Topic 3 Part 3

The Theory of Demand (Chapter 5)

Summary

- When the price of a good changes, its quantity demanded changes.

- The total effect on demand (TE) is due to **the sum of two effects**:
  – Income Effect (IE)
  – Substitution Effect (SE).

- That is, **TE = SE + IE.**

Summary

When the price of a good falls…

- Substitution Effect implies that the consumer will ALWAYS buy more of the good.
- Income Effect implies that the real income increases AND
  - the consumer will buy more of the good if it is a normal good.
  - the consumer will buy less of the good if it is an inferior good.

Summary

- To represent these two effects in the consumer choice diagram, **<u>economists adjust the BL with two steps</u>**:

1. **"PIVOT"** to represent the SE
2. **"SHIFT"** to represent the IE

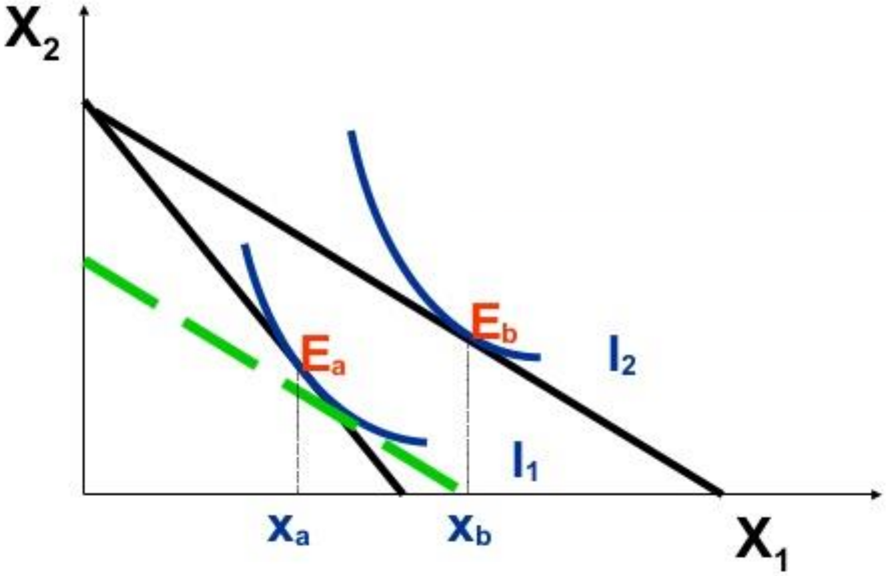
- Moreover, there are two approaches when we try to identify the SE by "pivoting" the BL.
- These approaches are
  - Hicksian Approach
  - Slutskian Approach

Summary

Hicksian Approach

- We find the SE by pivoting the BL (to reflect the price change) <u>along the **original indifference curve**</u>.
- We are essentially asking ourselves:
  Which <u>Bundle after the price change</u>, would keep the consumer at <u>the utility level that he had prior the price change</u>?
- Hicksian Definition of the SE:
  "the change in the demand for a good as its price changes, <u>holding the utility constant</u>".

