EE312 Macroeconomics, 2/2017 (Sec. 046402 - Sicha)
Problem Sets : Ch.11 Part 2 Endogenous Growth Model
Solution

*** Exam will consist of essay-type questions only.**

1. In the endogenous growth model, suppose that there are three possible uses of time. Let u denote the fraction of time spendt working. **Let s denote the fraction of time spent neither working nor accumulating human capital (call this unemployment)**, and $1-u-s$ the fraction of time spent accumulating human capital. Assume that $z=1$ and $b=4.2$. Also assume that the economy begins period 1 with 100 units of human capital.

   (a) Define the consumer's budget constraint, the law of motion of human capital, the firm's production function and the firm's profit function. (Fill in the blanks in the table below)

Consumers	**Firm**
$C = uwH^S$ $H^{s'} = (1-u-s)bH^s$	$Y = zuH^d$ $\pi = (z-w)uH^d.$
b = efficiency of human capital accumulation technology	The demand curve is infinitely elastic at w = z

   (b) Define equilibrium consumption and growth of capital accumulation. (Fill in the blanks below.)
   The market clears at $w = ...z...$ where $uH^d = ..uH^s...$
   Equilibrium consumption and growth of human capital accumulation: (Define H' , $\frac{H'}{H}$ and $\frac{H'}{H}-1$.)

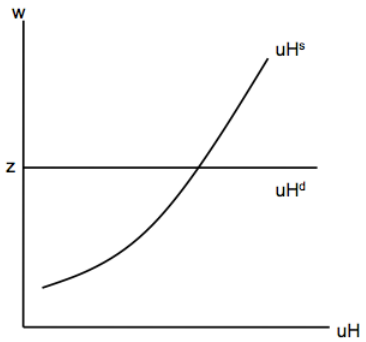
$$H' = (1-u-s)bH$$

$$\frac{H'}{H} = (1-u-s)b$$

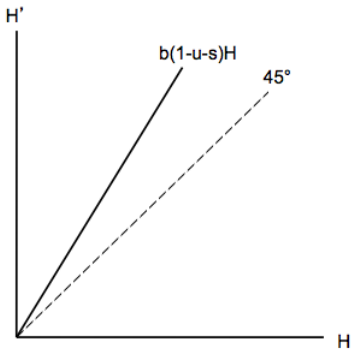
$$\frac{H'}{H}-1 = (1-u-s)b-1$$

   (c) From question (b), draw two diagrams; one to show equilibrium in the labour market and the other to show growth of human capital.

   - Equilibrium in the labour market



   - Growth of human capital



   - $H'>$ H. Slope = b(1-u-s) = rate of growth of human capital.

(d) Suppose that for periods 1,2,3, ..., 10, $u = 0.7$ and $s = 0.05$. Calculate aggregate consumption, output, and the quantity of human capital in each of these periods. (You may use a computer program or a calculator. Make sure you understand the calculations and the intuitions behind.)

$\frac{H'}{H} = (1 - u - s)b = (1 - 0.7 - 0.05) \times 4.2 = 1.05.$

t	H	Y	C
1	100.00	70.00	70.0
2	105.00	73.50	73.5
3	110.25	77.18	77.18
4	115.76	81.03	81.03
5	121.55	85.09	85.09
6	127.63	89.34	89.34
7	134.42	93.81	93.81
8	140.71	98.50	98.50
9	147.75	103.42	103.42
10	155.13	108.59	108.59

(e) Suppose that, in period 11, $u = 0.6$ and $s = 0.15$. Then, in period 12, 13, 14,...., $u = 0.7$ and $s = 0.05$. Calculate aggregate consumption, output, and quantity of human capital inperiods 11, 12, 13, ...,20.(You may use a computer program or a calculator. Make sure you understand the calculations and the intuitions behind.)

