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EE481

Expected Learning Outcomes (ELOs)

After reading and studying this chapter, you will be able to:

- Explain the role of strategies and payoffs in a game.
- Identify dominant and dominated strategies in a game.
- Explain the difference between a pure strategy and a mixed strategy.
- Describe a Nash equilibrium.

Expected Learning Outcomes (ELOs)

- Solve for the Nash equilibria in one-shot games and repeated games.
- Solve for the Nash equilibria in simultaneous-move games and sequential games.
- Explain why some kinds of games can lead players to cooperate, while other kinds do not.
- Explain how limiting your options can have strategic value.

Game Theory Overview

What is Game Theory?

Game Theory is the study of mathematical models of strategic interaction among rational decision-makers.

The key pioneers of game theory were mathematicians John von Neumann and **John Nash**. 1994

What is a Game?

Games are defined as strategic situations in which there are multiple participants. Furthermore, the outcome of the decision any individual makes is dependent on the decisions made by all of the other participants.

Game Elements (P-S-O-P)

Players: Agents participating in the game (Man, Woman) $N \geq 2$

Strategies: Actions that each player may take under any possible circumstance (Boxing, Shopping)

Outcomes: The various possible results of the game (four, each represented by one cell of matrix)

Payoffs: The benefit that each player gets from each possible outcome of the game (the numbers entered in each cell of the matrix)

For example, if both players choose Boxing, (2, 1) will be the outcome with 2 being the payoff of Man and 1 being the payoff of Woman.

Types of Games

Types of Games

- One-shot: a game that is played once.
- Repeated: a game that is played more than once.

finitely repeated game

infinitely repeated game.

- Simultaneous-move: players make decisions at the same time.
- Sequential-move: players takes turns to make decisions.

Most games that we will study are 2-player games, but they can be extended to n-player game (i.e. more than 2 players).

Representation of Games

w/ 2-person & 2-strategy game :

column player (or Player 2)

		LEFT	RIGHT
ROW PLAYER (or Player 1)	UP	(π_1, π_2)	$(\cdot, \cdot)$
	DOWN	$(\cdot, \cdot)$	$(\cdot, \cdot)$

1's Payoffs

2's Payoffs

PAYOFF MATRIX

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Capacity Expansion Game

What is the likely outcome of this game?

We study Game Theory to answer such question.

unit: mn USD

		Toyota	
		Build a new plant	Do not Build
Honda	Build a new plant	16,16	20,15
	Do not Build	15,20	18,18

not stable outcomes (why?)

- one-shot game
- simultaneous move game

Consider Player 1 (Honda)

- if T "buids", H's best response is "Build"

  as π^H when it builts > π^H when it doesn't
  (16) (15)

- if T "does not build", H's best response is "Build"

  as π^H when it builts > π^H when it doesn't.
  (20) (18)

Consider Player 2 (Toyota)

- If H buids, T's best response is "Build"
- If H "does not build", T's best response is "Build"

- Nash equilibrium is (Build, Build)

In equilibrium, H gets 16 mn USD

T gets 16 mn USD

Nash Equilibrium

Definition: A **Nash Equilibrium (NE)** occurs when each player chooses a strategy that gives him/her the highest payoff, given the strategy chosen by the other player(s).

NE of Toyota vs. Honda Game: (Build, Build).

- Given Toyota builds a new plant, Honda's best response is to Build.
- Given Honda builds a new plant, Toyota's best response is to

- Given Toyota builds a new plant, Honda's best response is to Build.
- Given Honda builds a new plant, Toyota's best response is to Build.

Why is the Nash Equilibrium plausible?

It is "**self enforcing**", and no player would want to deviate from the NE. If a player deviates, he/she will get a lower payoff.

The Prisoners' Dilemma

Definition: A game situation in which there is a conflict between the collective interest of all of the players and the self-interest of individual players is called a **Prisoner's Dilemma**.

Pareto Inferior outcome

David

Ron		Confess	Don't Confess
	Confess	-5,-5	0,-10
	Don't Confess	-10,0	-1,-1

Pareto superior outcome

Dominant and Dominated Strategies

Definition: A **dominant strategy** is a strategy that is better than any other strategy that a player might choose, no matter what strategy the other player follows.