THE HICKSIAN METHOD

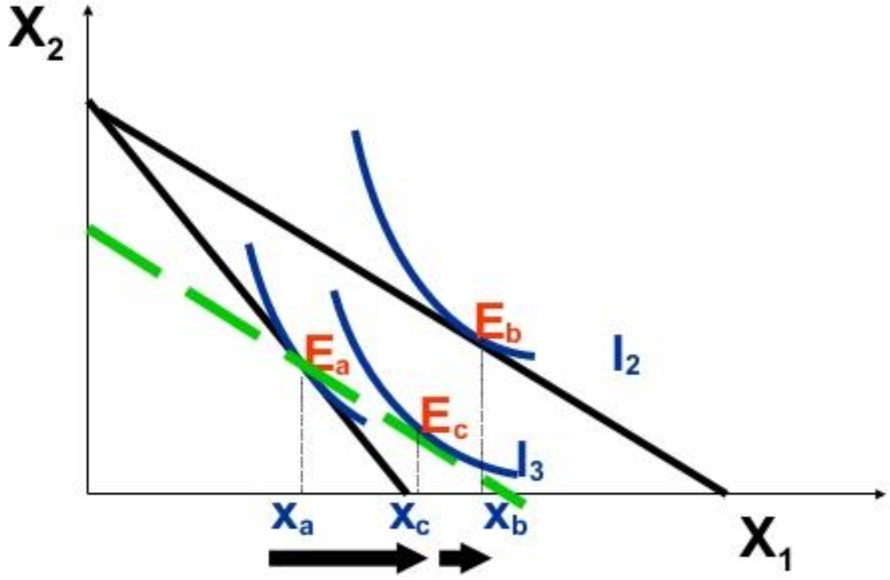


Summary

Slutskian Approach

- We find the SE by pivoting the BL (to reflect the price change) around the **original bundle**.
- We are essentially asking ourselves:
  Which Bundle after the price change, would keep the consumer at the purchasing power that he had prior the price change?
- Slutskian Definition of the SE:
  "the change in the demand for a good as its price changes, holding the purchasing power constant".

THE SLUTSKY METHOD



Slutskian Method Example

Suppose that we have the **demand function for X**:

$$X(Px, I) = 10 + \frac{I}{10Px}.$$

$X(Px, I)$ is the demand of X as a function of Px and I.

Note that

