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Question 0: (Three-variable optimization)

Suppose that there are three firms. Each firm produces one single product that is imperfectly substitutable to the other two products, i.e. differentiated products. Each of the firm *strategically* competes with each other, and faces the following market demand equations given by,

$$Q_1 = 80 - 2P_1 + P_{23}$$

$$Q_2 = 80 - 2P_2 + P_{13}$$

$$Q_3 = 80 - 2P_3 + P_{12}$$

where P_{23} is the average of the prices charged by firms 2 and 3, P_{13} is the average of the prices charged by firms 2, P_{12} is the average of the prices charged by firms 1 and 2 [e.g., $P_{12} = 0.5(P_1 + P_2)$]. Suppose that each of the three firms, as indexed by j, has the cost function given by $C^j(Q_j) = c_j Q_j$.

Consider the following problems.

a) Set up the profit function of each individual firm, and derive the profit-maximizing condition of each of the three firms when each of them *optimally* sets for the level of price that maximizes its own profit. (**Hint:** *Derive the best response function of each firm, i.e. optimal contingent plan or reaction function.*)

$$\pi_1 = TR_1 - TC_1$$
$$= P_1 Q_1 - c_1 Q_1$$
$$= P_1(80 - 2P_1 + P_{23}) - c_1(80 - 2P_1 + P_{23})$$
$$\frac{d\pi_1}{dP_1} = P_1(-2) + (80 - 2P_1 + P_{23}) + 2c_1 = 0$$
$$80 - 4P_1 + P_{23} + 2c_1 = 0$$
$$P_1 = \frac{80 + P_{23} + 2c_1}{4}$$
$$BR_1(P_2, P_3) \quad P_1 = \frac{80 + 0.5(P_2 + P_3) + 2c_1}{4}$$

$$\pi_2 = TR_2 - TC_2$$
$$= P_2 Q_2 - c_2 Q_2$$
$$= P_2(80 - 2P_2 + P_{13}) - c_2(80 - 2P_2 + P_{13})$$
$$\frac{d\pi_2}{dP_2} = P_2(-2) + (80 - 2P_2 + P_{13}) + 2c_2 = 0$$
$$80 - 4P_2 + P_{13} + 2c_2 = 0$$
$$P_2 = \frac{80 + P_{13} + 2c_2}{4}$$
$$BR_2(P_1, P_3) \quad P_2 = \frac{80 + 0.5(P_1 + P_3) + 2c_2}{4}$$

$$\pi_3 = TR_3 - TC_3$$
$$= P_3 Q_3 - c_3 Q_3$$
$$= P_3(80 - 2P_3 + P_{12}) - c_3(80 - 2P_3 + P_{12})$$
$$\frac{d\pi_2}{dP_2} = P_3(-2) + (80 - 2P_3 + P_{13}) + 2c_3 = 0$$
$$80 - 4P_3 + P_{13} + 2c_3 = 0$$
$$P_3 = \frac{80 + P_{12} + 2c_3}{4}$$
$$BR_3(P_1, P_2): \quad P_3 = \frac{80 + 0.5(P_1 + P_2) + 2c_3}{4}$$

$$4P_1 - 0.5P_2 - 0.5P_3 = 80 + 2c_1$$
$$-0.5P_1 + 4P_2 - 0.5P_3 = 80 + 2c_2$$
$$-0.5P_1 - 0.5P_2 + 4P_3 = 80 + 2c_3$$

$$\begin{bmatrix} 4 & -0.5 & -0.5 \\ -0.5 & 4 & -0.5 \\ -0.5 & -0.5 & 4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 80+2c_1 \\ 80+2c_2 \\ 80+2c_3 \end{bmatrix}$$

