B.E. International Program

Faculty of Economics, Thammasat University

Semester: 1/2014
EE325 Introductory Econometrics
Homework#1 (Due on 16 September 2014 in class)

Choose and answer only 5 questions.

1. Given **dependent variable** as **wages per month** of employed labor in Thailand's labor force, show **explanatory/independent variables** that can be used to explain wages in econometrics model. Provide reasons and expected sign (direction) that those explanatory variables affect wages

2. Let OUTPUT be the quantity demanded for rice in Thailand (tons). Express quantity demanded for rice as a function of independent variables, explain and indicate the relationship between quantity demanded for rice and those independent variables.

3. Solve for the result:

3.1 $\sum_{i=1}^{100}\left(a^2+bx_i\right)$

3.2 $\sum_{x=0}^{3}(x+2)^2$

3.3 $\sum_{i=4}^{8}(2)^{(1-i)}x_i y_i$

3.4 $\sum_{i=1}^{5}[i(i^2+i+1)]$

3.5 $\sum_{x=0}^{2} f(x,y)$

3.6 $\sum_{x=2}^{4}\sum_{y=1}^{2}(x+2y)y$

4. Express the following terms in summation notation:

4.1 $x_1+x_2+x_3+x_4+\ldots+x_{100}$

4.2 $x_2y_3+x_3y_4+x_4y_5+x_5y_6$

4.3 $(x_1-y_1)-(x_2-y_2)-(x_3-y_3)$

4.4 $x_2y_2^2+x_3y_3^2+y_3x_3^2+y_4x_4^2$

4.5 $x_1^2+x_2^4+x_3^8+x_4^{16}+x_5^{32}$

4.6 $4x_1+8x_2+16x_3$

Simplify the following terms:

4.7 $\sum_{i=1}^{4}\left(a+bx_i+cy_i\right)$

4.8 $\sum_{i=4}^{7}(-1)^{-i}x_i y_i$

4.9 $\sum_{x=0}^{4} f(x,y)$

4.10 $\sum_{x=2}^{4}\sum_{y=1}^{2}(x+2y)$

5. Given *c* and *d* are constant. *Y* is a discrete random variable with probability P_1 and P_2. The Probability Mass Function (PMF) of this variable is

$$f(y) = \begin{cases} P_1 & , y = c + d \\ P_2 - P_1 & , y = c \\ 1 - P_2 & , y = c - d \end{cases}$$

5.1 Explain the meaning of **random variable** and provide some example

5.2 Find the value of *E(Y)* and *var(Y)*

5.3 Find the value of *E(Y)* and *var(Y)* when $d = \sqrt{2}, P_1 = 0.3$ and $P_2 + P_1 = 0.8$

6. Let *X* be the discrete random variable with the probability density function as follows:

X	-2	-1	0	1	2	3	4
$f(x)$	0.5a	a	2.25a	2a	1.5a	0.5a	0.25a

If *a* is a constant, find:

6.1 What is the value of a and why?

6.2 Find $P(X \leq 3)$

6.3 Find $P(-2 \leq X \leq 2)$

6.4 Find $P(X \geq 0)$

7. The following table gives data on probability distribution of Econometrics' grades in each year from past information.

x	F (0.0)	D (1.0)	D+ (1.5)	C (2.0)	C+ (2.5)	B (3.0)	B+ (3.5)	A (4.0)
$f(x)$	0.071	0.112	0.160	0.183	0.195	0.118	0.107	0.053

7.1 Plot a PDF of *f(x)*

7.2 Find the probability that students will get grade B, B+ and A in each year

7.3 Find the probability that students will get grade higher than D in each year

7.4 Find the expected value, variance, and standard deviation of the random variable X

7.5 Find the expected value and variance of Y=3X+5

8. Let X and Y be continuous random variables and their joint probability distribution function is

$$f(x,y) = \frac{3}{2}xy^2 \text{ , where } 0 \le x \le 2, \quad 0 \le y \le 1$$

Find E(X) and Var(X)

