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Group Homework 5 & 6

Semester 2/2021 EE320 Introductory mathematical economics

Due date: April 7th 2022 (before midnight / B.E. moodle).

Q1. Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

a. To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

b. Under the assumption used in (a), show that the production function satisfies the law of diminishing returns.

c. Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).

d. Show that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.

e. Under the condition(s) assumed in (a), is the production function satisfied with the *global* concave property?

Suppose that $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

f. Show that Q is increasing over time.

g. Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.

Q2. A monopolist faces the market demand given by $P = Q^{-c}$ where "c" is a parameter with positive value, "P" is the price per unit output and "Q" is the amount of output. Suppose that monopolist's production technology is given by $Q = K^{\frac{1}{3}}L^{\frac{2}{3}}$ where "K" and "L" are the level of capital used and the number of labor employed, respectively. Assume that the unit price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a. What type of the return to scale technology does the production function exhibit?

From now on, assume that c = ¼ . Consider the following problems.

b. Construct the profit function of the monopolist. (Hint: your profit function should be expressed in terms of K and L.)

c. The firm wants to maximize profit and seek for combination of the two factor inputs . Derive the demand for factor inputs, capital and labor.

d. How does the demand for labor vary with respect to w and r? Show your result by using partial derivative.

e. Confirm your answer with the second-order condition.

Q3. In a duopoly market, suppose that firms 1 and 2 face market demand, $p = 100 - (q_1 + q_2)$. Firm costs are $c_1 = 10q_1$ and $c_2 = q_2^2$.

(a) Calculate market price under the Cournot environment. How much is the profit that each firm yields in the equilibrium?

(b) Calculate the market price under the joint production decision, i.e. collusive equilibrium. How much is the profit that each firm yields in the equilibrium?

(c) Do you think that the collusive equilibrium will sustain? What would be the profit of firm 1 if firm 1 chooses to deviate from the collusive allocation?

Q4. Consider a simple two-market model where demand and supply for each market is given below. (Notationally, let's name the two markets as A and B, respectively.)

Market A:

Demand: $p_A = 10 - 2Q_A$

Supply: $p_A = 1 + Q_A$

Q1. Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

a. To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?

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Suppose that $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

f. Show that Q is increasing over time.

g. Compute $\frac{dQ/Q}{dt}$ when t = 0, i.e. growth of output in the initial period.

Q1)

a) $Q = f(K,L) = A[K^n + L^n]$

[illegible] K, L [illegible] a [illegible]

$Q^n = A[(aK)^n + (aL)^n]$

$Q^n = a^n[A[K^n + L^n]]$

$Q^n = a^n Q$ [illegible]

Decreasing

$Q^n < Q$

$n < 1$ [illegible] $n < 1$ [illegible] a^n [illegible] $Q_n < Q$ [illegible]

b) $Q = A[K^n + L^n]$

Law of diminishing return

sol) Law of diminishing return [illegible] K; $\frac{dQ}{dK} = AnK^{n-1}$

[illegible] a) ; $n < 1$

$\therefore \frac{dQ}{dK} = AnK^{-(1-n)}$

$\frac{dQ}{dK} = \frac{An}{K^{1-n}}$

$\therefore$ K↑ [illegible] $\frac{dQ}{dK}$ ↓

$\therefore$ [illegible] Law of diminishing #

Sol 2

$MPL = \frac{dQ}{dL} = AnL^{n-1}$

$MPL = \frac{An}{L^{1-n}}$

a) increase RTS

$af(K,L) > f[aK, aL]$

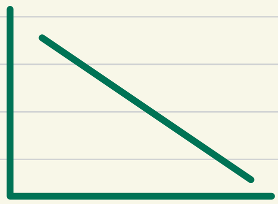
Output > input

Decresing RTS

$af(K,L) < f[aK, aL]$

Output < input

b) Diminishing



slope (−)

increasing

slope (+)