$$Ax = D$$

$$A^{-1} = \frac{\underline{adj(A)}}{|A|} \qquad x = A^{-1}D$$

$$adj(A) = [cof(A)]^T$$

$$cof(A) = \begin{bmatrix} (-1)^2(16-0.25) & (-1)^3(-2-0.25) & (-1)^4(0.25+2) \\ (-1)^3(-2-0.25) & (-1)^4(16-0.25) & (-1)^5(-2-0.25) \\ (-1)^4(0.25+2) & (-1)^5(-2-0.25) & (-1)^6(16-0.25) \end{bmatrix} = \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$[cof(A)]^T = \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$det(A) = (64 + .125 + .125) - (1+1+1) = 61.25 \quad A^{-1} = \frac{1}{61.25} \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2571 & 0.0367 & 0.0367 \\ 0.0367 & 0.2571 & 0.0367 \\ 0.0367 & 0.0367 & 0.2571 \end{bmatrix}$$

$$A^{-1}D = \begin{bmatrix} 0.2571 & 0.0367 & 0.0367 \\ 0.0367 & 0.2571 & 0.0367 \\ 0.0367 & 0.0367 & 0.2571 \end{bmatrix} \begin{bmatrix} 80+2c_1 \\ 80+2c_2 \\ 80+2c_3 \end{bmatrix} = \begin{bmatrix} 20.568 + 0.5142C_1 + 2.936 + 0.0734C_2 + 2.936 + 0.0734C_3 \\ 2.936 + 0.0734C_1 + 20.568 + 0.5142C_2 + 2.936 + 0.0734C_3 \\ 2.936 + 0.0734C_1 + 2.936 + 0.0734C_2 + 20.568 + 0.5142C_3 \end{bmatrix}$$

$$= \begin{bmatrix} 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3 \\ 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3 \\ 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3 \end{bmatrix}$$

$P_1^* = 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3$

$P_2^* = 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3$

$P_3^* = 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3$ #

b) Suppose that Based on (a), what would be the *equilibrium* level of price that each of the firms choose. Calculate the level of profit that each firm yields. (Hint: *Solve for the Bertrand equilibrium with differentiated product*)

$P_1^* = 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3$

$P_2^* = 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3$

$P_3^* = 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3$

$$\pi_1^* = (26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3 - C_1)(80 - 2P_1 + P_{23})$$
$$= (26.44 - 0.4858C_1 + 0.0734C_2 + 0.0734C_3)(80 - 2P_1 + P_{23})$$
$$\pi_2^* = (26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3 - c_2)(80 - 2P_2 + P_{13})$$
$$= (26.44 + 0.0734C_1 - 0.4858C_2 + 0.0734C_3)(80 - 2P_2 + P_{13})$$
$$\pi_3^* = (26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3 - c_3)(80 - 2P_3 + P_{12})$$
$$= (26.44 + 0.0734C_1 + 0.0734C_2 - 0.4858C_3)(80 - 2P_3 + P_{12})$$

c) How does the optimal price vary with respect to the size of marginal cost?

$$\frac{\partial P_1^*}{\partial c_1} = 0.5142$$

$$\frac{\partial P_2^*}{\partial c_2} = 0.5142$$

$$\frac{\partial P_3^*}{\partial c_3} = 0.5142$$

$\therefore$ When the marginal cost rises by one unit each of the firm's optimal price increases by 0.5142 unit

Now suppose that all the three firms agree to operate under a cartel (collusive) agreement. That is, they consider a joint pricing scheme that maximizes the joint profit function of the three firms combined. Consider the following problems.

d) Construct the joint profit function.

e) Given that $c_j = c,\ \forall j$. Calculate the level of optimal price setting that each of the firm will set under the joint profit function problem. Confirm your result with the second-order derivative test.

f) Is the Cartel agreement self-sustaining? Why?

d) $\pi_T = P_1Q_1 + P_2Q_2 + P_3Q_3 - c_1Q_1 - c_2Q_2 - c_3Q_3$

$= P_1(80-2P_1+P_{23}) + P_2(80-2P_2+P_{13}) + P_3(80-2P_3+P_{12}) - c_1(80-2P_1+P_{23}) - c_2(80-2P_2+P_{13}) - c_3(80-2P_3+P_{12})$