1. **X is a normal good because dX/dI > 0.**
2. **X is an ordinary good because dX/dPx < 0.**

Slutskian Method Example

$$X(Px, I) = 10 + \frac{I}{10Px}.$$

<u>Suppose I = 120 and Px = 3.</u>

We know that **X(3, 120)** = 10 + 120/30 = 14.

(X = 14 max. U when I = 120 and Px = 3)

<u>Now, what if Px falls from 3 to 2 per unit?</u>

We know that **X(2, 120)** = 10 + 120/20 = 16.

(X = 16 max. U when I = 120 and Px = 2)

Slutskian Method Example

The demand for X increases by 2 units.

That is, we buy more because X is cheaper.

This is the Total Effect (TE) of the price change.

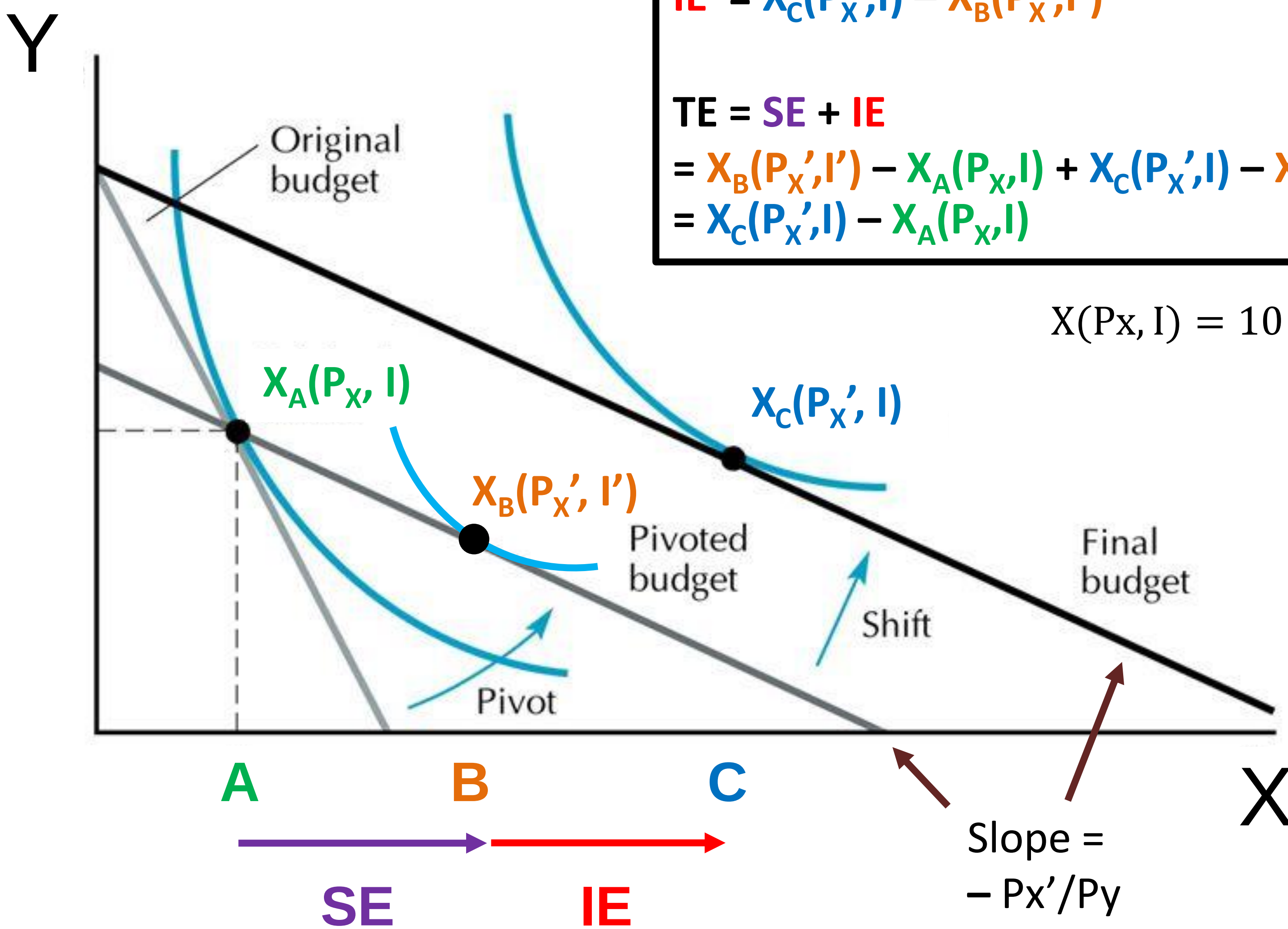
Now, we want to know how much the Income Effect (IE) and the Substitution Effect (SE) are.

