

1.)

3.2126

Student	Y_i	X_i	Y_i^2	X_i^2	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$X_i Y_i$	u_i	u_i^2	$(X_i - \bar{X})^2$
1	2.8	63	7.84	3969	-14.625	-0.4125	176.4	0.0862	0.0074	213.8906
2	3.4	72	11.56	5184	-9.625	0.1875	244.8	0.8792	0.1499	31.6406
3	3.0	78	9	6084	0.375	-0.2125	234	-0.2253	0.0508	0.1406
4	3.5	81	12.25	6561	3.375	0.2875	283.5	0.1724	0.0297	11.3906
5	3.6	87	12.96	7569	9.375	0.3875	313.2	0.0678	0.0046	87.8906
6	3.0	75	9	5625	-2.625	-0.2125	225	-0.1230	0.0151	6.8906
7	2.7	75	7.29	5625	-2.625	-0.5125	202.5	-0.4230	0.1789	6.8906
8	3.7	90	13.69	8100	12.375	0.4875	333	0.0655	0.0043	153.1406
$\bar{X} = 77.625$	25.7	621	85.59	48717	0	0	2,012.4		0.4347	511.8748
$\bar{Y} = 3.2125$	$\sum Y_i$	$\sum X_i$	$\sum Y_i^2$	$\sum X_i^2$	$\sum X_i$	$\sum Y_i$	$\sum X_i Y_i$		$\sum u_i^2$	$\sum X_i^2$

$$\bar{X} = \frac{\sum X_i}{n} \quad \hat{\beta}_1 = \bar{Y} - \beta_2 \bar{X}$$

$$\bar{Y} = \frac{\sum Y_i}{n} \quad \hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2} \quad \text{or} \quad \hat{\beta}_2 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$1.1) \quad \hat{\beta}_2 = \frac{8(2,012.4) - 15,959.7}{8(48,717) - 385,641} = \frac{16,099.2 - 15,959.7}{389,736 - 385,641} = \frac{139.5}{4095} = 0.0341 \quad \#$$

$$\hat{\beta}_1 = 3.2125 - 0.0341(77.625) = 0.5655 \quad \#$$

$Y_i = 0.5655 + 0.0341 X_i + u_i$, for any given level of total econometric exam points (X_i) the slope of estimated of GPA equal to 0.0341, with an estimated intercept in y-axis at 0.5655.

$$1.2) \quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i \quad Y_i = \hat{Y}_i + u_i \quad Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i \quad Y_i = \hat{Y}_i + \hat{u}_i \quad Y_i - \hat{Y}_i = \hat{u}_i$$

$$\hat{Y}_1 = 0.5655 + (0.0341)(63) = 2.7138$$

$$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$$

$$\hat{Y}_2 = 0.5655 + (0.0341)(72) = 3.0207$$

$$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$$

$$\hat{Y}_3 = 0.5655 + (0.0341)(78) = 3.2263$$

$$\hat{u}_3 = 3.0 - 3.2263 = -0.2263$$

$$\hat{Y}_4 = 0.5655 + (0.0341)(81) = 3.3276$$

$$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$$

$$\hat{Y}_5 = 0.5655 + (0.0341)(87) = 3.5322$$

$$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$$

$$\hat{Y}_6 = 0.5655 + (0.0341)(75) = 3.1230$$

$$\hat{u}_6 = 3.0 - 3.1230 = -0.1230$$

$$\hat{Y}_7 = 0.5655 + (0.0341)(75) = 3.1230$$

$$\hat{u}_7 = 2.7 - 3.1230 = -0.4230$$

$$\hat{Y}_8 = 0.5655 + (0.0341)(90) = 3.6345$$

$$\hat{u}_8 = 3.7 - 3.6345 = 0.0655$$

$$\sum \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.1230 - 0.4230 + 0.0655$$

$$= -0.0001$$

$$\sum \hat{u}_i = 0$$

$$1.3) \text{Var}(\hat{\beta}_1) = \frac{b_w^2 \sum x_i^2}{n \sum x_i^2} \quad b_w^2 = b_u^2 = \frac{\sum u_i^2}{n-2} = \frac{0.4347}{6} = 0.0725$$

$$\text{Var}(\hat{\beta}_2) = \frac{b_w^2}{\sum x_i^2}$$

$$\text{Var}(\hat{\beta}_1) = \frac{(0.0725)(48,717)}{8(511.8748)} = \frac{3,531.9825}{4094.9984} = 0.8625 \#$$

$$\text{Var}(\hat{\beta}_2) = (0.0725)/(511.8748) = 0.00014164 \approx 0.0001 \#$$

2.)

$$\bar{x} = 20 \quad \bar{y} = 9.1$$

X_i	Y_i	X_i^2	Y_i^2	$X_i Y_i$	$X_i - \bar{x}$	$(X_i - \bar{x})^2$	$Y_i - \bar{y}$	$(Y_i - \bar{y})^2$	$X_i Y_i$
10	0	100	0	0	-10	100	-9.1	82.81	91
12	2	144	4	24	-8	64	-7.1	50.41	86.8
14	5	196	25	70	-6	36	-4.1	16.81	24.6
16	6	256	36	96	-4	16	-3.1	9.61	12.4
18	7	324	49	126	-2	4	-2.1	4.41	4.2
22	10	484	100	220	2	4	0.9	0.81	19
24	10	576	100	240	4	16	0.9	0.81	3.6
26	15	676	225	390	6	36	5.9	34.81	35.4
28	16	784	256	448	8	64	6.9	47.61	55.2
30	20	900	400	600	10	100	10.9	118.81	109
200	91	4,440	1,195	2,214	0	440	0	366.9	394