$= (80-2P_1+P_{23})(P_1-c_1) + (80-2P_2+P_{13})(P_2-c_2) + (80-2P_3+P_{12})(P_3-c_3)$ #

e) $\frac{d\pi_T}{dP_1} = (80-2P_1+P_{23}) + (P_1-C)(-2) + (P_2-C)(0.5) + (P_3-C)(0.5) = 0$

$(80-2P_1+0.5P_2+0.5P_3) - 2P_1 + 2C + 0.5P_2 - 0.5C + 0.5P_3 - 0.5C = 0$

$80 - 4P_1 + P_2 + P_3 + C = 0$

$4P_1 - P_2 - P_3 = 80 + C$ —— ①

$\frac{d\pi_T}{dP_2} = (P_1-C)(0.5) + (80-2P_2+P_{13}) + (P_2-C)(-2) + (P_3-C)(0.5) = 0$

$0.5P_1 - 0.5C + (80-2P_2+0.5P_1+0.5P_3) - 2P_2 + 2C + 0.5P_3 - 0.5C = 0$

$80 + P_1 - 4P_2 + P_3 + C = 0$

$-P_1 + 4P_2 - P_3 = 80 + C$ —— ②

$\frac{d\pi_T}{dP_3} = (P_1-C)(0.5) + (P_2-C)(0.5) + (80-2P_3+P_{12}) + (P_3-C)(-2) = 0$

$0.5P_1 - 0.5C + 0.5P_2 - 0.5C + (80-2P_3+0.5P_1+0.5P_2) - 2P_3 + 2C = 0$

$80 + P_1 + P_2 - 4P_3 + C = 0$

$-P_1 - P_2 + 4P_3 = 80 + C$ —— ②

$4P_1 - P_2 - P_3 = 80 + 2C_1 - 0.5C_2 - 0.5C_3$

$-P_1 + 4P_2 - P_3 = 80 - 0.5C_1 + 2C_2 - 0.5C_3$

$-P_1 - P_2 + 4P_3 = 80 - 0.5C_1 - 0.5C_2 + 2C_3$

$① = ②$

$4P_1 - P_2 - P_3 = -P_1 + 4P_2 - P_3$

$5P_1 = 5P_2$

$P_1 = P_2$

Plug $P_1 = P_2$ into $①$ & $③$

$4P_1 - P_1 - P_3 = -P_1 - P_1 + 4P_3$

$3P_1 = -2P_1 + 5P_3$

$P_1 = P_3$

Plug $P_1 = P_2 = P_3$ into $①$

$4P_1 - P_1 - P_1 = 80 - c$

$P_3^* = P_2^* = P_1^* = \frac{80-c}{3}$ #

Second Order Derivative Test

$$H = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} -4 & 1 & 1 \\ 1 & -4 & 1 \\ 1 & 1 & -4 \end{bmatrix} \begin{matrix} -4 & 1 \\ 1 & -4 \\ 1 & 1 \end{matrix}$$

$|H_1| = -4$; $|H_1| < 0$; $\forall P_1, \forall P_2, \forall P_3$

$|H_2| = 16 - 1 = 15$; $|H_2| > 0$; $\forall P_1, \forall P_2, \forall P_3$

$|H_3| = (-64+1+1) - (-4-4-4) = -62 + 12 = -50$; $|H_3| < 0$; $\forall P_1, \forall P_2, \forall P_3$

$\therefore$ H is negative definite $\rightarrow d^2\pi < 0$

The function is globally concave

P_1^*, P_2^*, and P_3^* are thus the global solutions

f) Cartel agreement is not self-sustaining even though all three products of the firms are different. The quantity demanded of each good is interdependent on one another. If one of the firm were to set price a little bit higher than it is in the agreement, the quantity demanded for other products will drop. Accordingly, the cheated firm will gain relatively higher profit compared to the rest.

g) Revisit question (e) and assume that the marginal costs are varied with respect to firms. What would be the implication of the collusive equilibrium?