Note that TE = SE + IE.

$$SE = X_B(P_X',I') - X_A(P_X,I)$$
$$IE = X_C(P_X',I) - X_B(P_X',I')$$

$$TE = SE + IE$$
$$= X_B(P_X',I') - X_A(P_X,I) + X_C(P_X',I) - X_B(P_X',I')$$
$$= X_C(P_X',I) - X_A(P_X,I)$$

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

$SE = X_B(2, I') - 14$

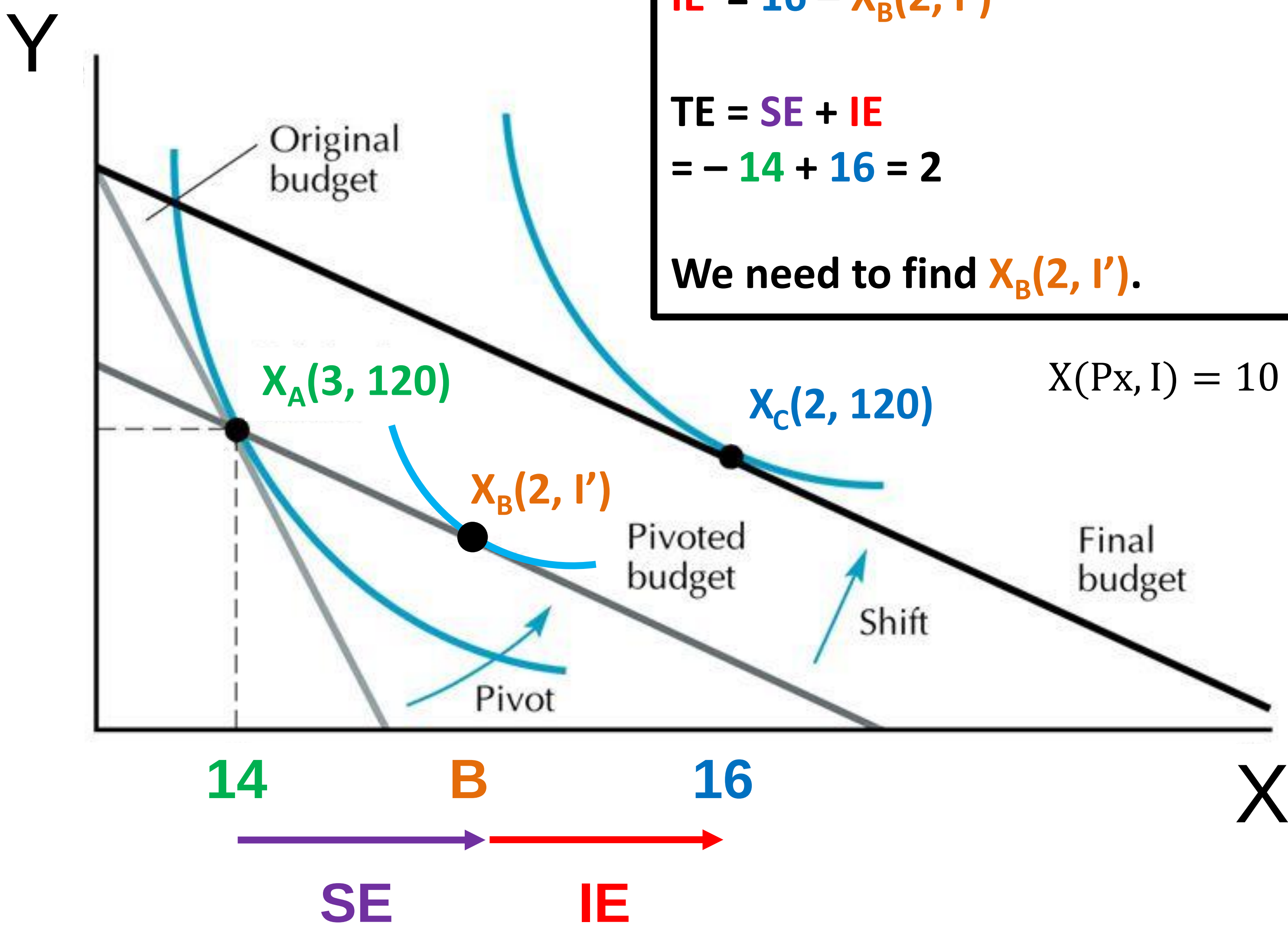
$IE = 16 - X_B(2, I')$

$TE = SE + IE$

$= -14 + 16 = 2$

We need to find $X_B(2, I')$.

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

Slutskian Method Example

Substitution Effect (holding purchasing power constant)

To find the SE,

1. Find the BL after Px changes, where the original bundle (X = 14 in this example) is still affordable.
2. Find the optimal bundle (Bundle B) on this new BL.

SE = Bundle B – Bundle A = $X_B(P_X', I') - X_A(P_X, I)$

Slutskian Method Example

To find the SE,

1. **Find the BL after Px changes, where the original bundle (X = 14 in this example) is still affordable.**

Original BL: $3X + P_yY = 120$ Pivoted BL: $2X + P_yY = I'$

We equate the two BL's to solve for I'.

$X = 120 - I'$ or $I' = 120 - X$.

We know that at both BL's, X = 14 is affordable.

Thus, $I' = 120 - 14 = 106$.

Slutskian Method Example

To find the SE,

2. Find the optimal bundle (Bundle B) on this new BL.

We now use the demand equation to find Bundle B.

