

OLS vs WLS vs GLS**Simulated Data**

```
. set obs 10
obs was 0, now 10

. set seed 123

. g x1=rnormal(10,100)

. g x2=rnormal(27,440)

. g x3=rnormal(-30,100)

. g u12=100*x3^2+rnormal(0,1)

. g u1=sqrt(u12)

. g y=150+0.1*x1-2.7*x2+u1
```

OLS

```
. reg y x1 x2
```

Source	SS	df	MS	Number of obs =	10
Model	23532457.5	2	11766228.8	F(2, 7) =	25.22
Residual	3265852.78	7	466550.397	Prob > F =	0.0006
				R-squared =	0.8781
				Adj R-squared =	0.8433
Total	26798310.3	9	2977590.03	Root MSE =	683.04

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x1	.3206024	1.574711	0.20	0.844	-3.402997 4.044202
x2	-3.480704	.5189972	-6.71	0.000	-4.707937 -2.253471
_cons	924.4407	239.2862	3.86	0.006	358.6187 1490.263

```
. predict uhat, r

. g uhat2=uhat^2

. g x32=x3^2

. reg uhat2 x32, noconst
```

Source	SS	df	MS	Number of obs =	10
Model	4.2430e+12	1	4.2430e+12	F(1, 9) =	71.22
Residual	5.3614e+11	9	5.9571e+10	Prob > F =	0.0000
				R-squared =	0.8878
				Adj R-squared =	0.8754
Total	4.7791e+12	10	4.7791e+11	Root MSE =	2.4e+05

uhat2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x32	30.81989	3.651869	8.44	0.000	22.55879 39.08099

```
. predict vhat2, xb

. g vhat=sqrt(vhat2)
```

WLS

```
. vwls y x1 x2, sd(vhat)
```

```
Variance-weighted least-squares regression      Number of obs   =      10
Goodness-of-fit chi2(7)      =      6.83         Model chi2(2)    =    416.60
Prob > chi2                  =    0.4467         Prob > chi2      =    0.0000
```

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1		-.512651	.2449598	-2.09	0.036	-.9927635 -.0325385
x2		-3.022738	.1489636	-20.29	0.000	-3.314702 -2.730775
_cons		591.2014	97.31571	6.08	0.000	400.4662 781.9367

Matrix Approach

```
. g one=1
. mkmat one x1 x2, matrix(X)
. mkmat y, matrix(Y)
```

OLS

```
. matrix b=inv(X'*X)*(X'*Y)
. matrix list b
b[3,1]
      y
one  924.44072
x1   .3206024
x2  -3.480704

. matrix u=Y-X*b
. matrix v=u*u'
. matrix u2=vecdiag(v)
. matrix u2t=u2'
. mkmat x32, matrix(X32)
. matrix b2=inv(X32'*X32)*X32'*u2t
. matrix u2hat=X32*b2
. matrix u2hatT=u2hat'
. matrix vhat=diag(u2hatT)
. matrix list vhat
```

```
symmetric vhat[10,10]
      r1      r2      r3      r4      r5      r6      r7      r8      r9
r10
r1  2063.3953
r2      0  62724.285
r3      0      0  281633.33
r4      0      0      0  27580.177
r5      0      0      0      0  1991510.4
r6      0      0      0      0      0  68397.079
r7      0      0      0      0      0      0  9538.9761
r8      0      0      0      0      0      0      0  158188.26
r9      0      0      0      0      0      0      0      0  348182.8
r10     0      0      0      0      0      0      0      0      0
204452.88
```

GLS – WLS

```
. matrix bglS=inv(X'*inv(vhat)*X)*(X'*inv(vhat)*Y)
```

```
. matrix list bglS
```

```
bglS[3,1]
```

```

      y
one   591.20145
x1    -.51265099
x2    -3.0227385

```

```
. vwls y x1 x2, sd(vhat)
```

```

Variance-weighted least-squares regression
Goodness-of-fit chi2(7)    =    6.83
Prob > chi2                =    0.4467
Number of obs              =    10
Model chi2(2)              =   416.60
Prob > chi2                =    0.0000

```

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
x1	-.512651	.2449598	-2.09	0.036	-.9927635	-.0325385
x2	-3.022738	.1489636	-20.29	0.000	-3.314702	-2.730775
_cons	591.2014	97.31571	6.08	0.000	400.4662	781.9367