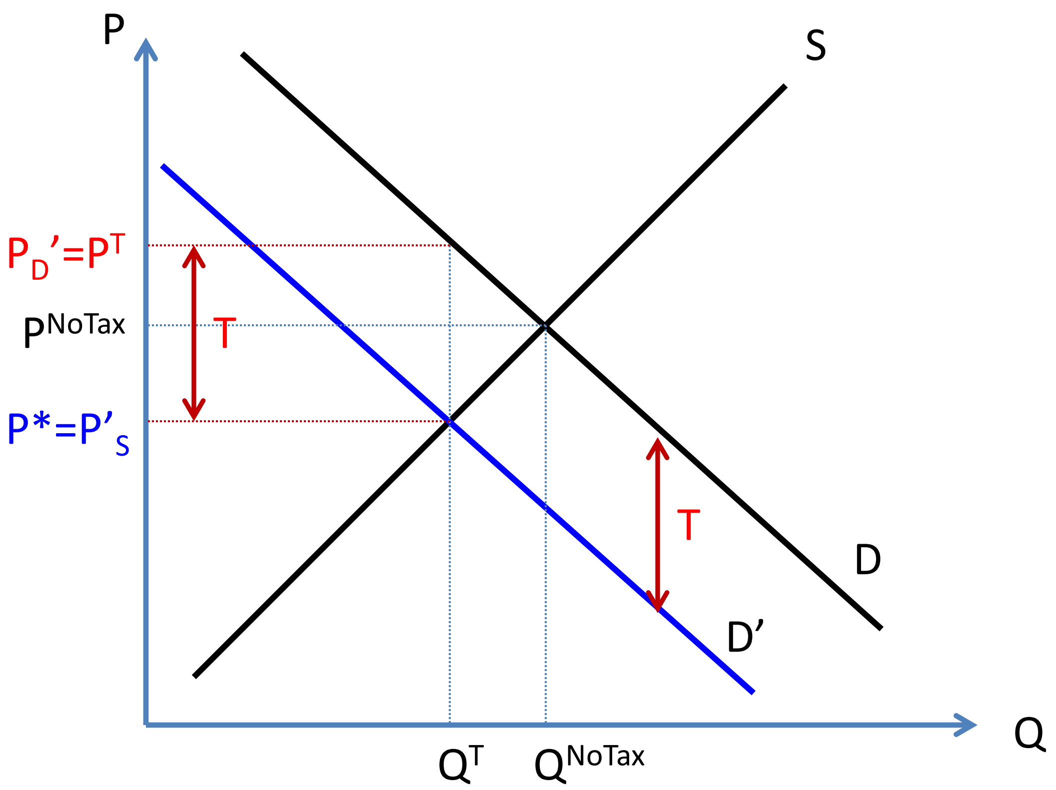
➔ $P^* = \dfrac{a + c - bT}{b + d} = P_S'$ : Price received by producer

➔ $P^t = P + T = \dfrac{a + c + dT}{b + d} = P_D'$ : Price (tax included) paid by consumer

# Specific Tax (imposed on consumers) – Cont'd

- Consumer tax burden: $P^{t} - P^{NoTax} = \dfrac{dT}{b+d}$
- Producer tax burden: $P^{NoTax} - P_{S}^{'} = -\dfrac{bT}{b+d}$
- Equilibrium quantity after tax: $Q^{*} = \dfrac{ad - bc - bdT}{b+d}$
- Tax revenue: $Tax = TQ^{*} = \dfrac{(ad - bc)T - bdT^{2}}{b+d}$

# Graph: Specific Tax (imposed on consumers)



# Specific Tax (imposed on producers)

- Suppose the government imposes a *T* baht specific tax on a good. Find the P*, Q*, $P^t$ (price excluding tax received by producer), and tax revenue.