$$X(Px, I) = 10 + \frac{I}{10Px}$$

$$X_B(2, 106) = 10 + \frac{106}{10(2)} = 15.3$$

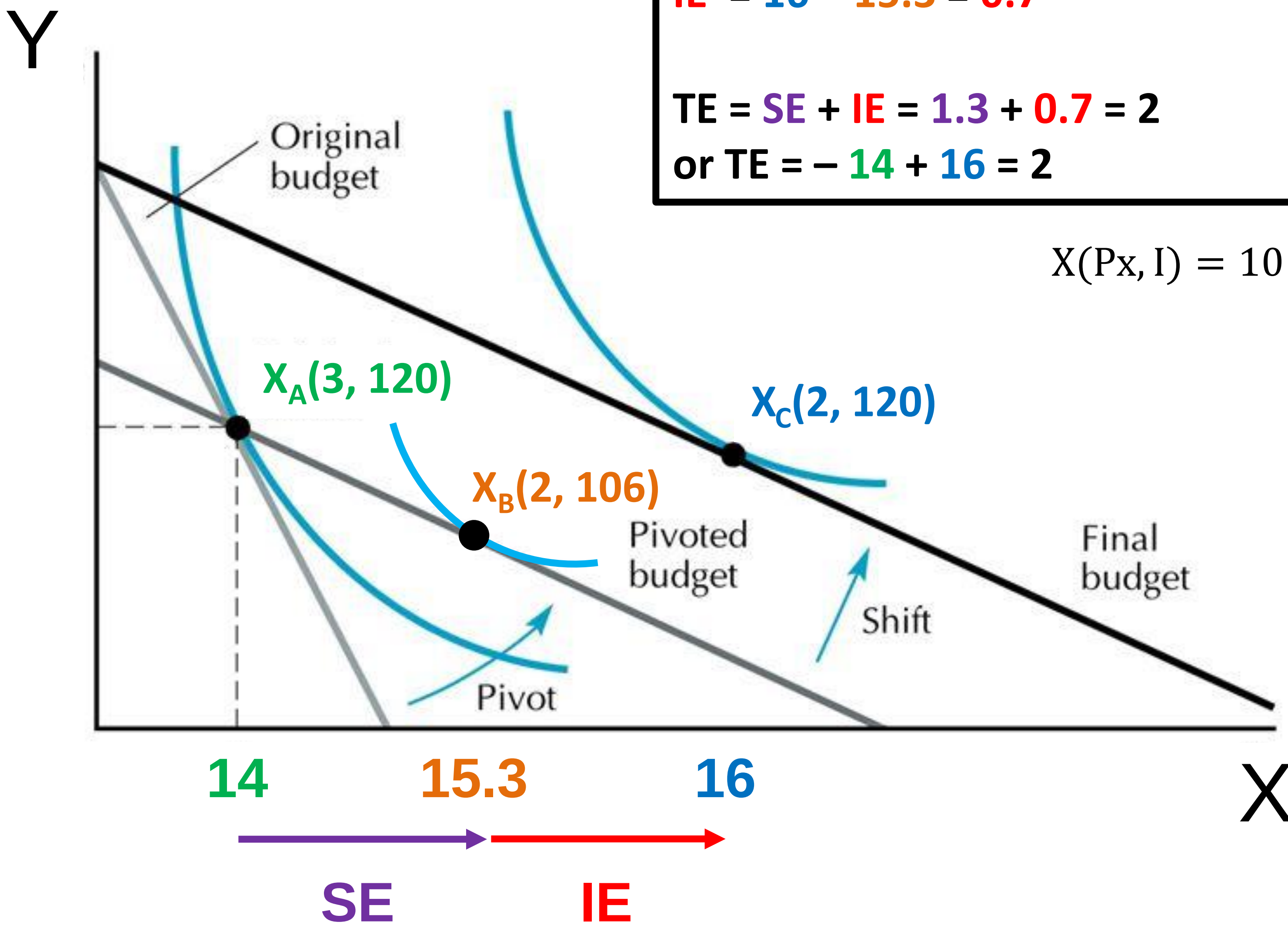
SE = 15.3 – 14 = 1.3

IE = 16 – 15.3 = 0.7

TE = SE + IE = 1.3 + 0.7 = 2

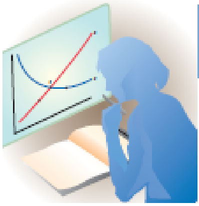
or TE = – 14 + 16 = 2

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

Hicksian Method Example



LEARNING-BY-DOING EXERCISE 5.4

Finding Income and Substitution Effects Algebraically

In Learning-By-Doing Exercises 4.2 and 5.2, we met a consumer who purchases two goods, food and clothing. He has the utility function $U(x, y) = xy$, where x denotes the amount of food consumed and y the amount of clothing. His marginal utilities are $MU_x = y$ and $MU_y = x$. Now suppose that he has an income of \$72 per week and that the price of clothing is $P_y = \$1$ per unit. Suppose that the price of food is initially $P_{x_1} = \$9$ per unit and that the price subsequently falls to $P_{x_2} = \$4$ per unit.