The implication of collusive equilibrium would be that if marginal costs are varied with respect to firms, they should set the price of each product based on marginal costs of all three goods. That is, if the price of product 1 has high marginal cost, holding all other things constant, the firm should sell it at relatively higher price than the product 2 and product 3 in order to maximize its profit.

At the end, the should satify the condition where $MR(Q_1^*) = MR(Q_2^*) = MR(Q_3^*) = MC(Q_{Total})$.

Question 2:

Pakorn's utility function depends on the consumption of two commodities, x and y, and it is given by

$$U(x, y) = 2xy$$

Suppose that his income is \$72, and the prices per unit of x and y are \$4 and \$6, respectively. Assume that Pakorn spends all of his income, and the values of x and y are both non-zero.

a. Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
b. Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
c. Suppose that the income is now \$73. Approximate the new maximum utility level.

a) $\max u(x,y) = 2xy$ s.t. $4x + 6y = 72$

$\mathcal{L}(x, y, \lambda) = 2xy + \lambda(72 - 4x - 6y)$

F.O.C of Lagrange f².

$[x] = \frac{\partial f}{\partial x} = 2y - 4\lambda = 0$ — (1)

$[y] = \frac{\partial f}{\partial y} = 2x - 6\lambda = 0$ — (2)

$[\lambda] = \frac{\partial f}{\partial \lambda} = 72 - 4x - 6y = 0$ — (3)

(1) × 3 ; $6y - 12\lambda = 0$ — (4)

(2) × 2 ; $4x - 12\lambda = 0$ — (5)

(4) = (5) ; $6y - 12\lambda = 4x - 12\lambda$

$6y = 4x$

Put $6y = 4x$ in (3) $72 - 4x - 4x = 0$

$x^* = 9$

So, $y^* = 6$ and $\lambda^* = 3$

S.O.C.

$$\bar{H} = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda x} & \mathcal{L}_{\lambda y} \\ \mathcal{L}_{x\lambda} & \mathcal{L}_{xx} & \mathcal{L}_{xy} \\ \mathcal{L}_{y\lambda} & \mathcal{L}_{yx} & \mathcal{L}_{yy} \end{bmatrix} = \begin{bmatrix} 0 & -4 & -6 \\ -4 & 0 & 2 \\ -6 & 2 & 0 \end{bmatrix} \begin{matrix} 0 & -4 \\ -4 & 0 \\ -6 & 2 \end{matrix}$$

$= [0 + 48 + 48] - [0 + 0 + 0]$

$= 96$

$|\bar{H}| > 0 \rightarrow$ "f" is concave along the constraint set

$\rightarrow$ solution from FOC represents max.

b) $u(x,y) = 2xy$ let $x^* = 9$, $y^* = 6$

So, $u(9,6) = 2(9)(6)$

$= 108$

Interpretion of Lagrange multiplier.

$\lambda^* = \frac{\partial u}{\partial m}$ when the income budget increased by \$1 , the maximum utility is increasing by \$3

c) $\max u(x,y) = 2xy$ s.t. $4x+6y=73$

$\mathcal{L}(x,y,\lambda) = 2xy + \lambda(73-4x-6y)$

F.O.C of Lagrange f^n.

$[x] = \frac{\partial f}{\partial x} = 2y - 4\lambda = 0$ —(1)

$[y] = \frac{\partial f}{\partial y} = 2x - 6\lambda = 0$ —(2)

$[\lambda] = \frac{\partial f}{\partial \lambda} = 73 - 4x - 6y = 0$ —(3)

(1) × 3 ; $6y - 12\lambda = 0$ —(4)

(2) × 2 ; $4x - 12\lambda = 0$ —(5)

(4) = (5) ; $6y - 12\lambda = 4x - 12\lambda$

$6y = 4x$

Put $6y = 4x$ in (3) $73 - 4x - 4x = 0$

$x^* = 73/8$

So, $y^* = 73/12$ and $\lambda^* = 73/24$

S.O.C.