2.1)

$$\hat{\beta}_2 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 9.1 - 0.8905(20) = -8.81$$

$$= \frac{10(2214) - 18,200}{10(4440) - (40,000)} = \frac{3,940}{4440} = 0.8955$$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i, \quad \hat{y}_i = -8.81 + 0.8905(x_i)$$

The regression model of estimated \hat{y}_i has a slope of 0.8955, and y-intercept at point every increase in x_i by one unit increase -8.81 by 0.8955 unit

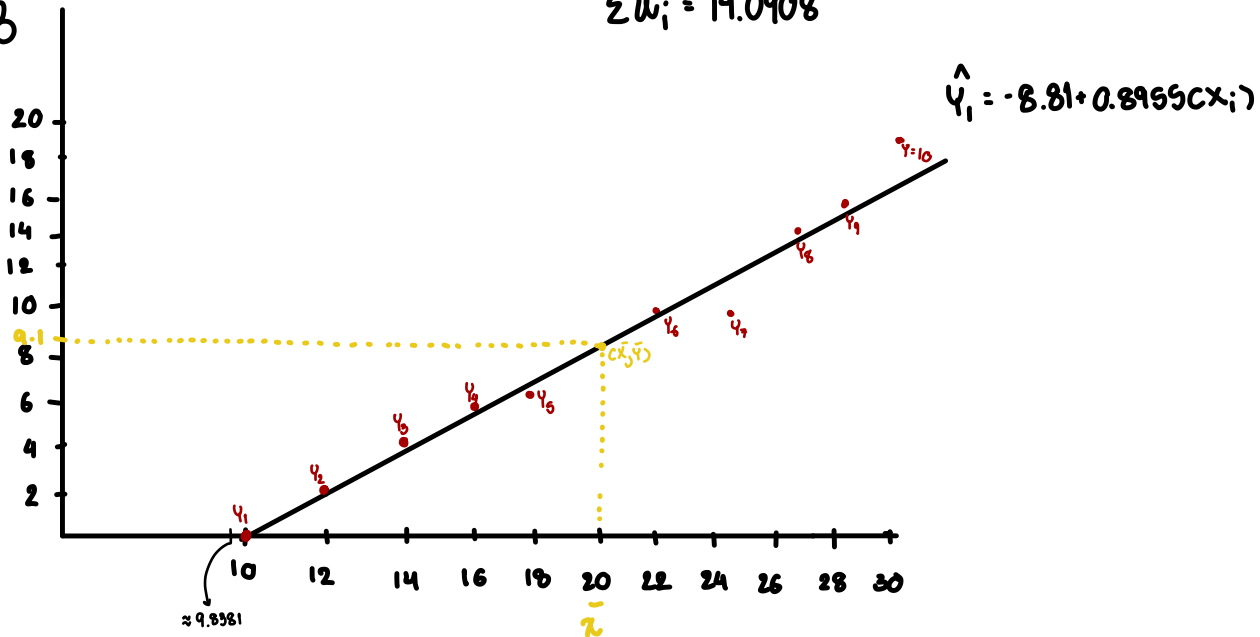
2.2

\hat{y}_1	0.146	\hat{u}_1	-0.149	\hat{u}_1^2	0.0210
\hat{y}_2	1.936	\hat{u}_2	0.064	\hat{u}_2^2	0.0041
\hat{y}_3	5.727	\hat{u}_3	1.273	\hat{u}_3^2	1.6209
\hat{y}_4	5.518	\hat{u}_4	0.482	\hat{u}_4^2	0.2323
\hat{y}_5	7.309	\hat{u}_5	-0.309	\hat{u}_5^2	0.0959
\hat{y}_6	10.891	\hat{u}_6	-0.891	\hat{u}_6^2	0.7939
\hat{y}_7	12.682	\hat{u}_7	-2.682	\hat{u}_7^2	7.1931
\hat{y}_8	14.473	\hat{u}_8	0.527	\hat{u}_8^2	0.2777
\hat{y}_9	16.264	\hat{u}_9	-0.264	\hat{u}_9^2	0.0697
\hat{y}_{10}	18.055	\hat{u}_{10}	1.946	\hat{u}_{10}^2	3.7830

$$\begin{aligned} \sum \hat{u}_i &= -0.149 + 0.064 + 1.273 + 0.482 \\ &\quad - 0.309 - 0.891 - 2.682 + 0.527 \\ &\quad - 0.264 + 1.946 = 0 \end{aligned}$$

$$\sum \hat{u}_i^2 = 14.0908$$

2.3



$$2.4) \text{ If } x_1 = 18, \hat{y}_1 = -8.81 + 0.8999(18)$$

$$\hat{y}_1 = 7.309 \#$$

$$2.5) \text{ Var } (\hat{\beta}_1) = \frac{b_w^2 \sum x_i}{n \sum x_i^2} = \frac{1.7614 \cdot 200}{10(440)} = \frac{352.28}{4400} = 0.0801 \#$$

$$\text{Var } (\hat{\beta}_2) = \frac{b_w^2}{\sum x_i^2} = \frac{1.7614}{440} = 0.0040 \#$$

$$b_w^2 = \frac{\sum u_i^2}{n-2} = \frac{14.0908}{8} = 1.7614 \#$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim \text{NIID}(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\hat{\beta}_1 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} = \bar{y} - \hat{\beta}_2 \bar{x}$$

$\hat{\beta}_1$ is an unbiased estimator

$$\begin{aligned} E(\hat{\beta}_1) &= E(\bar{y} - \hat{\beta}_2 \bar{x}) \\ &= E(\bar{y}) - \hat{\beta}_2 E(\bar{x}) \\ &= \beta_1 + \cancel{\beta_2 \bar{x}} - \bar{x} \cancel{\beta_2} \end{aligned}$$

$$E(\hat{\beta}_1) = \beta_1$$