Note: When a player has a dominant strategy, that strategy will be the player's Nash Equilibrium strategy.

Definition: A **dominated strategy** is a strategy such that the player has another strategy that gives a higher payoff, no matter what the other player does.

Note: Dominated strategies can be eliminated from consideration in order to solve the game more easily.

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David

Ron	C	DC
C	-5, -5	0, -10
DC	-10, 0	-1, -1

Politician

Army	corrupt	do not corrupt
coup d'tat		
do not coup d'tat		

Dominant Strategy

Definition: A **Dominant Strategy Equilibrium** occurs when each player uses a dominant strategy.

Toyota

	Build a new plant	Do not Build

	Build a new plant	Do not Build
Build a new plant	16,16	20,15
Do not Build	15,20	18,18

Honda

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Dominated Strategy

Toyota

Honda

	Build Large	Build Small	Do Not Build
~~Build Large~~	~~0, 0~~	~~12, 8~~	~~18, 9~~
Build Small	8, 12	16, 16	20, 15
Do Not Build	9, 18	15, 20	18, 18

"Build large" is dominated strategy **for each player**, so this strategy will not be chosen by both players. We can eliminate it from our consideration.

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This process is so called "Elimination of dominated strategies"

Dominated Strategy

Toyota

Honda

	Build small	Do not build
Build small	16, 16	20, 15
~~Do not build~~	15, 20	18, 18

Nash equilibrium: $(S_H^*, S_T^*) = (\text{Build small}, \text{Build small})$

"______________" is ______________**for each player**, so this strategy will not be chosen by both players. We can eliminate it from our consideration.

Finding the NE

LEARNING-BY-DOING EXERCISE 14.1

Finding the Nash Equilibrium: Coke versus Pepsi

Table 14.6 shows Coke's and Pepsi's profits for various combinations of prices that each firm might charge.

Problem Find the Nash equilibrium in this game.

Price Competition between Coke and Pepsi*

		Coke			
		$10.50	**$11.50**	**$12.50**	**$13.50**
Pepsi	**$6.25**	66, 190	68, 199	70, 198	73, 191
	$7.25	79, 201	82, 211	85, 214	89, 208
	$8.25	82, 212	86, 224	90, 229	95, 225
	$9.25	75, 223	80, 237	85, 244	91, 245

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Nash Equilibrium

$$(s'^{*}_{PEPSI}, s'^{*}_{COKE}) = (8.25, 12.50)$$

$$\pi^{*}_{PEPSI}(s'^{*}_{PEPSI}, s'^{*}_{COKE}) = 90$$

$$\pi^{*}_{COKE}(s'^{*}_{PEPSI}, s'^{*}_{COKE}) = 229$$

Nash Equilibrium Limitations

The Nash Equilibrium need not be unique.

The **Game of Chicken**, also known as the **Hawk–Dove Game**, is a model of conflict for two players in game theory.

The principle of the game is that while it is to both players' benefit if one player yields, the other player's optimal choice depends on what their opponent is doing: if the player opponent yields, they should not, but if the opponent fails to yield, the player should.

The game of chicken has TWO NEs, which show that one player will win and one player will lose, but we cannot say who will win.

Nash Equilibrium Limitations

The Nash Equilibrium need not be unique.

The name "chicken" has its origins in a game in which two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken", meaning a coward.

Game of Chicken

Smith

Adam

	Swerve	Stay
Swerve	0,0	A D -10,10
Stay	A D 10,-10	-100,-100

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Two Nash Equilibria:

(Stay , Swerve) , (Swerve , Stay)

Finding the NEs

TABLE 14.10 What Are the Nash Equilibria?

		Player 2		
		Strategy *D*	Strategy *E*	Strategy *F*
	Strategy *A*	4, 2	13, 6	1, 3
Player 1	Strategy *B*	11, 2	0, 0	15, 10
	Strategy *C*	12, 14	4, 11	5, 4

3 Nash Equilibria : { (C, D) , (A, E) , (B, F) }

Nash Equilibrium Limitations

Nash Equilibrium need not exist.