9. Given X as a continuous random variable. The probability function of this variable is

$$f(x) = -6x + 4, \quad 0 \le x \le 1$$

9.1 Plot a graph of Probability Density Function of *f(x)*

9.2 Find the area below the graph of *f(x)* within interval $0 \le x \le 1$

9.3 Find $P(X \ge 0.50)$

9.4 Find $P(0.25 \le X \le 0.50)$

10. Let X and Y be discrete random variables and their joint probability function $f(x,y)$ is as following:

		X	
		1	2
Y	0	$2/8$	$5/8$
	1	$1/8$	0

10.1 Find $E(X|Y=1)$ and $E(X^2|Y=1)$

10.2 Find $var(X|Y=1)$

10.3 Find Marginal pdf of *X* and Marginal pdf of *Y*

10.4 Are *X* and *Y* independent?

10.5 Find $var(X-Y)$

11. Given the following joint probability density function

$$f(x,y) = (0.32)^{x}(0.68)^{(1-x)}(0.45)^{y}(0.55)^{1-y}$$

where $x = 0,1$ and $y = 0,1$

11.1 Find *f(0,0), f(0,1), f(1,0),* and *f(1,1)*

11.2 Find the marginal density functions of X and Y [Find *f(x)* and *f(y)*]

11.3 Find cov(X,Y) and correlation coefficient between X and Y

11.4 Find $E(X|Y=1)$ and $\text{var}(X|Y=1)$

11.5 Are X and Y independent?

12. Let *X* be continuous random variable with probability density function as following:

$$f(x) = \begin{cases} \frac{6}{9} - \frac{2}{9}x, & 0 \le x \le 3 \\ 0 \quad , & \text{Otherwise} \end{cases}$$

12.1 Plot $f(x)$

12.2 Find $P(X=2)$

12.3 Find $P(1 \le X \le 2)$

12.4 Find the expected value of *X*

12.5 Find variance of X

13. Given X and Y are continuous random varibles. The joint probability distribution function(PDF) of X and Y is

$$f(x,y) = \begin{cases} x+y, & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & otherwise \end{cases}$$

Find the value of

13.1 $P\left(0 \le X \le \frac{1}{4}\right)$

13.2 $P(0.5 \le X \le 1)$

13.3 E(X)

13.4 Var(X)

13.5 E(Y)

13.6 Var(Y)

13.7 Cov(X,Y)

14. Given X_1, X_2, X_3 are independently distributed random variables from population with mean μ and variance σ^2. $\tilde{X}$ is estimator used to estimate mean value, $\tilde{X} = \frac{1}{3}X_1 + \frac{2}{3}X_2$.

14.1 Show that $\tilde{X}$ is a linear estimator of μ

14.2 Show that $\tilde{X}$ is an unbiased estimator

14.3 Given $\overline{X} = \sum_{i=1}^{3} X_i$ as another estimator of μ. Find and compare the value of Var $(\tilde{X})$ and Var$(\bar{X})$

14.4 Between $\tilde{X}$ and $\bar{X}$ which one is the better estimator? Why?

15. From midterm exam of EE325 last semester, the score was normally distributed with expected value of 50 and variance of 100, find:

15.1 Probability that a student in that class will receive the score lower than 40

15.2 Students will pass this course with the score of more than 30. Find the probability that a student will pass this course

15.3 If there is an adjustment of this midterm exam score by adding 5 points to each of the student, will this adjustment change the distribution, mean, and variance of the score? If yes, what are the new values?

15.4 Probability that a student will not pass the course after the score adjustment

16. Given the returns from stock A and stock B (R_A and R_B) are independently normally distributed with $R_A \sim N(0.03, 0.05^2)$ and $R_B \sim N(0.01, 0.02^2)$.

16.1 Find the probability that we will get loss if buy stock A. $[R_A < 0]$

16.2 Suppose we invest in a portfolio including both stock A and B If Stock A is 1/3 and stock B is 2/3 of the portfolio, what is the expected mean value of the investment?

16.3 Find the probability that we will get loss if invest in the portfolio above.

17. Let X_1, X_2 and X_3 be independent random variable with mean μ and variance σ^2. $\tilde{X}$ is the estimator for the mean where

$$\tilde{X} = \frac{1}{3}X_1 + \frac{2}{3}X_2$$

17.1 Show that $\tilde{X}$ is unbiased estimator of μ

17.2 Let $\bar{X} = \frac{\left(\sum_{i=1}^{3} X_i\right)}{3}$ be another estimator of mean. Show that $\bar{X}$ is an unbiased estimator of μ

17.3 Find $Var(\tilde{X})$, $Var(\bar{X})$, $MSE(\tilde{X})$ and $MSE(\bar{X})$

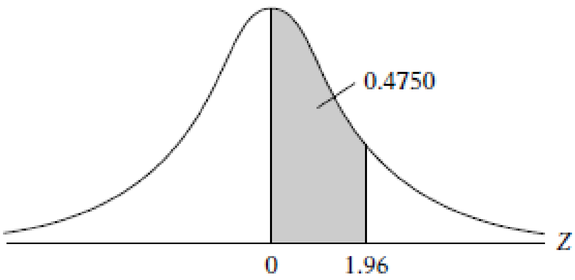
17.4 Between $\tilde{X}$ and $\bar{X}$, which one is better estimator and why?

TABLE D.1 AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION

Example

$\Pr(0 \leq Z \leq 1.96) = 0.4750$

$\Pr(Z \geq 1.96) = 0.5 - 0.4750 = 0.025$



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., $Z \geq 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $P(-1.96 \leq Z \leq 0) = 0.4750$. Therefore, $P(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$.