Problem Find the numerical values of the income and substitution effects on food consumption, and graph the results.

Note: Use the "Hicksian" Substitution Effect

To find the solution:

Step 1: Find Optimal Bundle before the price change (Bundle A)

Step 2: Find Optimal Bundle after the price change (Bundle C)

Step 3: Find Optimal Bundle after the price change (Bundle B) that still gives U = U(Bundle A), i.e. keeping U constant

Deriving Slutsky Equation

From TE = SE + IE, we can write it in a symbolic form.

$$\Delta X = \Delta X_{SE} + \Delta X_{IE}$$

…then..

$$\Delta X = \Delta X_{SE} - (\Delta X_{NIE})$$

where

$\Delta X = X_C(P_X' , I) - X_A(P_X , I)$

$\Delta X_{SE} = X_B(P_X' , I') - X_A(P_X , I)$ >>> Slutskian Method

$\Delta X_{IE} = X_C(P_X' , I) - X_B(P_X' , I')$

$\Delta X_{NIE} = - \Delta X_{IE} = X_B(P_X' , I') - X_C(P_X' , I)$ >>> "Negative" IE

Deriving Slutsky Equation

From $\Delta X = \Delta X_{SE} - \Delta X_{NIE}$,

we divide both sides with $\Delta Px = Px' - Px$.

$$\frac{\Delta \mathrm{X}}{\Delta \mathrm{Px}} = \frac{\Delta \mathrm{X_{SE}}}{\Delta \mathrm{Px}} - \frac{\Delta \mathrm{X_{NIE}}}{\Delta \mathrm{Px}}$$

In its derivative form,

$$\frac{\partial \mathrm{X}}{\partial \mathrm{Px}} = \frac{\partial \mathrm{X_{SE}}}{\partial \mathrm{Px}} - \frac{\partial \mathrm{X_{NIE}}}{\partial \mathrm{Px}}$$

Deriving Slutsky Equation

$$\frac{\partial \mathrm{X}}{\partial \mathrm{Px}} = \frac{\partial \mathrm{X}_{\mathrm{SE}}}{\partial \mathrm{Px}} - \frac{\partial \mathrm{X}_{\mathrm{NIE}}}{\partial \mathrm{Px}}$$

From I = PxX + PyY, we can write $\frac{\partial \mathrm{I}}{\partial \mathrm{Px}} = \mathrm{X}$ or $\partial \mathrm{Px} = \frac{\partial \mathrm{I}}{\mathrm{X}}$.

$$\boldsymbol{\frac{\partial \mathrm{X}}{\partial \mathrm{Px}} = \frac{\partial \mathrm{X}_{\mathrm{SE}}}{\partial \mathrm{Px}} - \frac{\partial \mathrm{X}_{\mathrm{NIE}}}{\partial \mathrm{I}} \cdot \mathrm{X}}$$

This is called the “Slutsky Equation”.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X}_{\mathbf{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X}_{\mathbf{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

$\frac{\partial X}{\partial Px}$ represents the TE.

$\frac{\partial X_{SE}}{\partial Px}$ represents the SE.

$-\frac{\partial X_{NIE}}{\partial I} \cdot X$ represents the IE.

We look at the "signs" of these derivatives when we interpret the Slutsky Equation because these signs tell us about the relationship between Px and X.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X_{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X_{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

Note that $\frac{\partial X_{SE}}{\partial Px} < 0$, always. That is, SE is negative.

If X is a normal good, $\frac{\partial X_{NIE}}{\partial I} > 0$. But, IE is negative.

If X is an inferior good, $\frac{\partial X_{NIE}}{\partial I} < 0$. But, IE is positive.

“Negative” refers to the negative relationship b/w Px and X.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X_{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X_{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

Conclusions

- If X is normal, SE and IE reinforce each other.
  - All effects are negative.
  - The Law of Demand holds, i.e. $\frac{\partial X}{\partial Px} < 0$.
- If X is inferior, SE and IE counteract each other.
  - SE is negative, but IE is positive.
  - The Law of Demand may not hold, e.g. Giffen good.