Matching pennies is a game, played by 2 players (A and B):

- Each player has a penny and must secretly turn the penny to heads or tails.
- The players then reveal their choices simultaneously.
- If the pennies match (both heads or both tails), then one player (e.g. Player B) keeps both pennies.
- If the pennies do not match (one head and one tail), then the other player (e.g. Player A) keeps both pennies.

Matching Pennies is a **zero-sum** game, which does not have NE. Another example of a zero-sum game is Rock-Paper-Scissors.

Nash Equilibrium Limitations

Nash Equilibrium need not exist.

		Player B	
		Heads	Tails
Player A	Heads	(1, -1)	(-1, 1)
	Tails	(-1, 1)	(1, -1)

Matching the pennies

Pure Strategies and Mixed Strategies

Pure Strategy – A specific choice of a strategy from the player's possible strategies in a game.

Note: The games with the NE(s) so far has a pure strategy NE, i.e. players select their strategies from the choices they have.

Mixed Strategy – An assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy.

Note: A game without a pure strategy NE, e.g. matching pennies, will always have at least one NE that involves a mixed strategy.

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Finding the Mixed Strategy NE

This involves at least one player randomizing between the pure strategies, i.e. he/she will assign probabilities to the choices.

In matching pennies, each player will play Head with 50% probability and Tail with 50% probability.

Example: Matching Pennies

Repeated Prisoner's Dilemma

So far, we considered only "one-shot" games, where players cannot cooperate, cooperation can arise in "repeated" games.

Cooperation can result from self-interested behaviour on the part of each player under certain circumstances:

"Game Theory 101"

"Grim Trigger" Strategy – One-time cheating by one player triggers the grim prospect of a permanent breakdown in cooperation for the remainder of the game.

"Tit-for-Tat" Strategy – A strategy in which you do to your opponent in this period what your opponent did to you in the last period.

	1	2	3	4	5	6
Player 1	D	C	D	C	D	C
Player 2	C	D	C	D	C	D

D = Defect

C = Cooperate

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Repeated Prisoner's Dilemma

finitely repeated play

in Finitely repeated play.

TABLE 14.12 Prisoners' Dilemma Game

		Player 2	
		Cheat	Cooperate
Player 1	Cheat	5, 5	14, 1
	Cooperate	1, 14	10, 10

N E is (Cheat, Cheat) in this one shot game

N.E.

I'm doing the best I could,
given what you are doing
AND
you are doing the best you could
given what I am doing.

Repeated Prisoner's Dilemma

EE 415 , EE 481

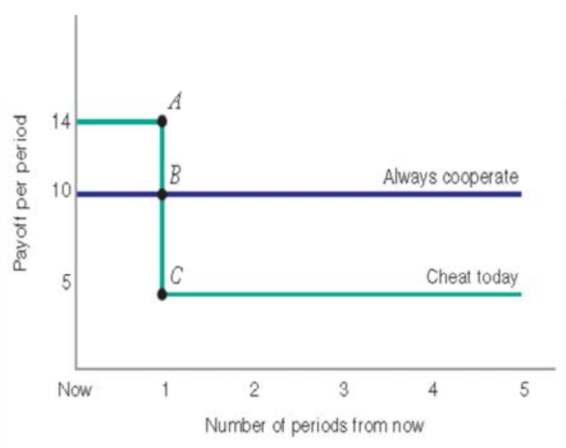

FIGURE 14.1 Payoffs in the Repeated Prisoners' Dilemma under the "Grim Trigger" Strategy
If Player 1 cheats today, he receives a stream of payoffs given by the light line. If he cooperates today and in the future he can ensure himself a stream of payoffs given by the dark line. The distance of line segment *AB* represents the one-time gain to Player 1 from cheating. The distance of line segment *BC* represents the reduction in each of Player 1's subsequent payoffs because Player 2 retaliates against Player 1's cheating.