K ; MPK

$MPK = \frac{dQ}{dK}$

$Q = AK^n + AL^n$

$MPK = \frac{dQ}{dK} = n \cdot A \cdot K^{n-1}$

slope MPK

$\frac{dMPK}{dK} = \text{slope}$

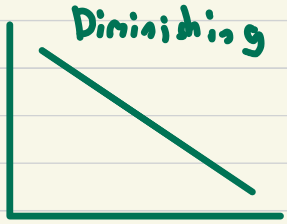
$\text{slope MPK} = (n-1)\, n \cdot AK^{n-2}$

$n < 1$

$(n-1) \cdot n \cdot A \cdot K^{n-1} < 0$

slope MPK < 0

Diminishing



L , MPL

$MPL = \frac{dQ}{dL} = nAL^{n-1}$

slope MPL

$\frac{dMPL}{dL} = \text{slope}$

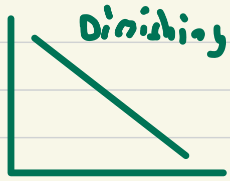
$\text{slope MPL} = (n-1)\, n \cdot AL^{n-2}$

$n < 1$

$(n-1) \cdot n \cdot A \cdot L^{n-1} < 0$

slope MPL < 0

Diminishing



c) $MRTS = \frac{MP_L}{MP_K}$

$MPL = \frac{dQ}{dL}$ $\quad AnL^{n-1}$

$MPK = \frac{dQ}{dK} = n \cdot A \cdot K^{n-1}$

$\frac{A n \cdot L^{n-1}}{nA\, K^{n-1}} = \frac{L^{n-1}}{K^{n-1}}$

$MPL = \frac{dQ}{dL}$

$Q = A(K^n + L^n)$

$Q = AK^n + AL^n$

$MPL = \frac{dQ}{dL} = nAL^{n-1}$

$MPK = \frac{dQ}{dK}$

$Q = A(K^n + L^n)$

$Q = AK^n + AL^n$

$MPK = AnK^{n-1}$

$MRTS = \frac{MPL}{MPK}$

$= \frac{nAL^{n-1}}{AnK^{n-1}}$

$= \left(\frac{L}{K}\right)^{n-1}$ #

d)

$$MRTS = \left(\frac{L}{K}\right)^{n-1} = \frac{L^{n-1}}{K^{n-1}}$$

$$\frac{dMRTS}{dL} = (n-1)\frac{L^{n-2}}{K^{n-1}}$$

$n < 1$

$n-1 \rightarrow (-)$

$\frac{dMRTS}{dL}$ ⊖ : $L\uparrow \rightarrow MRTS\downarrow$

$L\downarrow \rightarrow MRTS\uparrow$

a. What type of the return to scale technology does the production function exhibit?

From Cobb-Douglas production function $Q = K^{\alpha} \cdot L^{\beta}$

If $\alpha + \beta < 1$, there will be decreasing return to scale

If $\alpha + \beta = 1$, " constant "

If $\alpha + \beta > 1$, " increasing "

Consequently, $Q = K^{\frac{1}{3}} L^{\frac{2}{3}}$ $\alpha + \beta = \frac{1}{3} + \frac{2}{3} = 1$

∴ The production function is constant return to scale

b. Construct the profit function of the monopolist. (Hint: your profit function should be expressed in terms of K and L.)

$P = Q^{-c}$ $Q = K^{\frac{1}{3}} L^{\frac{2}{3}}$

from market demand : $P = Q^{-c}$

substitute $c = \frac{1}{4} \rightarrow P = Q^{-\frac{1}{4}}$

Profit = Total Revenue − Total Cost

$\pi = P \cdot Q$ − [Total Cost of capital + Total cost of labor]