$$\bar{H} = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda x} & \mathcal{L}_{\lambda y} \\ \mathcal{L}_{x\lambda} & \mathcal{L}_{xx} & \mathcal{L}_{xy} \\ \mathcal{L}_{y\lambda} & \mathcal{L}_{yx} & \mathcal{L}_{yy} \end{bmatrix} = \begin{bmatrix} 0 & -4 & -6 \\ -4 & 0 & 2 \\ -6 & 2 & 0 \end{bmatrix} \begin{matrix} 0 & -4 \\ -4 & 0 \\ -6 & 2 \end{matrix}$$

$= [0 + 48 + 48] - [0 + 0 + 0]$

$= 96$

$|\bar{H}| > 0 \rightarrow$ "f" is concave along the constraint set

$\rightarrow$ solution from FOC represents max.

when income increased by \$1, the maximum utility increased by \$3

So, utility level when income is \$73 equal to $108 + 3 = 111$

Question 4:

A consumer has the utility function $U = \ln C + \ln(24 - N)$, where C is consumption and N is labor supply. Her budget constraint is $pC = \bar{M} + wN$, where p is the price of the consumption good, w the wage rate, and $\bar{M}$ the consumer's non-wage income.

(a) Formulate the problem of utility maximization subject to the budget constraint, and derive the first-order conditions, using the Lagrange multiplier approach and ignoring the nonnegativity constraints.

$$\max_{C,N} U = \ln C + \ln(24-N) \quad \text{s.t.} \quad pC = \bar{M} + wN$$

$$L(C,N,\lambda; p,w,\bar{M}) = \ln C + \ln(24-N) + \lambda(\bar{M} + wN - pC)$$

$$\frac{\partial L}{\partial C} = \frac{1}{C} - \lambda p = 0$$

$$C = \lambda p$$

$$\frac{\partial L}{\partial N} = -\frac{1}{24-N} + \lambda w = 0$$

$$\lambda w = \frac{1}{24-N}$$

$$\lambda w(24-N) = 1$$

$$24\lambda w - 24N = 1$$

$$N = \frac{24\lambda w - 1}{24}$$

$$\frac{\partial L}{\partial \lambda} = \bar{M} + wN - pC = 0$$

$$0 = \bar{M} + w\left(\frac{24\lambda w - 1}{24}\right) - p(\lambda p)$$

$$\lambda p^2 = \bar{M} + w\left(\frac{24\lambda w - 1}{24}\right)$$

$$24\lambda p^2 = 24\bar{M} + 24\lambda w^2 - w$$

$$24\lambda p^2 - 24\lambda w^2 = 24\bar{M} - w$$

$$\lambda(24p^2 - 24w^2) = 24\bar{M} - w$$

$$\lambda^* = \frac{24\bar{M} - w}{24p^2 - 24w^2}$$

$$N^* = \frac{w\left(\frac{24\bar{M} - w}{p^2 - w^2}\right) - 1}{24} \qquad C^* = \frac{p}{24}\left(\frac{24\bar{M} - w}{p^2 - w^2}\right)$$

(b) Find the demand function $C = C^*(p, w, \bar{M})$ and the labor supply function $N = N^*(p, w, \bar{M})$ (i.e., express C and N in terms of p, w, and $\bar{M}$). Show that N^* and C^* are homogeneous of degree zero in $(p, w, \bar{M})$.

Demand Function: $C^* = \frac{p}{24}\left(\frac{24\bar{M} - w}{p^2 - w^2}\right)$

Labor Supply Function: $N^* = \frac{w\left(\frac{24\bar{M} - w}{p^2 - w^2}\right) - 1}{24}$

Increase $p, w, \bar{M}$ by t proportion

$$C^*(tp, tw, t\bar{M}) = tp\left(\frac{24t\bar{M} - tw}{24(tp)^2 - 24(tw)^2}\right)$$

$$= tp\left(\frac{t}{t^2}\left(\frac{24\bar{M} - w}{24p^2 - 24w^2}\right)\right)$$

$$= 1 \cdot p\left(\frac{24\bar{M} - w}{24p^2 - 24w^2}\right) = C^*(p, w, \bar{M})$$