Repeated Prisoner's Dilemma

Likelihood of cooperation increases under these conditions:

1. The players are patient.
2. Interactions between the players are frequent.
3. Cheating is easy to detect.
4. The one-time gain from cheating is relatively small.

Likelihood of cooperation diminishes under these conditions:

1. The players are impatient.
2. Interactions between the players are infrequent.
3. Cheating is hard to detect.
4. The one-time gain from cheating is large in comparison to the eventual cost of cheating.

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Sequential-Move Games

Sequential-Move Games are games in which one player (the first mover) takes an action before another player (the second mover). The second mover observes the action taken by the first mover before deciding what action it should take.

A **game tree** shows the different strategies that each player can follow in the game and the order in which those strategies get chosen.

Backward Induction is a procedure for solving a sequential-move game by starting at the end of the game tree and finding the optimal decision for the player at each decision point.

Sequential-Move Games

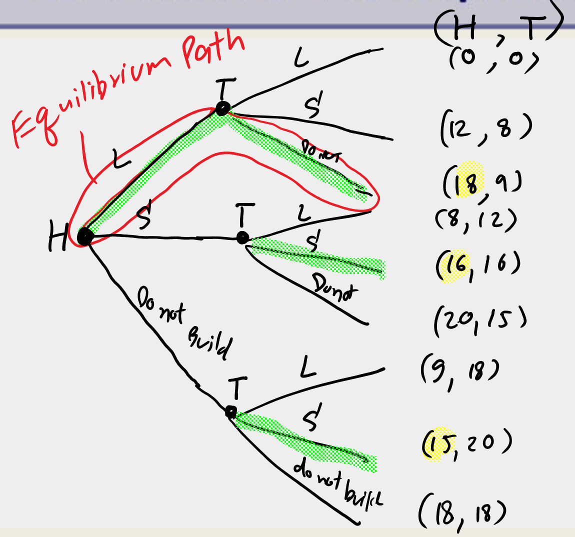
Here, Honda moves first. After Honda chooses its strategy, Toyota then decides its strategy.

TABLE 14.13 Capacity Expansion Game between Toyota and Honda*

		Toyota Build Large	Build Small	Do Not Build
	Build Large	0, 0	12, 8	18, 9
Honda	**Build Small**	8, 12	16, 16	20, 15
	Do Not Build	9, 18	15, 20	18, 18

*Payoffs are in millions of dollars.

Game Tree for the Sequential-Move Game



Backward Induction:

Thinking ahead and reasoning backward

In equilibrium:

(LARGE, DO NOT BUILD)

$$(\pi_H, \pi_T) = (18, 9)$$

backward induction A procedure for solving a sequential-move game by starting at the end of the game tree and finding the optimal decision for the player at each decision point.

Game Tree for the Sequential-Move Game

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Sequential-Move Games

Sometimes, in sequential-move games, the first mover can adopt "**strategic moves**" which are actions taken in an early stage of a game that can alter the second mover's behavior.

In the above example, the strategic move is for Honda to make commitment to "Build Large" when Toyota were to choose "Do Not Build".

First Mover's Advantage

In short, strategic moves that seemingly limit options (i.e. by having the commitment) can actually make a player better off.

For a strategic move to work, it must be visible, understandable, and hard to reverse.

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LEARNING-BY-DOING EXERCISE 14.3

An Entry Game

Avinash Dixit and Barry Nalebuff, authors of a delightful book on game theory, *Thinking Strategically*, have written, "It takes a clever carpenter to turn a tree into a table; a clever strategist knows how to turn a table into a tree."[18] In this exercise, we illustrate their point in the context of a simple entry game.

Suppose you own a firm that is considering entry into the digital camera business, where you will compete head to head with Kodak (which, let's say, currently has a monopoly). Kodak can react in one of two ways: It can start a price war or it can be accommodating. You can enter this business on a large scale or a small scale. Table 14.14 shows the payoffs you and Kodak are likely to get under the various scenarios that could unfold.

Problem Should you enter this business on a large scale or a small scale?

		Kodak	
		Accommodate	**Price War**
You	**Small**	4, 20	1, 16
	Large	8, 10	2, 12

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