$\pi = Q^{-\frac{1}{4}} \cdot Q - [Kr + WL]$

The unit price of K is equal to r → 1 unit price = r

K unit price = Kr

The unit price of L is equal to W → 1 unit price = W

L unit price = WL

$\pi = Q^{-\frac{1}{4}} \cdot Q^{1} - [Kr + WL]$

$\pi = Q^{-\frac{1}{4}} \cdot Q^{\frac{4}{4}} - Kr - WL$

$\pi = Q^{\frac{3}{4}} - Kr - WL$

From $Q = K^{\frac{1}{3}} \cdot L^{\frac{2}{3}}$

$\pi = [K^{\frac{1}{3}} \cdot L^{\frac{2}{3}}]^{\frac{3}{4}} - Kr - WL$

$\pi = (K^{\frac{1}{3}})^{\frac{3}{4}} \cdot (L^{\frac{2}{3}})^{\frac{3}{4}} - Kr - WL$

$\pi = K^{\frac{1}{4}} \cdot L^{\frac{1}{2}} - Kr - WL$

$f(K, L) = K^{\frac{1}{4}} \cdot L^{\frac{1}{2}} - Kr - WL$ *

c. The firm wants to maximize profit and seek for combination of the two factor inputs . Derive the demand for factor inputs, capital and labor.

F.O.C $\frac{\partial f}{\partial k} = 0$; $\frac{\partial f}{\partial L} = 0$

$$\frac{\partial f}{\partial k} = \frac{1}{4} K^{\frac{1}{4}-\frac{4}{4}} \cdot L^{\frac{1}{2}} - (1) r$$

$$\frac{\partial f}{\partial k} = \frac{L^{\frac{1}{2}}}{4k^{\frac{3}{4}}} - r$$

$$0 = \frac{L^{\frac{1}{2}}}{4k^{\frac{3}{4}}} - r$$

$$r = \frac{L^{\frac{1}{2}}}{4k^{\frac{3}{4}}}$$

$$4r = \frac{L^{\frac{1}{2}}}{K^{\frac{3}{4}}} \quad \text{--- ①}$$

$$\frac{\partial f}{\partial L} = K^{\frac{1}{4}} \cdot \left[\frac{1}{2} L^{\frac{1}{2}-\frac{1}{2}} \right] - 0 - W \quad (1)$$

$$\frac{\partial f}{\partial L} = K^{\frac{1}{4}} \cdot \frac{1}{2} L^{-\frac{1}{2}} - W$$

$$\frac{\partial f}{\partial L} = K^{\frac{1}{4}} \left(\frac{1}{2L^{\frac{1}{2}}} \right) - W$$

$$0 = \frac{K^{\frac{1}{4}}}{2L^{\frac{1}{2}}} - W$$

$$2W = \frac{K^{\frac{1}{4}}}{L^{\frac{1}{2}}} \quad \text{--- ②}$$

① × ② ; $\left[\frac{L^{\frac{1}{2}}}{K^{\frac{3}{4}}} \right] \left[\frac{K^{\frac{1}{4}}}{L^{\frac{1}{2}}} \right] = (4r)(2W)$

$$\frac{K^{\frac{1}{4}}}{K^{\frac{3}{4}}} = 8rW$$

$$K^{\frac{1}{4}-\frac{3}{4}} = 8rW$$

$$\frac{1}{8rW} = k^{\frac{1}{2}}$$

$$(k^{\frac{1}{2}})^2 = \left(\frac{1}{8rW}\right)^2$$

$$k = \frac{1}{64 r^2 W^2} \quad \text{--- ③}$$

Then substitute K in ①

$$\frac{L^{\frac{1}{2}}}{K^{\frac{3}{4}}} = 4r$$

$$L^{\frac{1}{2}} = K^{\frac{3}{4}} (4r)$$

$$L^{\frac{1}{2}} = \left[\frac{1}{64r^2W^2} \right]^{\frac{3}{4}} (4r)$$

$$L^{\frac{1}{2}} = \frac{1}{(64)^{\frac{3}{4}} (r^2)^{\frac{3}{4}} (W^2)^{\frac{3}{4}} \cdot 4r}$$

$$L^{\frac{1}{2}} = \frac{4r}{(2^6)^{\frac{3}{4}} \cdot r^{\frac{3}{2}} \cdot W^{\frac{3}{2}}}$$