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_S' = P - T = P^t$ and $P_D = P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d(P - T)$$

➔ $$P^* = \frac{a + c + dT}{b + d} = P_D'$$ : Price (tax included) paid by consumer

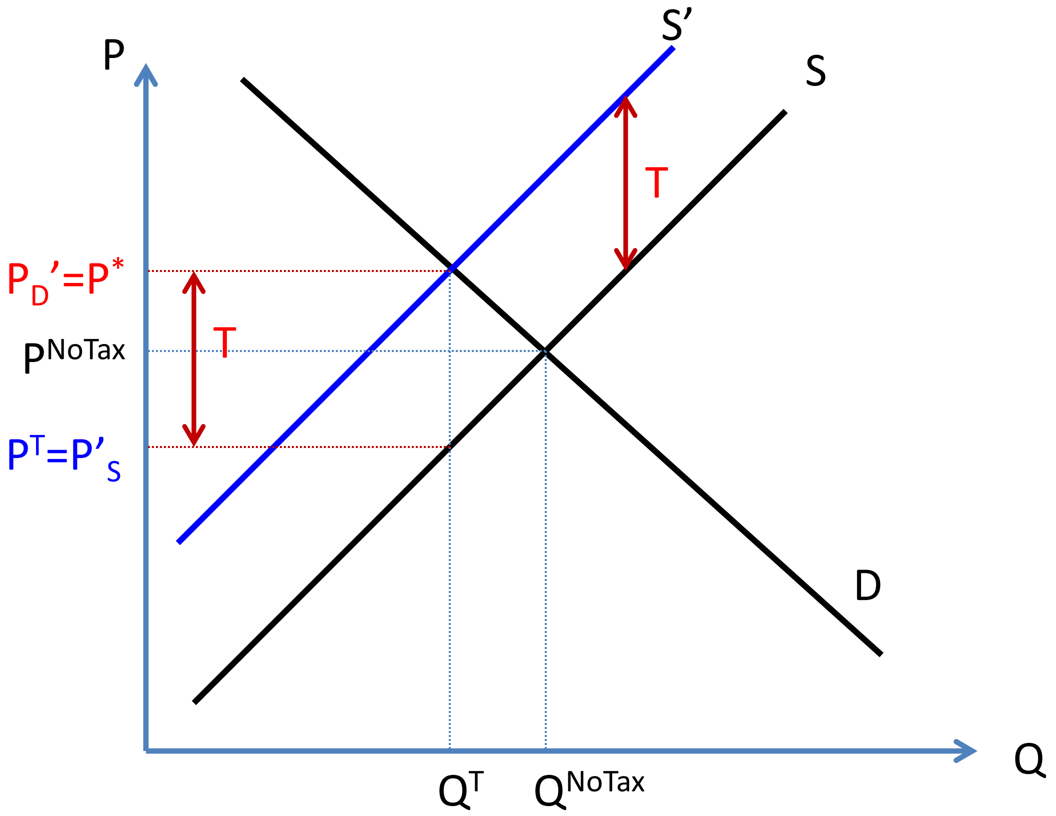
➔ $$P^T = P_D' - T = \frac{a + c - bT}{b + d} = P_S'$$ : Price received by producer

# Specific Tax (imposed on producers) – Cont'd

- Consumer tax burden: $P_D^{'} - P^{NoTax} = \frac{dT}{b+d}$
- Producer tax burden: $P^{NoTax} - P_S^{'} = -\frac{bT}{b+d}$
- Equilibrium quantity after tax: $Q^* = \frac{ad - bc - bdT}{b+d}$
- Tax revenue: $Tax = TQ^* = \frac{(ad-bc)T - bdT^2}{b+d}$

➢ The results (PD', PS', Q*, Tax) are exactly the same as regardless of whom the government imposes the specific tax on.

# Graph: Specific Tax (imposed on producers)



# Ad-Valorem Tax (imposed on consumers)

- Suppose the government imposes a *t%* ad-valorem tax on a good. Find the P*, Q*, and $P^t$ (market price).

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D' = P + tP = (1+t)P = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b[(1+t)P] = -c + dP$$

➔ $P^* = \dfrac{a+c}{b(1+t)+d} = P_S'$ : Price received by producer

➔ $P^t = P_S' + T = \dfrac{(1+t)(a+c)}{b(1+t)+d} = P_D'$ : Price (tax included) paid by consumer

➔ Equilibrium quantity after tax: $Q^* = \dfrac{ad-(1+t)bc}{b(1+t)+d}$

# Ad-Valorem Tax (imposed on producers)

- Suppose the government imposes a *t%* ad-valorem tax on a good. Find the P*, Q*, and $P^t$ (price excluding tax received by producer).