Period 11: $\frac{H'}{H} = (1 - u - s)b = (1 - 0.6 - 0.15) \times 4.2 = 1.05.$

Period 12 onwards : $\frac{H'}{H} = (1 - u - s)b = (1 - 0.7 - 0.05) \times 4.2 = 1.05.$

t	H	Y	C
11	162.89	97.73	97.73
12	171.03	119.72	119.72
13	179.59	125.71	125.71
14	188.56	132.00	132.00
15	197.99	138.60	138.60
16	207.89	145.52	145.52
17	218.29	152.80	152.80
18	229.20	160.44	160.44
19	240.66	168.46	168.46
20	252.70	176.89	176.89

(f) Suppose alternatively that in period 11, $u = 0.6$ and $s = 0.05$. Then, in period 12, 13, 14,...., $u = 0.7$ and $s = 0.05$. Again, calculate aggregate consumption, output, and the quantity of human capital in periods 11, 12, 13, ..., 20. (You may use a computer program or a calculator. Make sure you understand the calculations and the intuitions behind.)

Period 11: $\frac{H'}{H} = (1 - u - s)b = (1 - 0.6 - 0.05) \times 4.2 = 1.47.$

Period 12 onwards : $\frac{H'}{H} = (1 - u - s)b = (1 - 0.7 - 0.05) \times 4.2 = 1.05.$

t	H	Y	C
11	162.89	97.73	97.73
12	239.45	167.61	167.61
13	251.42	175.99	175.99
14	263.99	184.79	184.79
15	277.19	194.03	194.03
16	291.05	203.73	203.73
17	305.60	213.92	213.92
18	320.88	224.62	224.62
19	336.93	235.85	235.85
20	353.77	247.64	247.64

(g) Now Suppose that in period 11, $u = 0.6$ and $s = 0.10$. Then, in period 12, 13, 14,...., $u = 0.7$ and $s = 0.05$. Calculate aggregate consumption, output and quantity of humand capital in period 11, 12, 13, ..., 20.(You may use a computer program or a calculator. Make sure you understand the calculations and the intuitions behind.)

Period 11: $\frac{H'}{H} = (1 - u - s)b = (1 - 0.6 - 0.1) \times 4.2 = 1.26.$

Period 12 onwards : $\frac{H'}{H} = (1 - u - s)b = (1 - 0.05 - 0.7) \times 4.2 = 1.05.$

t	H	Y	C
11	162.89	97.73	97.73
12	205.24	143.67	143.67
13	215.50	150.85	150.85
14	226.28	158.39	158.39
15	237.59	166.31	166.31
16	249.47	174.63	174.63
17	261.94	183.36	183.36
18	275.04	192.53	192.53
19	288.89	202.16	202.16
20	303.23	212.26	212.26

(h) What do you conclude from your results in parts (d)-(g). Discuss. [Hint: in question (e), (f) and (g) there is one-time drop in employment, call this recession. The difference is in what the unemployment do.]

" In (d), we see an economy that has a sustained growth in output of 5% a period. In all subsequent cases, there is a one-time drop in employment, call this a recession. The difference is in what the unemployed do. In case (e), the unemployed sit idle. There is a one-time drop in output and consumption compared to scenario (d), but no other changes. Indeed, the accumulation of human capital is unaffected.

In (f) and (g), the unemployed go to school, which triggers a burst in human capital accumulation for one period, and this has an impact over all subsequent periods. In (f), all go to school, which lead to output and consumption being 40% above scenario (d) in period 12 and any thereafter. In (g), only half of the unemployed go to school, and the increase in output is only half as large.

This is an illustration of how recessions can have a positive side in the future if the unemployed take the opportunity to improve their skills."

2. In the endogenous growth model, the competitive equilibrium is given by

$$Y = C = zuH$$

$$H' = b(1-u)H$$

where Y = output; C = consumption, z = total factor productivity; H and H' = levels of current nd future human capital, respectively; b = the efficiency of human capital accumulation technology; and u - time allocated to producing output.

What is the most important source of economic growth in the model? What are the growth rates of consumption, output and human capital? Explain why economic growth is unbound in the model as well as compare the source of growth with Solow model. (If the space provided is not enough, please attach a separate paper.)

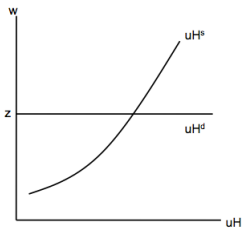
Solution

- Note that there are 3 sub-questions. In the exam, do not forget to answer all questions. Carefully show the steps and carefully write down all relevant details.
  (1) What are the growth rates of consumption, output and human capital?
  (2) What is the most important source of economic growth in the model?
  (3) Explain why economic growth is unbound in the model as well as compare the source of growth with Solow model.