$$L^{\frac{1}{2}} = \frac{4r}{2^{\frac{9}{2}} \cdot r^{\frac{3}{2}} \cdot W^{\frac{3}{2}}}$$

$$(L^{\frac{1}{2}})^2 = \left(\frac{4r}{2^{\frac{9}{2}} \cdot r^{\frac{3}{2}} \cdot W^{\frac{3}{2}}} \right)^2$$

$$L = \frac{16r^2}{2^9 \cdot r^3 \cdot W^3}$$

$$L = \frac{1}{32 r W^3} \quad \text{--- ④}$$

from the production function

$$Q = K^{\frac{1}{3}} L^{\frac{2}{3}}$$

$$Q = \left(\frac{1}{64r^2w^2} \right)^{\frac{1}{3}} \left(\frac{1}{32rw^3} \right)^{\frac{2}{3}}$$

$$Q = \frac{1}{(64)^{\frac{1}{3}} (r^2)^{\frac{1}{3}} (W^2)^{\frac{1}{3}} \cdot (32)^{\frac{2}{3}} (r)^{\frac{2}{3}} \cdot W^2}$$

$$Q = \frac{1}{4 \cdot r^{\frac{2}{3}} W^{\frac{2}{3}} \sqrt[3]{32 \cdot 32} \cdot r^{\frac{2}{3}} \cdot W^2}$$

$$Q = \frac{1}{4r^{\frac{4}{3}} W^{\frac{8}{3}} \cdot \sqrt[3]{8 \cdot 4 \cdot 8 \cdot 4}}$$

$$Q = \frac{1}{4r^{\frac{4}{3}} W^{\frac{8}{3}} \cdot \sqrt[3]{4 \cdot 2 \cdot 4 \cdot 4 \cdot 2 \cdot 4}}$$

$$Q = \frac{1}{4r^{\frac{4}{3}} W^{\frac{8}{3}} \cdot 4 \sqrt[3]{2 \cdot 2}}$$

$$Q = \frac{1}{\sqrt{32 \cdot \sqrt[3]{2}}\, r^{\frac{4}{3}} W^{\frac{8}{3}}}$$

d. How does the demand for labor vary with respect to w and r? Show your result by using partial derivative.

from $L = \frac{1}{32rw^3}$

$= \frac{1}{32} \cdot r^{-1} \cdot w^{-3}$

$\frac{\partial L}{\partial r} = \frac{1}{32} \cdot (-1r^{-2}) \cdot w^{-3}$

$\frac{\partial L}{\partial r} = \frac{-1}{32r^2w^3}$ #

$\frac{\partial L}{\partial w} = \frac{1}{32} r^{-1} [-3w^{-4}]$

$= \frac{-3}{32rw^4}$ #

e. Confirm your answer with the second-order condition.

S.O.S find $f_{KK}, f_{KL}, f_{LK}, f_{LL}$

$f_{KK} = \frac{1}{4}K^{-\frac{3}{4}} L^{\frac{1}{2}} - r$

$= \frac{1}{4}\left[\frac{-3}{4} K^{-\frac{3}{4}-\frac{4}{4}}\right] L^{\frac{1}{2}} - 0$

$f_{KK} = \frac{-3}{16} K^{-\frac{7}{4}} L^{\frac{1}{2}}$

$f_{KK} = \frac{-3L^{\frac{1}{2}}}{16K^{\frac{7}{4}}}$ #

$f_L = \frac{1}{2}K^{\frac{1}{4}} L^{-\frac{1}{2}} - w$

$f_{LK} = \frac{1}{2}\left[\frac{1}{4}K^{\frac{1}{4}-\frac{4}{4}}\right]L^{-\frac{1}{2}} - 0$

$f_{LK} = \frac{K^{-\frac{3}{4}} L^{-\frac{1}{2}}}{8}$

$f_{LK} = \frac{1}{8K^{\frac{3}{4}} L^{\frac{1}{2}}}$ #

$f_{LL} = \frac{1}{2}K^{\frac{1}{4}}\left[-\frac{1}{2}L^{-\frac{1}{2}-\frac{2}{2}}\right] - 0$