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D = P$ and $P_S' = P - tP = (1-t)P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d[(1-t)P]$$

➔ $P^* = \dfrac{(a+c)}{b+d(1-t)} = P_D'$ : Price paid by consumer

➔ $P^t = P_D' - T = \dfrac{(1-t)(a+c)}{b+d(1-t)} = P_S'$ : Price received by producer

➔ Equilibrium quantity after tax: $Q^* = \dfrac{ad(1-t) - bc}{b+d(1-t)}$

# MARKET EQUILIBRIUM

MARKET EQUILIBRIUM IN NATIONAL-INCOME ANALYSIS

# Simplest Keynesian National-Income Model

- Consider a *closed* economy (i.e. no trade).
- The equilibrium is: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY$ $(a > 0, 0 < b < 1)$

➢ At equilibrium, $Y = a + bY + I_0 + G_0$

$$\rightarrow \quad Y^* = \frac{a + I_0 + G_0}{1 - b}$$

➢ Corresponding consumption: $C^* = a + bY^*$

$$\rightarrow \quad C^* = \frac{a + b(I_0 + G_0)}{1 - b}$$

# Closed Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $Y_d = Y - T$

  $T = tY \quad (0 < t < 1)$

➢ At equilibrium, $Y = a + bY_d + I_0 + G_0$

$Y = a + b(1\text{-}t)Y + I_0 + G_0$

➔ $$Y^* = \frac{a + I_0 + G_0}{1 - b(1 - t)}$$

➔ $$C^* = \frac{a + b(1 - t)(I_0 + G_0)}{1 - b(1 - t)}$$

# Open Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 + X_0 - M = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $T = tY \quad (0 < t < 1)$

  $M = mY$

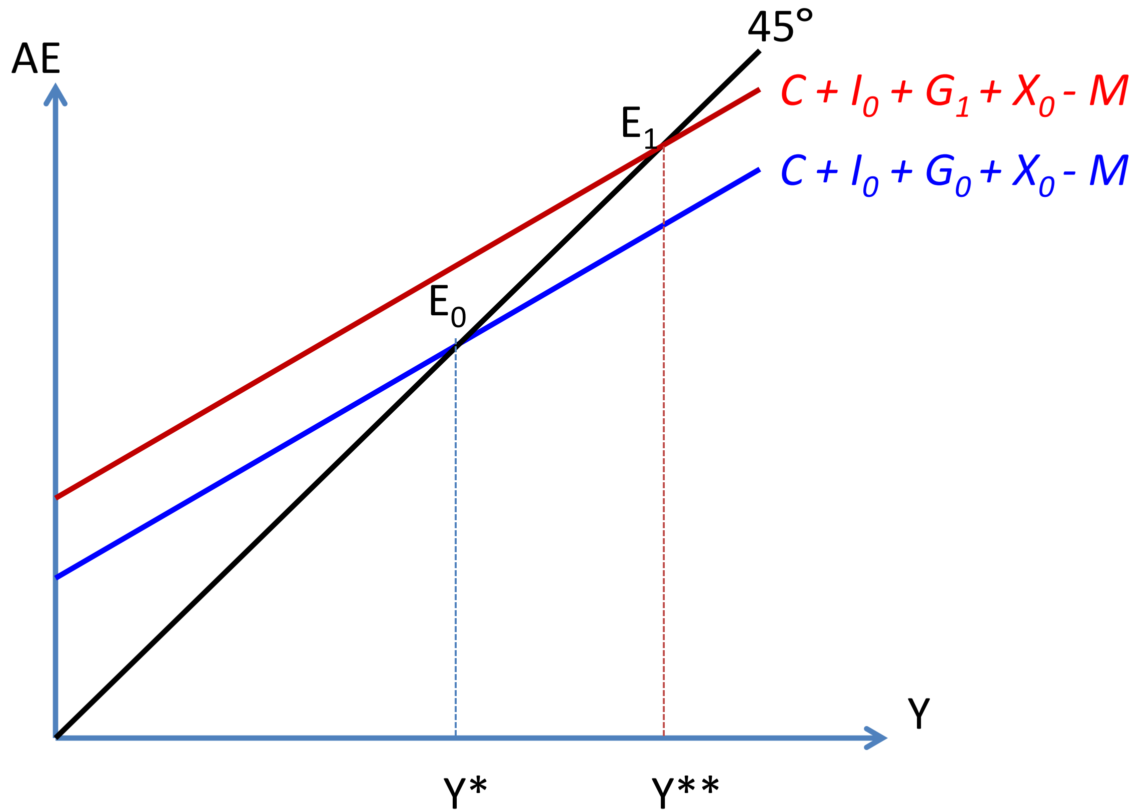
- At equilibrium, $Y = a + b(1-t)Y + I_0 + G_0 + X_0 - mY$