- To answer the question, **define the behaviour of the agents (consumer and firm)** in the model as follows.

Consumer	**Firm**
$C = uwH^S$ $H^{s'} = (1-u)bH^s$	$Y = zuH^d$ $\pi = (z-w)uH^d$
b = efficiency of human capital accumulation technology	The demand curve is infinitely elastic at w = z

  - Then, **define competitive equilibrium** in the labour market. The market clears at w = z where uH^d = uH^s.



  - Therefore, at equilibrium, $H^s = H^d = H$.
  - We can **write down rquilibrium consumption and growth of human capital accumulation** as follows:
    $H' = b(1-u)H$.
    $\frac{H'}{H} - 1 = b(1-u) - 1$, where $b(1-u) - 1$ is constant.
  - **Consumption grows at the same rate as human capita**l;
    $\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1 = \frac{H'}{H} - 1 = b(1-u) - 1.$
  - Since there is no saving in the model. Consumption is always equal to output. Therefore, **output grows at the same rate as consumption (and human capital)**.

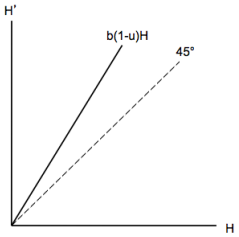
– Then, we have **the balanced growth path**.

$$\frac{H'}{H} = \frac{C'}{C} = \frac{Y'}{Y} = b(1-u)-1$$

$$g_H = g_C = g_Y = b(1-u)-1$$

The growth rates of consumption, output and human capital are equal to $b(1-u)-1$. **ANS2**

– **Let's assume $H' > H$, allowing human capital to grow over time**.



∗ The slope of the graph H' is equal to = b(1-u) = rate of growth of human capital.

∗ From $H' > H$, $b(1-u) > 1$.

– **Ans 1** From the balanced growth path **the growth rates of consumption, output and human capital** are equal to $b(1-u)-1$. Therefore, the most important source of economic growth in the model is **accumulation of human capital**, **which promotes efficiency of output production in the model. The higher human capital, the more efficiency the production of human capital has**.

∗ **Human capital is an investment** : sacrifice of current consumption → more future production and consumption.

∗ **More time spent on education**, **increase in efficiency of human capital accumulation** promotes growth in human capital; $\frac{H'}{H} - 1 = b(1-u)-1$, $b \uparrow$ *or* $u \downarrow \Rightarrow \frac{H'}{H} \uparrow$.

– **Ans 3** Economic growth is unbound in the endogenous growth model. This is different from Solow's model. The economic reasons and the comparison between the two model are as follows.

∗ Production function :
Solow's model : $Y = zF(K,N)$, $y = zf(k)$.

Solow's model	**An endogenous growth model (A Simplified version of Lucas's)**
production function : $Y = zF(K,N)$ $y = zf(k)$ Production function is **subject to diminishing marginal returns** in capital per worker.	$Y = uzH^d$ Production function **does not subject to diminishing marginal returns** in human capital.
Source of growth is physical capital accumulation.	**Source of growth is human capital accumulation**
There is **a limit for growth** as physical capital per worker increases. Output will go to steady state output. After it reaches the steady state, it will stay there. Eventually, growth of per capita output goes down to zero.	**Output continues to grow at a constant rate.** as human capital increases. There is **no limit** for growth. Output growth is always positive. (growth rate is equal to $b(1-u)-1$).
Economic reason for bounded growth : Physical capital is **rivalry** (once one company is using it the other cannot). production function is **subject to diminishing marginal returns** in capital per worker As phyical capital increases, output increases at a decreasing rate. Therefore, we cannot always generate growth by raising physical capital.	Economic reason for unbounded growth : This is because knowledge is '**non-rivalry**', production function **does not subject to diminishing marginal returns** in human capital As human capital increases, output increases at a constant rate. Therefore, we can always generate growth by raising human capital.