$f_{LL} = \frac{-K^{\frac{1}{4}} L^{-\frac{3}{2}}}{4}$

$f_{LL} = \frac{-K^{\frac{1}{4}}}{4L^{\frac{3}{2}}}$

$f_{KL} = \frac{1}{4}K^{-\frac{3}{4}}\left[\frac{1}{2}L^{-\frac{1}{2}}\right] - 0$

$f_{KL} = \frac{1}{8K^{\frac{3}{4}} L^{\frac{1}{2}}}$ #

$H = \begin{bmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{bmatrix}$

$H = \begin{bmatrix} \frac{-3L^{\frac{1}{2}}}{16K^{\frac{7}{4}}} & \frac{1}{8K^{\frac{3}{4}} L^{\frac{1}{2}}} \\ \frac{1}{8K^{\frac{3}{4}} L^{\frac{1}{2}}} & \frac{-K^{\frac{1}{4}}}{4L^{\frac{3}{2}}} \end{bmatrix}$

Substitute $k = \frac{1}{64r^2w^2}$

$$L = \frac{1}{32rw^3}$$

$$L^{\frac{1}{2}} = \left(\frac{1}{32rw^3}\right)^{\frac{1}{2}}$$

$$= \frac{1}{32^{\frac{1}{2}} r^{\frac{1}{2}} w^{\frac{1}{2}}}$$

$$= \frac{1}{2^{\frac{5}{2}} r^{\frac{1}{2}} w^{\frac{3}{2}}}$$

$$k^{\frac{7}{4}} = \left(\frac{1}{64r^2w^2}\right)^{\frac{7}{4}}$$

$$= \frac{1}{64^{\frac{7}{4}} r^{\frac{7}{2}} w^{\frac{7}{2}}}$$

$$= \frac{1}{2^{\frac{21}{2}} r^{\frac{7}{2}} w^{\frac{7}{2}}}$$

$$k^{\frac{3}{4}} = \left(\frac{1}{64r^2w^2}\right)^{\frac{3}{4}}$$

$$= \frac{1}{64^{\frac{3}{4}} r^{\frac{3}{2}} w^{\frac{3}{2}}}$$

$$= \frac{1}{2^{\frac{9}{2}} r^{\frac{3}{2}} w^{\frac{3}{2}}}$$

Q3. In a duopoly market, suppose that firms 1 and 2 face market demand, $p = 100 - (q_1 + q_2)$. Firm costs are $c_1 = 10q_1$ and $c_2 = q_2^2$.

(a) Calculate market price under the Cournot environment. How much is the profit that each firm yields in the equilibrium?

(b) Calculate the market price under the joint production decision, i.e. collusive equilibrium. How much is the profit that each firm yields in the equilibrium?

(c) Do you think that the collusive equilibrium will sustain? What would be the profit of firm 1 if firm 1 chooses to deviate from the collusive allocation?

Market B:

Demand: $p_B = 20 - Q_B$

Supply: $p_B = 2 + 2Q_B$

a. Derive the market equilibrium
b. Suppose the government imposes unit tax on both markets at the rate of t_A and t_B. Solve for the after-tax equilibrium as the function of t_A and t_B.
c. How much revenue can the government collect from the taxation?
d. Determine the level of t_A and t_B that maximizes government's revenue.