  ➔ $$Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

  ➔ $$C^* = \frac{a(1+m) + b(1-t)(I_0 + G_0 + X_0)}{1 - b(1-t) + m}$$

- If $M = mY_d$, $Y^* = ?$ (own practice!)

# Graph: Keynesian National-Income Model




Suppose G increases. What is the new value of Y*?

➔ Will show the answer on slide 50.

# IS-LM Model

- The previous Keynesian model only deals with the equilibrium in the goods (or commodity) market. It seeks to determine the equilibrium level of national income.
- The IS-LM model deals with not only the commodity market but also the money market. It determines the equilibrium levels of national income and interest rate in the economy.
- The IS curve represents the equilibrium in the commodity market.
  - Planned "investment" = Planned "savings"
- The LM curve represents the equilibrium in the money market.
  - Liquidity preference (money demand) = Money supply

# IS-LM Model: Closed Economy & No Tax

- Commodity Market:

  $Y = C + I + G_0$

  $C = a + bY$ $(a > 0,\ 0 < b < 1)$

  $I = I_0 - ir$ ($I_0, i > 0$, and $r$ is interest rate)

  ➢ $Y = a + bY + I_0 - ir + G_0$ ➔ $Y = \frac{(a + I_0 + G_0) - ir}{1 - b}$ : *IS*

- Money Market:

  $M^S = M_0$

  $M^D = kY - hr$ $(k, h > 0)$

  ➢ At equilibrium, $M_0 = kY - hr$ ➔ $Y = \frac{M_0}{k} + \frac{h}{k} r$ : *LM*

➢ From the two equilibrium conditions, we can solve for $Y^*$ and $r^*$:

$$Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h(1 - b)}$$

and

$$r^* = \frac{(a + I_0 + G_0)k - (1 - b)M_0}{ik + h(1 - b)}$$

# IS-LM Model: Closed Economy with Tax

- Commodity Market:

$Y = C + I + G_0$

$C = a + bY_d$ $(a > 0, 0 < b < 1)$

$I = I_0 - ir$ $(I_0, i > 0)$

$Y_d = Y - T$ where $T = tY$ $(0 < t < 1)$

➔ $Y = a + b(1-t)Y + I_0 - ir + G_0$ ➔ $Y = \frac{(a + I_0 + G_0) - ir}{1 - b(1-t)}$ : IS

- Money Market:

$M^S = M_0$

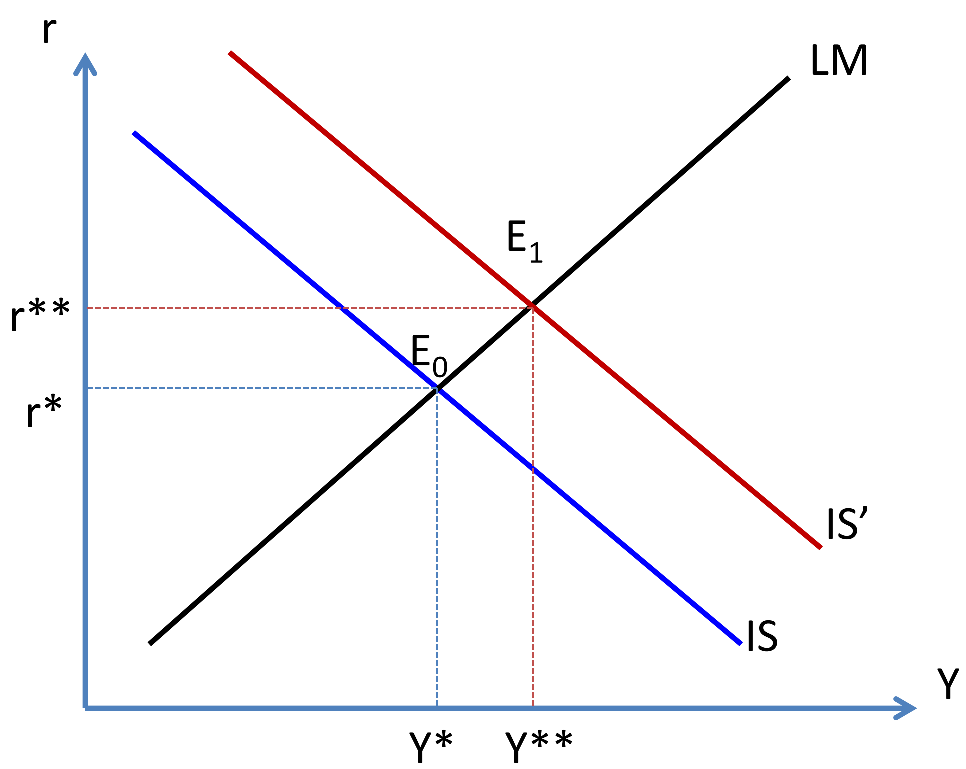
$M^D = kY - hr$ $(k, h > 0)$

➔ $M_0 = kY - hr$ ➔ $Y = \frac{M_0}{k} + \frac{h}{k}r$ : LM

➢ Equilibrium values for $Y^*$ and $r^*$:

$Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$ and $r^* = \frac{(a + I_0 + G_0)k - [1 - b(1-t)M_0]}{ik + h[1 - b(1-t)]}$

# Graph: Equilibrium in IS-LM Model




- When the government increases its expenditure, what's the impact on the equilibrium Y and r? (➔ Will show the answer on slide 50.)

# COMPARATIVE STATICS ANALYSIS

# Comparative Static Analysis

- Comparative statics is concerned with the comparison of different equilibrium states as a result of a change in the value of some parameters or exogenous variables.
- Example: Partial market equilibrium (slides 17-18)

Initially, the demand and supply functions were $Q_d = a_0 - bP$ and $Q_s = -c + dP$. Suppose now the demand function is $Q_d = a_1 - bP$, everything else remains the same. What are the changes in P* and Q*?

# Comparative Statics Analysis:
## Simple Macroeconomic Model

- Suppose government increases its expenditure, what's the impact on the equilibrium national income (i.e. ΔY*/ΔG=?)
- Recall: $Y^* = \dfrac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$ , and suppose $\Delta G = G_1 - G_0$.

➢ $$Y^{*1} - Y^{*0} = \frac{a + I_0 + G_1 + X_0}{1 - b(1-t) + m} - \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

➢ $$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{\Delta G}{1 - b(1-t) + m}$$

➔ $$\frac{\Delta Y^*}{\Delta G} = \frac{1}{1 - b(1-t) + m} > 0$$

Thus, if $-b(1-t) + m < 1$, then $\Delta Y^*/\Delta G > 1$.