Increase $p, w, \bar{M}$ by t proportion

$$N^*(tp, tw, t\bar{M}) = \frac{tw\left(\frac{24t\bar{M} - tw}{(tp)^2 - (tw)^2}\right) - 1}{24}$$

$$= \frac{\frac{t^2}{t^2}w\left(\frac{24\bar{M} - w}{p^2 - w^2}\right) - 1}{24}$$

$$= \frac{w\left(\frac{24\bar{M} - w}{p^2 - w^2}\right) - 1}{24} = N^*(p, w, \bar{M})$$

$\therefore$ N^* and C^* are homogeneous of degree zero in $(p, w, \bar{M})$

(c) Let $U^* = \ln[C^*(p, w, \bar{M})] + \ln[24 - N^*(p, w, \bar{M})]$. Show that $\partial U^*/\partial \bar{M} > 0$ and $\partial U^*/\partial p < 0$. Show that U^* is concave in $\bar{M}$ and convex in p. What is the relationship between $\partial U^*/\partial \bar{M}$ and the Lagrange multiplier?

$$U^* = \ln\left[p\left(\frac{24\bar{M} - w}{24p^2 - 24w^2}\right)\right] + \ln\left[24 - \left(\frac{24w\left(\frac{24\bar{M} - w}{24p^2 - 24w^2}\right) - 1}{24}\right)\right]$$

$$U^* = \ln\left[\frac{24\bar{M}p - wp}{24p^2 - 24w^2}\right] + \ln\left[25 - \left(\frac{24\bar{M}w - w^2}{24p^2 - 24w^2}\right)\right]$$

$$\frac{\partial U^*}{\partial \bar{M}} = \left(\frac{24p^2 - 24w^2}{24\bar{M}p - wp}\right)\left[\frac{24p}{24p^2 - 24w^2}\right] + \left[\frac{1}{25 - \left(\frac{24\bar{M}w - w^2}{24p^2 - 24w^2}\right)}\right]\left[\frac{-24w}{24p^2 - 24w^2}\right]$$

$$= \left(\frac{24p^2 - 24w^2}{24\bar{M}p - wp}\right)\left[\frac{p}{p^2 - w^2}\right] + \left[\frac{1}{25 - \left(\frac{24\bar{M}w - w^2}{24p^2 - 24w^2}\right)}\right]\left[\frac{-w}{p^2 - w^2}\right] > 0$$

U^* is concave in $\bar{M}$

$$\frac{\partial U^*}{\partial p} = \left(\frac{24p^2 - 24w^2}{24\bar{M}p - wp}\right)\left[\frac{(24p^2 - 24w^2)(24\bar{M} - w) - (24\bar{M}p - wp)(48p)}{(24p^2 - 24w^2)^2}\right] + \left[\frac{1}{25 - \left(\frac{24\bar{M}w - w^2}{24p^2 - 24w^2}\right)}\right]\left[-\left(\frac{(24p^2 - 24w^2)(0) - (24\bar{M}w - w^2)(48p)}{(24p^2 - 24w^2)^2}\right)\right]$$

$$= \left(\frac{24p^2 - 24w^2}{24\bar{M}p - wp}\right)\left[\frac{(24p^2 - 24w^2)(24\bar{M} - w) - (24\bar{M}p - wp)(48p)}{(24p^2 - 24w^2)^2}\right] + \left[\frac{1}{25 - \left(\frac{24\bar{M}w - w^2}{24p^2 - 24w^2}\right)}\right]\left[\frac{(24\bar{M}w - w^2)(48p)}{(24p^2 - 24w^2)^2}\right] < 0$$

U^* is convex in p

The relationship between the LaGrange Multiplier and $\frac{\partial U^*}{\partial \bar{M}}$ is positive.

$$\frac{\partial U^*}{\partial \bar{M}} = \frac{1}{\lambda p}\left[\frac{p}{p^2 - w^2}\right] + \left[\frac{1}{25 - \frac{1}{\lambda w}}\right]\left[\frac{-w}{p^2 - w^2}\right]$$