a) Cournot Environment

$Profit_1 = P \cdot q_1 - C_1$

$= (100 - (q_1 + q_2))q_1 - 10q_1$

$= 100q_1 - q_1^2 - q_1q_2 - 10q_1$

$\Pi_1 = 90q_1 - q_1^2 - q_1q_2$

$Profit_2 = P \cdot q_2 - C_2$

$= (100 - (q_1 + q_2))q_2 - q_2^2$

$= 100q_2 - q_1q_2 - q_2^2 - q_2^2$

$\Pi_2 = 100q_2 - q_1q_2 - 2q_2^2$

Max Π_1

$\frac{d\Pi_1}{dq_1} = 90 - 2q_1 - q_2 = 0$

$q_1 = 45 - 0.5q_2$ —①

Max Π_2

$\frac{d\Pi_2}{dq_2} = 100 - q_1 - 4q_2 = 0$

$q_1 = 100 - 4q_2$ —②

Derivative

$y = 4x_1 + 3x_2 + 2x_1x_2 + x_1^2 + x_2^2$

$\frac{\partial y}{\partial x_1} = 4 + 0 + 2x_2 + 2x_1 + 8$

$\frac{\partial y}{\partial x_2} = 8 + 3 + 2x_1$

$100 - 4q_2 = 45 - 0.5q_2$

$55 = 3.5q_2$

$q_2 = 15.714$

$q_1 = 37.143$

$Q = 52.857$

$P = 100 - 52.857 = 47.143$

b) $\Pi_{รวม} = \Pi_1 + \Pi_2$

$\Pi_{รวม} = P \cdot Q - C_1 - C_2$

$\Pi_{รวม} = (100 - q_1 - q_2)(q_1 + q_2) - 10q_1 - q_2^2$

$\Pi_{รวม} = 100q_1 - q_1^2 - q_1q_2 + 100q_2 - q_1q_2 - q_2^2 - 10q_1 - q_2^2$

$\Pi_T = 90q_1 - 2q_1q_2 - 2q_2^2 - q_1^2 + 100q_2$

$\frac{d\Pi}{dq_1} = 90 - 2q_2 - 2q_1 = 0$

$q_1 = 45 - q_2$ —①

$\frac{d\Pi}{dq_2} = -2q_1 - 4q_2 + 100 = 0$

$q_1 = 50 - 2q_2$ —②

① และ ②;

$45 - q_2 = 50 - 2q_2$

$q_2 = 5$

$q_1 = 40$

$Q = 45$

$P = 100 - 45 = 55$

$\Pi_1 = P \cdot q_1 - C_1 = 55(40) - 10(40) = 1800$

$\Pi_2 = P \cdot q_2 - C_2 = 55(5) - 5^2 = 250$

c) $\Pi_1 = 2513$

$\Pi_2 = 492.98$

$\Pi_1 = 1800$

$\Pi_2 = 250$

∴ callusion ไม่ stable

Q4. Consider a simple two-market model where demand and supply for each market is given below. (Notationally, let's name the two markets as A and B, respectively.)

Market A:

Demand: $p_A = 10 - 2Q_A$

Supply: $p_A = 1 + Q_A$

Market B:

Demand: $p_B = 20 - Q_B$

Supply: $p_B = 2 + 2Q_B$

a. Derive the market equilibrium
b. Suppose the government imposes unit tax on both markets at the rate of t_A and t_B. Solve for the after-tax equilibrium as the function of t_A and t_B.
c. How much revenue can the government collect from the taxation?
d. Determine the level of t_A and t_B that maximizes government's revenue.

a.) Market equilibrium

Market A: D = S

$10 - 2Q_A = 1 + Q_A$

$Q_A = 3$

$P_A = 4$

Market B: D = S

$20 - Q_B = 2 + 2Q_B$

$Q_B = 6$

$P_B = 14$

b. $t = P^D - P^S$

Market A

$t_A = [10 - 2Q_A] - [1 + Q_A]$

$t_A = 9 - 3Q_A$

$Q_A = \frac{9 - t_A}{3}$ $\quad P_A = 1 + \frac{9 - t_A}{3}$

Market B

$t_B = (20 - Q_B) - (2 + 2Q_B)$

$t_B = 18 - 3Q_B$

$Q_B = \frac{18 - t_B}{3}$ $\quad P_B = 2 + 2\left[\frac{18 - t_B}{3}\right]$

c.) tax revenue $= t_A + t_B$

$\text{Tax R} = t_a Q_A + t_b Q_B$

$\text{Tax R} = t_a\left[\frac{9 - t_a}{3}\right] + t_b\left[\frac{18 - t_b}{3}\right]$

d.) Max

ต้องการ Tax R ;