# Comparative Statics Analysis: IS-LM Model

- In a closed economy with tax, suppose G increases ($\Delta G = G_1 - G_0$), $\Delta Y^*/\Delta G = ?$ and $\Delta r^*/\Delta G = ?$
- Recall: $Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$ and $r^* = \frac{(a + I_0 + G_0)k - [1 - b(1-t)]M_0}{ik + h[1 - b(1-t)]}$

$$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{h}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta Y^*}{\Delta G} = \frac{h}{ik + h[1 - b(1-t)]} > 0}$$

$$\Delta r^* = r^{*1} - r^{*0} = \frac{k}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta r^*}{\Delta G} = \frac{k}{ik + h[1 - b(1-t)]} > 0}$$

# Tax Incidence and Price Elasticity (1)

- Recall the concept of price elasticity:

$$E_p = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

- When $\Delta$ is very small, we use notation *d* instead of $\Delta$: $E_p = \frac{dQ}{dP} \cdot \frac{P}{Q}$
- Price elasticity of demand:

$$\eta = -\frac{dQ_d}{dP} \cdot \frac{P}{Q_d}$$

- Price elasticity of supply:

$$\varepsilon = \frac{dQ_s}{dP} \cdot \frac{P}{Q_s}$$

# Tax Incidence and Price Elasticity (2)

- Case 1 - Specific tax imposed on consumers.
- Consumer's price: $P^t = \frac{a+c+dT}{b+d}$ . Find $dP^t/dT$ = ?

Suppose $\Delta T = T_1 - T_0$. We want to know $\Delta P^t$ = ?

- $P^{t0} = \frac{a+c+dT_0}{b+d}$ and $P^{t1} = \frac{a+c+dT_1}{b+d}$
- $\Delta P^t = P^{t1} - P^{t0} = \frac{a+c+dT_1}{b+d} - \frac{a+c+dT_0}{b+d}$
- $\Delta P^t = \frac{d}{b+d}(T_1 - T_0) = \frac{d}{b+d}\Delta T$

➔ The impact of tax change on equilibrium price when tax is imposed on consumer is: $\frac{\Delta P^t}{\Delta T} = \frac{d}{b+d}$ -- (1)

# Tax Incidence and Price Elasticity (3)

- Rewrite the impact of tax change on equilibrium price, when tax is imposed on consumers:
  - Recall $Q_D = a - bP$ and $Q_S = -c + dP$.
  - Thus, $\Delta Q_D/\Delta P = -b$ and $\Delta Q_S/\Delta P = d$.
- From (1), $\dfrac{\Delta P^t}{\Delta T} = \dfrac{d}{b+d} = \dfrac{\Delta Q_S/\Delta P}{-\Delta Q_D/\Delta P + \Delta Q_S/\Delta P}$ -- (2)
- Recall $\eta = \dfrac{\Delta Q_d}{\Delta P}\cdot\dfrac{P}{Q_d}$ and $\varepsilon = \dfrac{\Delta Q_s}{\Delta P}\cdot\dfrac{P}{Q_s}$
- Rearrange (2) and we obtain:

$$\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{\varepsilon}{\eta + \varepsilon}$$

- If $\varepsilon = 0$, then $dP^t/dT = 0$.
- If $\eta = 0$, then $dP^t/dT = 1$.

# Tax Incidence and Price Elasticity (4)

- Case 2 - Specific tax imposed on producers.
- Producer's price: $P^t = \dfrac{a+c-bT}{b+d}$ . Find $dP^t/dT$ = ?
- Suppose $\Delta T = T_1 - T_0$. We want to know $\Delta P^t$ = ?

➢ $P^{t0} = \dfrac{a+c-bT_0}{b+d}$ and $P^{t1} = \dfrac{a+c+dT_1}{b+d}$

➢ $\Delta P^t = \dfrac{-b}{b+d}(T_1 - T_0) = \dfrac{-b}{b+d}\Delta T$

➔ The impact of tax change on equilibrium price when tax is imposed on producer is: $\dfrac{\Delta P^t}{\Delta T} = \dfrac{-b}{b+d}$ -- (3)

# Tax Incidence and Price Elasticity (5)

- Rewrite the impact of tax change on equilibrium price, when tax is imposed on producers:

From (3) and following the same method, we get:

$$\frac{\Delta P^t}{\Delta T} = \frac{-b}{b+d} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P}$$

$$\frac{\Delta P^t}{\Delta T} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P} \cdot \frac{P/Q}{P/Q}$$

$$\Rightarrow \boxed{\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{-\eta}{\eta + \varepsilon}}$$

- If $\varepsilon = 0$, then $dP^t/dT = -1$.
- If $\eta = 0$, then $dP^t/dT = 0$.