$\text{Tax R} = 3t_a - \frac{t_a^2}{3} + 6t_b - \frac{t_b^2}{3}$

$\frac{d\,\text{Tax R}}{dt_a} = 3 - \frac{2}{3}t_a = 0 \quad t_a = 4.5$

$\frac{d\,\text{Tax R}}{dt_b} = 6 - \frac{2}{3}t_b = 0 \quad t_b = 9$

tax A = 4.5

B = 9

Q5. (Monopolist and the Optimal advertising)

Consider a linear demand equation faced by a monopolist.

$$Q = 2000 + 4\sqrt{A} - 20P,$$

where A is the dollar amount of the expenditure on advertisement, P is the unit price, and Q is the amount of quantity demanded by consumers. Both θ and β are strictly positive.

The demand equation is an extended version of the traditional one. The proposed function captures an idea that advertising can boost the total demand in the market as potential customers get more information about the product.

Suppose that the monopolist cost function is given by $C(Q, A) = c(Q) + A$ where $c(Q) = 2Q + 1000$

a) Construct the profit function.

$\Pi = P.Q - C(Q) - A$

$= P(2{,}000 + 4\sqrt{A} - 20P) - 2Q - 1{,}000 - A$

$= 2{,}040P + 4PA^{\frac{1}{2}} - 20P^2 - 5{,}000 - 8A^{\frac{1}{2}} - A$

b) Determine the optimal pricing (P*) and optimal advertisement (A*) that maximize the profit of the monopolist.

optimal pricing $(P^*) \rightarrow 2040 + 4A^{\frac{1}{2}} - 40P = 0$

$40P - 4A^{1/2} = 2{,}040$

$P^* = \frac{2040 + 4A^{1/2}}{40}$

optimal advertisement $(A^*) \rightarrow 2PA^{-1/2} - 4A^{-1/2} - 1 = 0$

$2P - 4 = A^{1/2}$

$4P^2 - 16 = A^*$

c) Confirm the result with the second-order differential test, i.e. hessian-matrix method.

$H = \begin{bmatrix} \Pi_{PP} & \Pi_{PA} \\ \Pi_{AP} & \Pi_{AA} \end{bmatrix} = \begin{bmatrix} -40 & 2A^{-1/2} \\ 2A^{-1/2} & A^{-3/2}(2-P) \end{bmatrix}$ → negative definite

$|H_1| = -40 < 0 \rightarrow \forall (P, A)$

$|H_2| = -40(A^{-3/2}(2-P)) - (2A^{-1/2})^2 < 0 \rightarrow \forall (P, A)$

$\therefore P^*, A^*$ are global solution

Market B:

Demand: $p_B = 20 - Q_B$

Supply: $p_B = 2 + 2Q_B$

a. Derive the market equilibrium
b. Suppose the government imposes unit tax on both markets at the rate of $t_A$and t_B. Solve for the after-tax equilibrium as the function of t_A and t_B.
c. How much revenue can the government collect from the taxation?
d. Determine the level of t_A and t_B that maximizes government's revenue.

Q5. (Monopolist and the Optimal advertising)

Consider a linear demand equation faced by a monopolist.

$$Q = 2000 + 4\sqrt{A} - 20P,$$

where A is the dollar amount of the expenditure on advertisement, P is the unit price, and Q is the amount of quantity demanded by consumers. Both θ and β are strictly positive.

The demand equation is an extended version of the traditional one. The proposed function captures an idea that advertising can boost the total demand in the market as potential customers get more information about the product.

Suppose that the monopolist cost function is given by $C(Q, A) = c(Q) + A$ where $c(Q) = 2Q + 1000$

Consider the following problem.

a) Construct the profit function.
b) Determine the optimal pricing (P*) and optimal advertisement (A*) that maximize the profit of the monopolist.
c) Confirm the result with the second-order differential test, i.e. hessian-matrix method.
